Both Criterions are based on:

1. State of stress can be completely described by the magnitude and direction of the principal stresses. For an isotropic material the principal stress directions are unimportant.

2. Experiments show that hydrostatic state of stress does NOT effect yielding.

I. MISES YIELD CRITERION

Yielding begins in a 3-D stress state when the octahedral shear stress \( \tau_h \) is equal to its value at yield initiation in the uniaxial tension test.

\[
\begin{align*}
\tau_h \bigg|_{3-D} &= \tau_h \bigg|_{1-D} \\
\tau_h \bigg|_{1-D} &= \frac{\sqrt{2}}{3} \sigma_y \\
\tau_h \bigg|_{3-D} &= \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{9}} \\
\end{align*}
\]

(1) \hspace{1cm} (2) \hspace{1cm} (3) \hspace{1cm} (4)

Note that YIELDING BEGINS when (1) is true. However for design purposes this should not happen. It is wanted that

\[
\tau_h \bigg|_{3-D} < \frac{\sqrt{2}}{3} \sigma_y
\]

(5)

Mises stress is a method to predict yielding in 3-D state of stress and it is defined as:

\[
\begin{align*}
\sigma_M &= \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \\
\sigma_M &= \sqrt{\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2} \quad (\sigma_3 = 0) \\
\sigma_M &= \sqrt{\sigma_2^2 - \sigma_2 \sigma_3 + 3\tau_{zs}^2} \quad (\sigma_n = 0)
\end{align*}
\]

(6) \hspace{1cm} (7) \hspace{1cm} (8)

In the problems solved for this course, \( \sigma_s = 0 \), leading to the following result

\[
\sigma_M = \sqrt{\sigma_z^2 + 3\tau_{zs}^2}
\]

(9)
II. MAXIMUM SHEAR STRESS CRITERION

Yielding begins in a 3-D stress state when the maximum shear stress $\tau_{max}$ is equal to its value at the initiation of yielding in the tension test.

$$\tau_{max}\bigg|_{3-D} = \tau_{max}\bigg|_{1-D}$$ (10)

$$\tau_{max}\bigg|_{1-D} = \frac{\sigma_y}{2}$$ (11)

$$\tau_{max}\bigg|_{3-D} = \frac{\sigma_{max} - \sigma_{min}}{2}$$ (12)

Note that YIELDING BEGINS when (10) is true. However for design purposes this should not happen. It is wanted that

$$\tau_{max}\bigg|_{3-D} < \frac{\sigma_y}{2}$$ (13)

Maximum Shear Stress is another method to predict yielding in 3-D state of stress and it is defined as:

$$\sigma_{max} - \sigma_{min} = \pm\sigma_y$$ (14)

Three different cases with $\sigma_3 = 0$ are considered.

First Case

\[
\begin{align*}
\sigma_3 < \sigma_2 < \sigma_1 \\
\sigma_2 < \sigma_1 \\
\sigma_1 = \pm\sigma_y
\end{align*}
\]

$$\sigma_{max} - \sigma_{min} = \pm\sigma_y$$

$$\sigma_1 - \sigma_3 = \pm\sigma_y$$

$$\sigma_1 = \pm\sigma_y$$
Second Case

\[
\sigma_2 < \sigma_3 < \sigma_1
\]

\[
\sigma_{max} - \sigma_{min} = \pm \sigma_y
\]
\[
\sigma_1 - \sigma_2 = \pm \sigma_y
\]
\[
\sigma_1 = \sigma_2 \pm \sigma_y
\]

Third Case

\[
\sigma_2 < \sigma_1 < \sigma_3
\]

\[
\sigma_{max} - \sigma_{min} = \pm \sigma_y
\]
\[
\sigma_3 - \sigma_2 = \pm \sigma_y
\]
\[
\sigma_2 = \mp \sigma_y
\]