

**FRACTURE OF CRACKED MEMBERS**

1. The presence of a crack in a structure may weaken it so that it fails by fracturing in two or more pieces.

2. Fracture can occur at stresses below the material’s yield strength, where failure would not normally be expected.

**FRACTURE MECHANICS**

1. Study of crack propagation in bodies

2. Methodology used to aid in selecting materials and designing components to minimize the possibility of fracture.

3. It begins with the assumption that all real materials contain cracks of some size—even if only submicroscopically

4. Based on three types of displacement modes:

   ![Diagram of fracture modes](image)

   - **MODE I**
     - OPENING
     - *Tension*
   - **MODE II**
     - SLIDING
     - *In-plane shear*
   - **MODE III**
     - TEARING
     - *Out-of-plane shear*

**FRACTURE TOUGHNESS**

1. In fracture mechanics, one does not attempt to evaluate an effective stress concentration, rather a stress intensity factor $K$

2. After obtaining $K$, it is compared with a limiting value of $K$ that is necessary for crack propagation in that material, called $K_c$

3. The limiting value $K_c$ is characteristic of the material and is called fracture toughness

4. Toughness is defined as the capacity of a material to resist crack growth
1. The stress intensity factor $K_I$ characterizes the magnitude of the stresses in the vicinity of an ideal sharp crack tip in a linear-elastic and isotropic material under mode I displacement.

2. Near the crack tip the dominant terms in the stress field are:

\[
\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + \cdots
\] (1)

\[
\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + \cdots
\] (2)

\[
\sigma_z = \begin{cases} 
0 & \text{(plane stress)} \\
\nu(\sigma_x + \sigma_y) & \text{(plane strain)}
\end{cases}
\] (4)

\[
\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} + \cdots
\] (3)

\[
\tau_{yz} = \tau_{xz} = 0
\] (5)

3. The dimensional units of $K_I$ are: [stress\(\sqrt{\text{length}}\)], i.e., [MPa\(\sqrt{\text{m}}\)] or [ksi\(\sqrt{\text{in}}\)]

4. $K_I$ measures the severity of the crack and it is generally expressed as:

\[
K_I = F S \sqrt{\pi a}
\] (6)

$F$ is a dimensionless quantity accounting for the plate/specimen geometry and relative crack size, $S$ is the stress ($\sigma_y$) if no crack were present, $a$ is half crack length
It can also be observed that as $a \to b$, the plate fractures into two pieces.

Dowling, in his book "Mechanical Behavior of Materials" (Prentice-Hall, 1999, pp. 301-304; or 1993, pp. 292), gives the following expression for a center-cracked plate with any $\alpha = a/b$:

$$F = \frac{1 - 0.5\alpha + 0.326a^2}{\sqrt{1 - \alpha}} \quad \frac{h}{b} \geq 1.5$$

From the above expression it can be shown that $F = 1$ for an infinite plate ($b \to \infty$) and for $0 \ll \frac{a}{b} \ll 1$. However, for a center-cracked plate with $\alpha \leq 0.4$, when taking $F = 1$, the result is accurate within 10%.

**CRITICAL STRESS INTENSITY FACTOR**

1. The calculated $K_I$ is compared to the critical stress intensity factor or fracture toughness $K_{IC}$:
   - $K_I < K_{IC}$, material will resist crack growth without brittle fracture (safe)
   - $K_I = K_{IC}$, crack begins to propagate and brittle fracture occurs (fracture)

2. The critical value $K_{IC}$ is defined as $K_{IC} = FSC\sqrt{\pi a}$

The following figure shows the relationship between the critical value of the remote stress and the crack length.

![Diagram](https://via.placeholder.com/150)

$a_t$ is the transition crack length, and it is defined as the approximate length above which strength is limited by brittle fracture; and $\sigma_o = \sigma_y$. In other words, $a_t$ is the crack length where $S_C = \sigma_y$:

$$a_t = \frac{1}{F^2\pi} \left( \frac{K_{IC}}{\sigma_o} \right)^2 = \frac{1}{\pi} \left( \frac{K_{IC}}{\sigma_o} \right)^2 \quad (F = 1)$$

When $a > a_t$ strength is limited by fracture, and when $0 < a < a_t$ yielding dominates strength. Materials with:

1. high $K_{IC}$ and low $\sigma_o$ implies long $a_t$; therefore small cracks are not a problem.
2. low $K_{IC}$ and high $\sigma_o$ implies short $a_t$; therefore small cracks can be a problem.
Sec. 8.4 Application of $K$ to Design and Analysis

Values for small $a/b$ and limits for 10% accuracy:

(a) $K = \frac{P}{t \sqrt{\pi a}}$ \hspace{1cm} (a/b \leq 0.3), \hspace{1cm} (b) $K = \frac{2.60P}{t \sqrt{\pi a}}$ \hspace{1cm} (a/b \leq 0.08)

Expressions for any $\alpha = a/b$:

(a) $F_p = \frac{1.297 - 0.297 \cos \frac{\pi \alpha}{2}}{\sqrt{\sin \pi \alpha}}$ \hspace{1cm} (h/b \geq 2)

(b) $F_p = \frac{0.92 + 6.12 \alpha + 1.68(1 - \alpha)^2 + 1.32 \alpha^2(1 - \alpha)^2}{\sqrt{\pi \alpha(1 - \alpha)^{3/2}}}$ \hspace{1cm} (large h/b)

(c) $F_p = \frac{(2 + \alpha)}{(1 - \alpha)^{3/2}} (0.886 + 4.64 \alpha - 13.32 \alpha^2 + 14.72 \alpha^3 - 5.6 \alpha^4)$ \hspace{1cm} (a/b \geq 0.2)

Figure 8.15 Stress intensity factors for three cases of concentrated load. Case (c) is the ASTM standard compact specimen. (Equations from [Tada 85] pp. 2.23 and 2.25, and [Srawley 76].)
Sec. 8.2 Preliminary Discussion

### TABLE 8.1 FRACTURE TOUGHNESS AND CORRESPONDING TENSILE PROPERTIES FOR REPRESENTATIVE METALS AT ROOM TEMPERATURE

<table>
<thead>
<tr>
<th>Material</th>
<th>Toughness $K_{lc}$</th>
<th>Yield $\sigma_y$</th>
<th>Ultimate $\sigma_u$</th>
<th>Elong. $100%$</th>
<th>Red. Area %RA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPa $\sqrt{m}$</td>
<td>MPa</td>
<td>MPa</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td></td>
<td>(ksi $\sqrt{in}$)</td>
<td>(ksi)</td>
<td>(ksi)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Steels</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AISI 1144</td>
<td>66 (60)</td>
<td>540 (78)</td>
<td>840 (122)</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>ASTM A470-8 (Cr-Mo-V)</td>
<td>60 (55)</td>
<td>620 (90)</td>
<td>780 (113)</td>
<td>17</td>
<td>45</td>
</tr>
<tr>
<td>ASTM A517-F</td>
<td>187 (170)</td>
<td>760 (110)</td>
<td>830 (121)</td>
<td>20</td>
<td>66</td>
</tr>
<tr>
<td>AISI 4130</td>
<td>110 (100)</td>
<td>1090 (158)</td>
<td>1150 (167)</td>
<td>14</td>
<td>49</td>
</tr>
<tr>
<td>18-Ni maraging air melted</td>
<td>123 (112)</td>
<td>1310 (190)</td>
<td>1350 (196)</td>
<td>12</td>
<td>54</td>
</tr>
<tr>
<td>18-Ni maraging vacuum melted</td>
<td>176 (160)</td>
<td>1290 (187)</td>
<td>1345 (195)</td>
<td>15</td>
<td>66</td>
</tr>
<tr>
<td>300-M 650°C temper</td>
<td>152 (138)</td>
<td>1070 (156)</td>
<td>1190 (172)</td>
<td>18</td>
<td>56</td>
</tr>
<tr>
<td>300-M 300°C temper</td>
<td>65 (59)</td>
<td>1740 (252)</td>
<td>2010 (291)</td>
<td>12</td>
<td>48</td>
</tr>
</tbody>
</table>

(b) Aluminum and Titanium Alloys (L-T Orientation)

<table>
<thead>
<tr>
<th>Material</th>
<th>Toughness $K_{lc}$</th>
<th>Yield $\sigma_y$</th>
<th>Ultimate $\sigma_u$</th>
<th>Elong. $100%$</th>
<th>Red. Area %RA</th>
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<tr>
<td></td>
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</tr>
<tr>
<td>2014-T651</td>
<td>24 (22)</td>
<td>415 (60)</td>
<td>485 (70)</td>
<td>13</td>
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<tr>
<td>2024-T351</td>
<td>34 (31)</td>
<td>325 (47)</td>
<td>470 (68)</td>
<td>20</td>
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</tr>
<tr>
<td>2219-T851</td>
<td>36 (33)</td>
<td>350 (51)</td>
<td>455 (66)</td>
<td>10</td>
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<tr>
<td>7075-T651</td>
<td>29 (26)</td>
<td>505 (73)</td>
<td>570 (83)</td>
<td>11</td>
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</tr>
<tr>
<td>7475-T7351</td>
<td>52 (47)</td>
<td>435 (63)</td>
<td>505 (73)</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Ti-6Al-4V annealed</td>
<td>66 (60)</td>
<td>925 (134)</td>
<td>1000 (145)</td>
<td>16</td>
<td>34</td>
</tr>
</tbody>
</table>

Sources: Data in [Barsom 87] p. 172, [Boyer 85] pp. 6.34, 6.35, and 9.8, [MILHDBK 94] pp. 3.10 to 3.12 and 5.3, and [Ritchie 77].
FRACTURE UNDER COMBINED LOADING

In many cases, the structure is not only subjected to tensile stress $\sigma_x$ but also a contour shear stress $\tau_{xs}$, such as web 1 in the wing box of homework problem 4. Thus, the crack is exposed to tension and shear which leads to mixed mode cracking; i.e., a mixture of mode I and mode II.

Whenever the crack length $2a$ is small with respect to the web length $b$, the geometric factor $F$ is unity in formulas for the stress intensity factors from LEFM. That is, $K_I = \sigma_x \sqrt{\pi a}$ and $K_{II} = \tau_{xs} \sqrt{\pi a}$, where $\sigma_x$ and $\tau_{xs}$ are the normal and shear stresses in the web if there were no crack present.

Mixed mode fracture is complicated by the fact that the crack extension takes place at an angle with respect to the original crack direction. If a crack propagates in the direction of the original crack, it is called self-similar crack growth. Under mixed mode fracture the crack growth is, in general, not self-similar. Various mixed mode criteria for crack growth have been proposed based on experiments. A common mixed mode criterion, at the initiation of the fracture, is

$$\left( \frac{K_I}{K_{Ic}} \right)^2 + \left( \frac{K_{II}}{K_{IIc}} \right)^2 = 1$$  \hspace{1cm} (7)

where $K_{Ic}$ is the fracture toughness for mode I loading only, and $K_{IIc}$ is the fracture toughness for mode II loading only. The plane strain fracture toughness for mode I loading is usually readily available in the literature, but the mode II fracture toughness is not usually available.

Tests for mode II are more difficult to design than for mode I. To estimate $K_{IIc}$ knowing the value of $K_{Ic}$ we use the maximum principal stress criterion\(^1\). The maximum principal stress criterion postulates that crack growth will occur in the direction perpendicular to the maximum principal stress in the vicinity of the crack tip. Using this criterion it is possible to estimate $K_{IIc}$ (see Fig. 14.16 in Broek) as

$K_{IIc} = \frac{\sqrt{3}}{2} K_{Ic} = 0.866 K_{Ic}$ \hspace{1cm} (8)

The mixed mode criterion given by Eq. (7) is plotted in the following figure:

Under proportional loading, the stresses, and in turn the stress intensity factors, are proportional to the magnitude of the total lift acting on the wing. The stress intensity factors at the 80% limit load specified for the damage design condition determine the coordinates of the required strength in the

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plot, which is represented by the ray $0 - f$.

The quantity $f$ denotes the dimensionless required strength. The excess strength with respect to fracture is represented by $1 - f$, and if it is divided by the required strength we get the margin of safety as

$$MS = \frac{1 - f}{f}$$

(9)

$$f = \sqrt{\left(\frac{K_I}{K_{Ic}}\right)^2 + \left(\frac{K_{II}}{K_{IIc}}\right)^2}$$

(10)

The crack is predicted not to propagate if $0 \leq f < 1$, and the initiation of fracture is predicted if $f = 1$. The margin of safety is positive if $0 < f < 1$, and the margin of safety is zero if $f = 1$. 