PLASTIC ZONE SIZE IN CRACKED METAL PLATES

The in-place stress are

\[ \sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + \text{Higher Order Terms} \]  

\[ \sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + \text{H.O.T.} \]  

\[ \tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} + \text{H.O.T.} \]  

\[ \frac{K_I}{\sqrt{2\pi r}} = S \sqrt{\frac{a}{2r}} \]  

\[ \sigma_y = \frac{K_I}{\sqrt{2\pi r}} \]  

@ \theta = 0 \rightarrow x = r, y = 0

Neglecting higher order terms,
\( r_p^* \) is the estimate of plastic zone, and we can define it as

\[
r_p^* = \frac{1}{2\pi} \left( \frac{K_{Ic}}{\sigma_{\text{yield}}} \right)^2
\]

From experiments and analysis it was found that the plastic zone size \( r_p \) is \( r_p^* \). We define plastic zone size as

\[
r_p = c \left( \frac{K_{Ic}}{\sigma_{\text{yield}}} \right)^2
\]

\( c \) = constant of proportionality

PLANE STRESS

For this particular case \( \sigma_z = 0 \) and the yielding occurs about \( \sigma_{\text{yield}} = \sigma_o \)

\[
r_p^* = r_{o\sigma}
\]

\[
r_p = 2r_{o\sigma} = \frac{1}{\pi} \left( \frac{K_{Ic}}{\sigma_o} \right)^2
\]

where the constant of proportionality is

\[
c_{o\sigma} = \frac{1}{\pi}
\]

PLANE STRAIN

For this particular case \( \sigma_z = \nu(\sigma_x + \sigma_y) \) and the yielding occurs about \( \sigma_{\text{yield}} = \sqrt{3} \sigma_o \):

\[
r_p^* = r_{o\varepsilon}
\]

\[
r_p = 2r_{o\varepsilon} = \frac{1}{\pi} \left( \frac{K_{Ic}}{\sqrt{3}\sigma_o} \right)^2
\]

where the constant of proportionality is

\[
c_{o\varepsilon} = \frac{1}{3\pi}
\]

Note that \( r_{o\varepsilon} = \frac{r_{o\sigma}}{3} \)
PLASTICITY LIMITATIONS ON LEFM

Plastic zone size is characterized by $K_I$ only if first term in $\sigma_y$ dominates (recall H.O.T. were neglected in the in-plane stress). If plasticity spreads further then $K_I$ cannot be used to characterize the plastic zone size and the use of Linear Elastic Fracture Mechanics (LEFM) is invalid. In the following situations $K_I$ cannot be used to characterize the stress field because the plastic zone is too large:

1. relative to crack

2. relative to uncracked ligament

3. relative to specimen height

Since $2r_{os} > 2r_{oe}$ an overall limit on the use of LEFM is:

$$a, (b - a), h \geq \frac{4}{\pi} \left( \frac{KQ}{\sigma_o} \right)^2$$  \hspace{1cm} (7)

This must be satisfied for all three of $a, (b - a), h$. 