Unified Framework for Modern Synthetic Aperture Imaging Algorithms

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ABSTRACT: Imaging using synthetic aperture techniques is a mature technique with a host of different reconstruction algorithms available. Often the same basic algorithm has a different name depending on where the particular algorithm is used, since it may have originated from the medical, nondestructive testing, geological, or remote sensing fields. All this adds to confusion for the nonspecialist. This article gives a short historical precise of active synthetic aperture imaging as it applies to airborne, spaceborne, and underwater remote sensing systems using either radar or sonar, then defines some generic imaging geometry and places all the usable synthetic aperture reconstruction algorithms in a unified framework. This is done by the introduction of mapping operators, which simplify the mapping or reformatting of data from one sampling grid to another. Using these operators, readers can see how strip-map synthetic aperture systems (both radar- and sonar-based) differ from spotlight synthetic aperture systems, how the various algorithms fit together, and how the chirp-scaling algorithm is likely to be the reconstruction algorithm of choice for most future strip-map systems, and just why that should be so. Multilook processing and methods to deal with undersampled apertures using postdetection digital spotlighting are put into the same unified framework, as both of these techniques are frequent adjuncts to synthetic aperture imaging. © 1997 John Wiley & Sons, Inc. Int J Imaging Syst Technol, 8, 343–358, 1997

Key words: synthetic apertures; sonar; synthetic aperture algorithms

I. HISTORICAL BACKGROUND OF ACTIVE SAR AND SAS

The synthetic aperture concept is attributed to C. A. Wiley of Goodyear Aircraft Corporation for his 1951 analysis of along-track spatial modulation (Doppler) in the returns of a forward-looking (squinted), pulsed airborne radar [1–3]. He patented the idea under the name Doppler beam-sharpening; the name reflects his frequency domain analysis of the effect. Independent of this work, L. J. Cutrona of the University of Michigan and C. W. Sherwin of the University of Illinois were developing the same ideas from an aperture (spatial) point of view [1–3]. During the summer of 1953, at the University of Michigan, a combined effort with the University of Illinois team was undertaken under the guise of Project Wolverine to develop a high-resolution combat-surveillance radar system [4]. The ability of SAR to remotely observe a scene through cloud cover during day and night and through light rain made it ideal for use as a tactical weapon. The development of the early work in synthetic aperture radar (or SAR, as it is more usually known) is retraced by Sherwin et al. in [1]; a review of his personal involvement is made by Wiley in [2]. Curlander and McDonough [3] also retraced developments from an insider’s point of view.

Early workers considered unfocused SAR [5]; however, at the 1953 meeting, Sherwin indicated that a fully focused synthetic aperture (SA) would produce finer resolution [6, p. 339]. In 1961, Cutrona et al. first published the fact that a fully focused synthetic aperture produces an along-track resolution that is independent of range and wavelength and depends only on the physical size of the illuminating antenna [1,7]. The major problem faced by the initial developers of a focused SAR processor was the range variance of the along-track focusing filters. This problem was overcome with the development of the first optical processor by Cutrona in 1957. In this system, the pulse compressed echoes were recorded on photographic film for subsequent ground processing by a system of lenses and optical filters [8]. In the optical processor, the range variant focusing filter conveniently translated to a conical lens. The reviews by Brown and Porcello [4] and Tomiyasu [9] covered optical processing in some detail.

The optical processor was the mainstay of SAR processing until the late 1960s, when digital technology had advanced to the point where it could handle some sections of the processing [3]. The first fully digital system was developed by Kirk in 1975 [10]; the system also included a digital motion compensation system [11]. Since that time, digital-optical hybrids and fully digital processors have become common. Digital implementation of synthetic aperture systems also heralded the introduction of squint mode and spotlight mode radar systems [10].

The initial SAR systems were developed as airborne SARs; the first space-borne SAR developed for terrestrial imaging was the Seasat-SAR launched in 1978. Subsequent space-borne SARs are described in Curlander and McDonough [3]. The space-borne SAR differs from the airborne SAR in that the effect of range migration is more severe, earth rotation causes an effect known
as range walk, satellite motion and orbit eccentricity must be accounted for, and ionospheric and tropospheric irregularities cause defocus of the final images [3,9,12]. A processor that can handle most of these effects meant a more complex SAR processor was required. One of the first of these processors was developed by C. Wu [3]. References [13] and [14] are the basis of most of the space-borne SAR processors described in 1991 by Curlander and McDonough [3, pp. 197–208]. These processors are termed range/Doppler processors. The range/Doppler processor data are obtained by Fourier transformation of the compressed raw data in the along-track direction.

Walker is credited with the concept of spotlight SAR in 1980 [15,16] (the systems mentioned by Kirk in 1975 [10] and Brookner in 1978 [17] have a fixed squint angle, and so are not true spotlight systems). In spotlight mode, the physical antenna is slewed so that the same small area of terrain remains illuminated while the platform traverses the synthetic aperture. In this mode, the along-track dimension of the image becomes limited by the beamwidth of the physical aperture, but the along-track resolution of the final processed image is improved beyond the limit of conventional strip-map systems. Walker interpreted the processing of the spotlight synthetic aperture data in terms of a Fourier synthesis framework. This insight led others such as Munsen [18], Soumekh [19], and Jakowitz et al. [20] to describe SAR processing in a more consistent signal-processing framework. In the Fourier synthesis view, the raw data can be shown to be samples of the Fourier transform of the image reflectivity at discrete locations in the three-dimensional (3D) Fourier space of the object. These discrete samples are interpolated onto an appropriate 2D rectangular grid appropriate for inverse Fourier transformation by the inverse fast-Fourier transform (IFFT). Initial developments in this and similar areas are reviewed by Ausherman et al. [5].

Until the early 1990s, the processing algorithms for strip-map SAR were largely based on the Fresnel approximation algorithm (itself a subset of the range/Doppler algorithm) [19, p. 308]. During the early 1990s, what is now referred to as Fourier-based multidimensional signal processing was applied to develop a more concrete theoretical principle for the inversion of SAR data [19,21,22]. The algorithms produced by this theory are now generically referred to as wavenumber algorithms. The development of these wavenumber algorithms presents a breakthrough as dramatic as the development of the original optical processor [23]. The initial wavenumber processors required interpolators to perform a remap of the raw data onto a rectangular grid suitable for Fourier processing. Developments of two groups at the International Geoscience and Remote Sensing Symposium in 1992 [24–26] led to the publication in 1994 of a new wavenumber inversion scheme known as the chirp-scaling algorithm [27]. Chirp-scaling removes the interpolator needed in the previous wavenumber inversions and represents a further advancement of SAR processing. It is the purpose of this article to place all these various algorithms in a unified framework with consistent notation.

The history of synthetic aperture sonar (SAS) is much shorter and of more recent origin. In 1969, a patent was issued to Raytheon [28] for an SAS intended for high-resolution seafloor imaging, and in 1971 [29] analyzed a potential system in terms of its resolution and signal-to-noise ratio. Cutrona [6,30] was the first well-known radar specialist to point out how the various aspects of SAR could be translated to an underwater SAS. Hughes [31] compared the performance of a standard side-looking sonar to an SAS equivalent and showed that the potential mapping rate (an important operational parameter) for SAS was significantly higher than for side-looking sonar. This claim was confirmed by Lee [32,33].

At the time it was felt that perhaps the instability of the environment, especially in the ocean, would prevent the formation of a synthetic aperture; however, this was disproved in some early experimental work [34,35] and verified at higher frequencies a little later [36]. The stability of the towed platform was also seen as a major problem, and some experimental SAS systems resorted to rail- or wire-guided bodies to remove that extra complication [37,38]. There have been a number of tank experimental SAS systems [39,40], some to look at interferometric SAS [41], and there are a host of medical ultrasonic systems that in essence are synthetic aperture sonars but fall outside the intended scope of this article. There are currently at least five operating underwater SAS systems: the extensive European ACID project [42–47], one based at the U.C. Santa Barbara [48], a French Navy/U.S. Navy collaborative SAS launched in July 1996, one built and operated by Alliant Techsystems, and our own Kiwi-SAS [49,50]. So far the only unclassified publication that has shown diffraction limited imagery is possible with an unconstrained towfish is that of Hawkins and Gough [51].

Regardless of whether it is a strip-map or spotlight system and whether it is based on radar or sonar, synthetic aperture techniques are in widespread use and the algorithms used by these systems to form the images need to be put in a unified framework.

II. SYSTEM GEOMETRY

With reference to Figure 1, let us propose that the surface to be imaged (whether it be the earth’s surface, as in SAR, or the seafloor, as in SAS) is substantially flat and has on it a collection of subwavelength reflectors collectively described as the object reflectivity function \( ff(x, y) \)—mostly referred to as just the object. (It is important to note that for comparative purposes, we have defined the origin of the coordinate system always from the center of the aperture plane, not the center of the object as it is defined in some wavenumber and many spotlight SAR geometries.) For strip-map SA, the limits on \( x \) are the inner and outer edges of the swath which has a width of \( 2X_o \), whereas the only limit on \( y \) is determined by how long you are prepared to go on recording, say, \( 2Y_o \). For spotlight SA, the diameter of the footprint area is \( 2X_o \).

Let us assume the object is comprised of a magnitude function more or less surrounding the point \((x, y) = (r_o, 0)\) multiplied by a highly random phase function, i.e.,

\[
ff(x, y) = |ff(x, y)|\exp[j\phi(x, y)].
\]  (1)

As \( \phi(x, y) \) is highly random, its Fourier transform is extremely broad and stretches to all parts of the Fourier transform domain. Since the Fourier transform of \( ff(x, y) \) is in essence a convolution of the Fourier transform of \(|ff(x, y)\|\) with the Fourier transform of \(\exp[j\phi(x, y)]\), the Fourier transform of \( |ff(x, y)| \) is “modulated” to all parts of the Fourier domain [52]. In this way, any window of the complete Fourier transform domain—defined as \( FF(k_x, k_y) \)—is able to reconstruct the object magnitude, \(|FF(k_x, k_y)|\), to the resolution determined by the size and shape of the window in the object Fourier domain.
Now all imaging systems are limited in some way as to their resolution or, equivalently, how much of the Fourier domain they record so we can define two further quantities: the complex diffraction-limited image \( \mathcal{f}(x, y) \) and its Fourier transform, \( \mathcal{F}(k_x, k_y) \). Common parlance calls the latter the offset Fourier data, since it is seldom centered about the origin of the \((k_x, k_y)\) domain. The diffraction-limited image is no more than our representation of the object \( \mathcal{f}(x, y) \) convolved with the point-spread function of the limiting aperture (the window) in the imaging system, but does not account for aberrations or impairments induced by turbulence, etc. Equally, the offset Fourier perpendicular of the flight path (systems that steer the radiation imaging system, but does not account for aberrations or impairments induced by turbulence, etc.) of the boresight (center) of the main lobe is close to the spread function of the limiting aperture (the window) in the main lobe of the real aperture radiation pattern is usually steered such that the boresight (center) of the main lobe is close to the perpendicular of the flight path (systems that steer the radiation pattern to a fixed angle off the perpendicular are termed squinted strip-map systems). The radiation patterns of the real apertures often have beamwidths of \(3^\circ-10^\circ\) depending on whether it is a satellite SAR, aircraft SAR, or boat-deployed SAS. In contrast, a spotlight system has a much narrower beamwidth (sometimes significantly \(<1^\circ\)) in which the antenna boresight is slewed to keep a small patch of the terrain illuminated over a very long synthetic aperture, much longer, in fact, than would be spanned by the real antenna radiation pattern if the boresight were fixed perpendicular to the flight path. At this stage, let us consider only broadband strip-map SA systems and delay the consideration of spotlight SA systems until later.

Much of the SAR literature develops SA processing on the basis of what appears to be temporal Doppler effects. This aspect

### Table I. Parameters of Seasat, a typical airborne spotlight SAR, and Kiwi-SAS.

<table>
<thead>
<tr>
<th></th>
<th>Seasat</th>
<th>SpotSAR</th>
<th>Kiwi-SAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c ) (m/s)</td>
<td>(3 \times 10^4)</td>
<td>(3 \times 10^4)</td>
<td>(1.5 \times 10^4)</td>
</tr>
<tr>
<td>Platform ( V_p ) (m/s)</td>
<td>7400</td>
<td>500</td>
<td>1</td>
</tr>
<tr>
<td>Carrier ( f_c ) (GHz)</td>
<td>1.3</td>
<td>10</td>
<td>30 kHz</td>
</tr>
<tr>
<td>Bandwidth ( B_o ) (MHz)</td>
<td>20</td>
<td>400 MHz</td>
<td>20 kHz</td>
</tr>
<tr>
<td>Antenna D (m)</td>
<td>10.7</td>
<td>3</td>
<td>0.30</td>
</tr>
<tr>
<td>Beamwidth</td>
<td>1.5(^\circ)</td>
<td>1(^\circ)</td>
<td>10(^\circ)</td>
</tr>
<tr>
<td>Depression angle</td>
<td>70(^\circ)</td>
<td>12(^\circ)</td>
<td>5(^\circ)</td>
</tr>
<tr>
<td>Stand-off ( r_0 ) (km)</td>
<td>700</td>
<td>70 km</td>
<td>100 m</td>
</tr>
<tr>
<td>Swathwidth</td>
<td>100 km</td>
<td>200 m</td>
<td></td>
</tr>
<tr>
<td>Patch size diameter</td>
<td>500 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ground resolution</td>
<td>25 (\times) 25 m</td>
<td>0.5 (\times) 0.5 m</td>
<td>0.05 (\times) 0.15 m</td>
</tr>
<tr>
<td>No. looks</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
of SAR literature is misleading to any nonspecialist. Synthetic aperture system models are generally developed based on the assumption that the platform can be considered to be stationary during the transmission of a pulse and the reception of the echoes from this pulse; then the platform jumps instantaneously to the next along-track sampling location, and the process repeats. This scenario is often referred to as the “stop-start” approximation or the “stop-and-hop” approximation. Aperture synthesis is then achieved by exploiting the modulation induced into the along-track signal of the 2D echo matrix (described above) by relative platform-target motion, i.e., by processing a target’s phase history. This modulation is identical in form to the temporal Doppler shift expected for a particular platform-target relative motion and pulse frequency; however, since the SA model was developed on the basis of the stop-and-hop scenario, no temporal Doppler effects can occur. The modulation induced by the relative platform-target motion is a geometric effect and should more correctly (or less ambiguously) be referred to as a spatial-Doppler effect. In summary, SA models assume temporal Doppler effects within the echo pulses can be ignored and then exploit spatial-Doppler effects that occur between pulses.

First, let us determine the form of the detected echoes given the strip-map system geometry shown in Figure 1(a) and the transmitted waveform \( p_m(t) \). In deriving this expression, many authors ignore the physical size of the real antennas and transducers in their description to maintain some mathematical simplicity. Here, we follow their example but later blend in the necessary functions to account for the finite size of the transmitting and receiving apertures. So with that proviso, and assuming the apertures are very much smaller than a wavelength in extent, the detected echoes may be approximated by

\[
e e_m(t, u) \approx \int_s \int_s f f(x, y) \cdot p_m \left[ t - \frac{2}{c} \sqrt{x^2 + (y - u)^2} \right] dx dy \quad (2)
\]

This expression, then, is where the various SA algorithms start, in that all inversion procedures take \( e e_m(t, u) \) (or its complex demodulated version at baseband \( ee_m(t, u) = e e_m(t, u) \exp(-j \omega t) \)) and attempt to produce the complex diffraction limited image \( f f(x, y) \) and its modulus \( |f f(x, y)| \).

It is worth a comment on broad-bandwidth systems. Most SAS and some SAR systems use a time-dispersed, broad-bandwidth chirp for \( p_m(t) \) which, although it has the same range resolution as a short-time CW pulse of the same bandwidth after pulse compression, makes \( e e_m(t, u) \) almost impossible to interpret visually. Consequently, it is quite common to pulse-compress the incoming echoes on the fly so that we actually record a short-time pulse equivalent of \( e e_m(t, u) \), which is written as

\[
ss_m(t, u) \approx \int_{t'} p_m(t' - t) \cdot e e_m(t', u) \, dt'
= p_m(t) \ast e e_m(t, u)
\]

where \( \ast \) denotes correlation. The pulse-compressed data represented by this expression are then the usual starting point for most of the SA reconstruction/processing algorithms.

At this point, it is prudent to comment on the double-functional notation. This notation is useful when describing synthetic aperture algorithms, because the processing algorithms often work on the data in pseudo-Fourier spaces. For example, as is seen shortly, the range/Doppler algorithm uses baseband pulse-compressed data, \( ss_m(t, u) \), that have undergone only a 1D Fourier transform in the along-track direction into the range/Doppler domain, i.e., \( SS_m(t, k) \). The double-functional notation allows the capitalization of the character corresponding to the transformed parameter: in this case, the along-track direction, \( u \), to wavenumber \( k \). The consistent use of this notation clarifies the mathematical description of the different SA imaging algorithms considerably.

Before we discuss the specifics of the various algorithms, it is helpful to look at two Fourier transforms of the raw echo data \( ee_m(t, u) \) specifically, \( EE_m(\omega, u) \) and \( EE_m(\omega, k) \). To determine the first, take a 1D temporal Fourier transform of \( ee_m(t, u) \) as defined in Equation (2) to produce

\[
EE_m(\omega, u) = P_m(\omega) \cdot \int_s \int_s f f(x, y) \cdot \exp(-j 2k \sqrt{x^2 + (y - u)^2}) \, dx \, dy \quad (4)
\]

where the modulated pulse spectrum \( P_m(\omega) \) is defined over a different set of coordinates to \( f f(x, y) \).

Now, using the principle of stationary phase or by performing a Fourier decomposition on the exponential in Equation (4) \([19,49]\), we obtain the 1D Fourier transform of Equation (4) along the \( u \) axis to give a new domain in coordinates of temporal frequency \( \omega \) and Doppler wavenumber \( k \):

\[
EE_m(\omega, k) = P_m(\omega) \cdot \int_s \int_s f f(x, y) \exp(-j 4k^2 - k^2_x - k^2_y) \, dx \, dy \quad (5)
\]

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\[
EE_m(\omega, u) = P_m(\omega) \cdot \int_x \int_y f f(x, y) \cdot \exp[-j 2k(x^2 + (y - u)^2)] \, dx \, dy \quad (4)
\]

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\]

where \( k = \omega / c \). If we now define a new set of coordinates such that

\[
k_x(\omega, k_x) = \sqrt{4k^2 - k_x^2}

\]

then \( EE_m(\omega, k_x) \) can be simply expressed as

\[
EE_m(\omega, k_x) \approx P_m(\omega) \cdot FF(k_x, k_x)
\]

and similarly, the 2D spectrum of the pulse-compressed data is

\[
SS_m(\omega, k_x) \approx |P_m(\omega)|^2 \cdot FF(k_x, k_x)
\]

The effect of the finite temporal bandwidth is to window the wavenumber domain in the \( \omega \) direction. Note that \( FF(k_x, k_x) \) is defined over a different set of coordinates to \( EE_m(\omega, k_x) \) or \( SS_m(\omega, k_x) \), which is fine until sampled data are used; but more about that later when we discuss the details of the various reconstruction algorithms.

### IV. EFFECTS OF FINITE TRANSMITTING AND RECEIVING APERTURES

It is now appropriate to replace the subwavelength physical apertures by ones with some extent and see how the system model can be improved. To do so, we need to consider the radiation pattern of the radiating aperture and the sensitivity pattern of the
detecting aperture. The appropriate geometry for our 2D model is shown in Figure 2. The signal received at a point \((x, y)\) from a transmitting aperture of physical length \(D\) having an illumination function \(I_T(v)\) and centered around the origin of the \((x, y)\) plane, while radiating a signal \(p_m(t)\), is well described by

\[
h_T(t, x, y) \approx \int_{-D/2}^{D/2} \frac{i_T(v)}{\sqrt{x^2 + (y - v)^2}} \times p_m \left[ t - \frac{1}{c} \sqrt{x^2 + (y - v)^2} \right] dv \tag{8}
\]

where the \(v\) axis is colocated with the \(y\) and \(u\) axis. The radiation pattern of the aperture is the temporal Fourier transform of Equation (8); that is,

\[
H_T(\omega, x, y) \approx P_m(\omega) \cdot \int_{-D/2}^{D/2} i_T(v) \exp \left[ -jk\sqrt{x^2 + (y - v)^2} \right] dv \tag{9}
\]

We can reference everything to the center of the real aperture, thus removing the dependence on \(v\), by replacing the exponential in Equation (9) by its Fourier decomposition and changing the order of integration [19,22,49]:

\[
H_T(\omega, x, y) \approx P_m(\omega) \cdot \int_{-D/2}^{D/2} i_T(v) \exp \left[ -jk\sqrt{x^2 + (y - v)^2} \right] dv \times \int_{-D/2}^{D/2} \exp \left[ \frac{j(k^2 - k_0^2) \cdot x - jk_0(y - v)}{\sqrt{k^2 - k_0^2}} \right] dk_0 \tag{10}
\]

where \(I_T(k)\) is a Fourier-like transform of \(i_T(v)\) with the normal forward-Fourier kernel of \(\exp(-jkv)\) replaced by \(\exp(jkv)\). In general, \(I(k)\) is complex with a slowly varying amplitude \(A(k)\) and a phase that determines whether the real aperture has any steering or focusing power. So,

\[
I(k) = A(k) \exp[-j \phi(k)]
\]

For an unsteered and unfocused real aperture, the illumination function \(I_T(v)\) is real and mostly just tapers or apodizes the real aperture. (This will mean that effective length of the aperture \(D_{eff}\) is smaller than its physical length, \(D\).) The integral in Equation (10) can be solved via the principle of stationary phase to give [49]

\[
H_T(\omega, x, y) \approx P_m(\omega) \cdot \int_{-D/2}^{D/2} i_T(v) \exp \left[ -\frac{j(k^2 - k_0^2) \cdot x - jk_0(y - v)}{\sqrt{k^2 - k_0^2}} \right] dv \times \exp \left[ \frac{j(k^2 - k_0^2) \cdot x - jk_0(y - v)}{\sqrt{k^2 - k_0^2}} \right] \right] dk_0 
\]

where the wavenumber \(k = k \sin \theta\) and \(\theta = \sin^{-1}(y/v^{1/2} + v^{1/2})\) is the aspect angle from the center of the aperture to the measurement location as indicated in Figure 2. The magnitude of the aperture’s amplitude pattern \(|A(\omega, x, y)|\) dictates its power distribution in the spatial domain at frequency \(\omega\). This function is also referred to as the aperture beam pattern and is often plotted versus angle \(\theta\), spatial frequency \(\sin \theta/\lambda\), or wavenumber \(k_0\). The \(\sqrt{x^2 + y^2}\) in the denominator of Equation (12) represents the one-way spreading losses.

Figure 2 also shows an example of the power distribution in the spatial domain at frequency \(\omega\) for an evenly illuminated transmitting aperture of length \(D\). The amplitude pattern is of the form \(A_T(k_0) = \sin[kD/2] / \sin[kD/2]\), i.e., \(A_T(\omega, x, y) = \sin[kD \sin \theta(\gamma)]\). In synthetic aperture systems, the combined transmitter/receiver amplitude pattern acts like a low-pass spatial filter, limiting the along-track bandwidth available for processing. In spotlight systems, slewing of the aperture to point it at the same patch at the expense of a smaller imaged field).

Using similar arguments to those used above, we can account for the sensitivity pattern of receiving aperture to produce an overall radiation pattern:

\[
H(\omega, x, y) = P_m(\omega) \cdot A(\omega, x, y) \times \exp \left[ \frac{-j2k\sqrt{x^2 + y^2}}{x^2 + y^2} \right] \tag{13}
\]

and since it is common to use time-varying gain to offset the decline in power due to the now two-way spreading losses, we drop the denominator \((x^2 + y^2)\) from here on.
The appearance of the factor 2\(k\) in the delay exponential means the rate of change of the phase in Equation (13) —i.e., the instantaneous Doppler frequency—increases at twice the rate of the single aperture function in Equation (12). This is why an active imaging system has twice the bandwidth of a bistatic arrangement where only one aperture moves or a passive system where only the receiver moves.

If we define the transmitting aperture impulse response as the inverse Fourier transform of the radiation amplitude pattern, i.e., \(a_t(t, x, y) = \mathcal{F}^{-1}\{A(\omega, x, y)\}\), and the receiving aperture impulse response as \(a_r(t, x, y) = \mathcal{F}^{-1}\{A(\omega, x, y)\}\), then the effect of using finite apertures is to modify the echo model in Equation (2) to give the more precise

\[
\begin{align*}
\varepsilon(t, t_0, u) &= \int_x \int_y \left( a_t(t, x, y - u) \odot a_r(t, x, y - u) \right) \times \mathcal{F}^{-1}\left\{ t - \frac{2}{c} \sqrt{x^2 + (y - u)^2} \right\} dx dy,
\end{align*}
\]

where \(\odot\) denotes convolution in time and where forward and inverse Fourier transforms are indicated by \(\mathcal{F}\{\cdot\}\) and \(\mathcal{F}^{-1}\{\cdot\}\) with a subscript that indicates the domain from which the transform occurred. In the temporal frequency/along-track domain, the approximation for \(E\varepsilon(t, t_0, u)\) as given by Equation (4) is replaced by

\[
E\varepsilon(t, t_0, u) = P_m(\omega) \cdot \int_x \int_y \left( A(\omega, x, y) A(\omega, x, y) \right) f(x, y) \times \exp\left( -t^2 / \omega^2 \right) dx dy,
\]

where \(\omega\) is the Doppler frequency. Here, to get better azimuth resolution, you make the physical aperture smaller!

The resolution of strip-map SA images in range (perhaps more correctly, the resolution in the slant-range direction) is

\[
\delta x_{s, a} = \frac{c}{2B_c}
\]

where \(c\) is the velocity of propagation and \(B_c\) is the (chirp) bandwidth in Hertz of the transmitted waveform [i.e., the equivalent bandwidth of \(P_m(\omega)\)].

The resolution of strip-map SA images in along-track direction is

\[
\delta y_{s, a} = \frac{D_{eff}}{2}
\]

where \(D_{eff}\) is the effective length of the real apertures used to transmit the waveform and receive the reflected echoes. [Note that if the aperture is tapered in any significant way, \(D_{eff}\) will be smaller than the length of the real apertures \(D\), and this should be taken into account. Without adding too much to the confusion, \(D\) is used here to mean both the physical length and the effective length of the real aperture (s).] The most important aspect to note is that the azimuth resolution is completely independent of the range and transmitted frequency. This fact runs counter to the usual idea that to get better azimuth resolution in a real antenna, you make the physical aperture larger or increase the carrier frequency. Here, to get better azimuth resolution, you make the physical aperture smaller!

There are several algorithms that start with the uncompressed or compressed echos and end with the diffraction limited image \(f(x, y)\) that has resolution of \(\delta x_{s, a}\) and \(\delta y_{s, a}\), and these are summarized in the remainder of this section.

### A. Exact Algorithm and Exact Transfer Function Algorithm

Given that we have recorded the pulse-compressed echos \(s_{\alpha}(t, u)\) reflected from an object described by \(f(x, y)\), let us now also assume we have a second (this time completely hypothetical) object in which there is only one reflecting point; without loss of generality, let it be in the center of the whole field of view, and so \(f(x, y) = \delta(r_0, 0)\). Its compressed echo, if it existed, would be:

\[
|s_{\alpha}(t, u)|_{f(x, y) = \delta(r_0, 0)} = a(t, r_0, u) \odot p_m \left( t - \frac{2}{c} \sqrt{r_0^2 + u^2} \right)
\]

As we need not concern ourselves about amplitude weighting, we can ignore the combined aperture impulse response \(a(t, r_0, u)\), and since the pulse-compressed signal is usually only a few
cycles long, we can approximate the baseband version of Equation (17) by the following delta function expression:

\[ s_{tb}(t, u) \approx \delta \left[ t - \frac{2}{c} \sqrt{u^2 + r_0^2} \right] \exp(-2k_0(\sqrt{u^2 + r_0^2} + u')). \] (18)

Let us define this as the point reflector function and note that there is a unique point reflector function for every pixel in the image. The original time domain algorithm (the exact algorithm) takes the baseband, pulse-compressed data along a locus scribed out by the point reflector function, multiplies it by the complex conjugate of the phase calculated from the point reflector function, and integrates the result. (Since the recorded data are sampled on a regular grid and the locus is a continuously varying function in both \( u \) and \( t \), this inversion scheme requires interpolation from sampled data surrounding the exact position of the locus at that \( u \) and \( t \).) The result of all this processing is the value for just one pixel in the image, and the whole process must be repeated for every pixel. Clearly, this is time-consuming.

There are alternatives to this purely time domain approach to the inversion problem, usually involving some form of block array processing. For example, we can use the set of point reflector functions to correlate against the data we actually recorded. Given the baseband, pulse-compressed data, \( s_{tb}(t, u) \), and one particular \( s_{tb}(t, u) \), any reflector in the object at the point defined by the point reflector function shows up as a large peak in the cross-correlation. This is the value of just one pixel in the image. Thus, the complete diffraction limited image must be reconstructed using the whole set of point variant cross-correlations in the time/space domain. The point variant cross-correlations are usually computed by a series of 2D multiplications in the temporal frequency/spatial wavenumber domain followed by inverse transformation into the image domain. This process is sometimes called the exact transfer function algorithm. Alternatively, the range signal is sometimes significantly oversampled as a form of time-domain interpolation.

Obviously, any of these approaches is time-consuming and costly; however, it is not as bad as it might seem, because the length of the aperture to be synthesized is not infinite, and so the imaging system has a nonzero depth of focus, meaning we might have to compute only a few different \( s_{tb}(t, u) \) to cover the whole swath. To illustrate the concept of a useful depth of focus, refer to Figure 3. Here, we have started with a simulated object field having four point reflectors at \((x, y) = \{(28, \pm 3), (30.5, 0), (32, 3)\}\). Using the Kiwi-SAS parameters given in Table I, the absolute value of the simulated baseband compressed echo field \( |s_{tb}(t, u)| \) is shown in Figure 3(a) and the real part of the spectral estimate obtained from these data, \(|{FF_\delta}(k_x, k_y)|\), is shown in Figure 3(b). The point reflector function used for the matched filter, i.e., \( s_{tb}(t, u) \), was chosen for a stand-off range of \( x = 30 \). The image domain shows a well-focused point at 30.5 m; however, the three targets located further from the focal point are slightly blurred. This blurring is not particularly significant at 2 m from the focal point, so the useful depth of focus of this array is around 2 m. Targets further from the focal point suffer more blurring.

The use of interpolators, oversampling, or Fourier techniques to implement any form of the exact algorithm, hardly leads to efficient processing. So, while the exact and exact transfer function algorithm are fine for system evaluation and calibration procedures (for isolated pointlike targets), what is really needed is some way of processing significant blocks of the echo domain data in an efficient way to produce an estimate of sections of the whole swath simultaneously. This desire led to the development of the range/Doppler, wavenumber, and chirp-scaling algorithms.

B. Range/Doppler Algorithm. The basis of the range/Doppler algorithm is a coordinate transformation (sometimes referred to as a coordinate warping, rescaling, or remapping) in \( t \) (the delay time proportional to range) and \( k_u \) (the along-track wavenumber or spatial-Doppler wavenumber) domain. In this unified treatment of SA algorithms, there are a considerable number of coordinate transformations where the information contained in the domain does not change, only the locations of the 2D sampling grid. We will indicate any coordinate transform with curly brackets, viz. \( \{ \cdot \} \), \( \{ F(\cdot) \} \), \( \{ {FF_\delta}(\cdot) \} \), and \( \{ \cdot' \} \). As has already been seen, we also use \( \{ \cdot \} \) and \( \{ \cdot' \} \) to indicate forward and reverse Fourier transforms, respectively.

On the assumption we have recorded \( s_{tb}(t, u) \), we compute the range/Doppler domain in coordinates of \( t \) and \( k_u \) using a 1D Fourier transform from \( t \) to \( k_u \) (the baseband signal is arbitrarily chosen here, as it is generally possible to use a much lower sampling rate to record a complex baseband signal):

\[ s_{\delta}(t, k_u) = \{ s_{tb}(t, u) \} \] (19)

Given the data in this form, the complete range/Doppler algorithm can be expressed as two multiplications and a coordinate transformation,
\[ f(F(x, k_x) = W(k_x) \cdot qQ(x, k_x) \cdot \mathcal{F} \{ sS_s(t, k_x) \} \] (20)

First, \( \mathcal{F} \{ \cdot \} \) defines a coordinate transformation given by
\[
x(t, k_x) = \frac{c}{2} t [1 - C(k_u)]
\]
\[
k_u(t, k_x) = k_u
\]

where the curvature factor is given as
\[
C(k_u) = \frac{1}{\sqrt{1 - \left( \frac{k_u}{2k_0} \right)^2}} - 1
\] (21)

After the coordinate transformation, there is a phase-only 2D azimuth compression function in \( x \) and \( k_x \)
\[
qQ(x, k_x) = \exp \left[ j \left( \sqrt{4k_0^2 - k_x^2} - 2k_0 \right) \right]
\] (22)

followed by a tapered window \( W(k_x) \) which may be any one of a number of 1D window functions that will ensure low sidelobes in the image along-track direction (windowing in the range dimension during pulse compression is also advised). The concept here is that the coordinate transformation specified in \( \mathcal{F} \{ \cdot \} \) puts a time advance into the domain that is both delay time \( t \) (proportional to range) and spatial wavenumber \( k_x \) (Doppler) dependent. This decouples the rows and the columns of the range/Doppler matrix. This is followed by a phase-only multiplication by \( qQ(x, k_x) \) which removes the Fresnel-like dependence of the reflected echoes in the range/Doppler domain, producing data corresponding to a pseudo-Fourier (1D Fourier) transform \( fF_s(x, k_x) \) of the diffraction-limited image \( f_s(x, y) \).

In cases where the standoff distance is small and the range migration is less than the resolution cell size of the image \([i.e., C(k_u) \approx 0]\), the coordinate transformation \( \mathcal{F} \{ \cdot \} \) does not require interpolation at all. This makes the algorithm very efficient, as only scaling and multiplication by the phase-only azimuth compression function \( qQ(x, k_x) \) are required. This variation of the range/Doppler algorithm is sometimes known as the Fresnel approximation-based algorithm \([19]\) and is the algorithm used in many airborne strip-map SAR systems. The use of the quadratic expansions of the curvature factor in Equation (21) and the along-track compression phase in Equation (22) caused little discernable difference in the final images for any of the simulations performed by the authors for both broadband SAR and SAS systems.

What are the inherent disadvantages of the range/Doppler algorithm? Basically, the coordinate transformation \( \mathcal{F} \{ \cdot \} \) normally requires interpolation between two sets of sampling grids, so many of the efficiency gains by doing full-swath processing are lost when forced to use a large kernel in the interpolation step. A second disadvantage starts to become apparent as the physical beamwidths increase. At larger angles between the bore-sight and the target and at large squint angles (if not using a broadside geometry), the spatial Doppler in azimuth is not fully decoupled from the range, which results in an additional linear frequency-modulated (LFM) chirp at large wavenumbers. This means that the pulse compression appropriate for \( k_x = 0 \) is no longer appropriate at the larger wavenumbers. The matching of this extra LFM is termed the secondary range compression (SRC). For simplicity of form, we have not included SRC in the mathematics of the range/Doppler algorithm, but it would need to be included for any broad-beamwidth system. Despite these disadvantages, the range/Doppler algorithm was the mainstay of most SAR and narrow-beamwidth and swathwidth SAS for several years, mainly because the system geometry was such that they could ignore the time-wasting coordinate transformation.

Figure 4 demonstrates the range/Doppler processing of the raw data shown in Figure 3(a). Figure 4(a) shows the pseudo-Fourier data obtained via a Fourier transform of the pulse-compressed data in the along-track direction, i.e., the range/Doppler data. Note how the loci of the two nearest targets overlay perfectly and interfere coherently. Note also the broadening of all the loci at extreme wavenumbers. This broadening is caused by the induced LFM component that is compensated for by SRC. Figure 4(b) shows how the mapping \( \mathcal{F} \{ \cdot \} \) has straightened the range-dependent loci and SRC has removed the range spreading. Figure 4(c) shows the spectral estimate, and Figure 4(d) shows the final image estimate. The impulse response for each target in the image estimate is identical.

**C. Wavenumber Algorithm.** Of all the extant algorithms, the wavenumber algorithm is by far the most elegant, at least conceptually. It masquerades under more than one alias depending on who invented it and what discipline is using it. The seismic migration algorithm comes from geophysics \([21]\), whereas the
\( \Omega - K \) algorithm comes from SAR [23]; however, we will refer to all these collectively as the wavenumber algorithm.

The algorithm starts with either the raw or pulse-compressed echo data and immediately performs a 2D Fourier transform to move the data into the 2D temporal frequency/spatial wavenumber domain. A basic reconstruction using a matched filter and a coordinate transform gives the following image spectral estimate:

\[
\hat{FF}'_{m}(k_{x}, k_{y}) = s \{ \exp [j(\sqrt{4k^{2} - k_{u}^{2}} - 2k) \cdot r_{0}] \\
\times A^{*}(k_{x}) \cdot P_{m}(\omega) \cdot EE'_{m}(\omega, k_{y}) \} \\
= s \{ \exp [j(\sqrt{4k^{2} - k_{u}^{2}} - 2k) \cdot r_{0}] \\
\times A^{*}(k_{x}) \cdot SS'_{m}(\omega, k_{y}) \} \quad (23)
\]

where the coordinate transformation, most often called the Stolt mapping \( s \{ \cdot \} \), is a transformation from coordinates of \( k_{u} \) and \( \omega \) into \( k_{x} \) and \( k_{y} \), defined by

\[
k_{x} = \sqrt{4k^{2} - k_{u}^{2}} \\
k_{y} = k_{u}. \quad (24)
\]

[Note that for this section, we display only the wavenumber reconstruction algorithm for the processing of raw, modulated data. When processing baseband data, the baseband pulse \( P_{b}(\omega) \) must be used for pulse compression and any phase functions must be calculated for the appropriate modulated frequencies.] The appearance of the phase factor in Equation (23) and the primes in both Equations (23) and (24) require an explanation: The phase factor is introduced by the choice of origin in the system model and the use of the fast-Fourier transform (FFT) during processing. An FFT of length \( N \) considers element \( (N/2 + 1) \) to be the axis origin, or alternatively, this element can be interpreted as being the phase reference point, i.e., linear phase functions in the frequency/wavenumber domain, representing linear displacement in the time/spatial domain, are referenced to the \( (N/2 + 1) \)th element of the transformed data (this is why the origin is usually chosen as the center element—it gets the phase in the frequency/wavenumber domain right). Now, as a strip-map image is going to consist of a swath of data out to the side of the platform, the valid data for processing are those temporal samples consistent with echos from targets located within a swath of width \( 2X_{0} \) centered on some range \( r_{0} \), i.e., the valid range data exist for times \( t \in [2/c \cdot (r_{0} - X_{0}, r_{0} + X_{0}) \) (gating is sometimes used so that these are the only data collected). The correct data format for the FFT is then equivalent to redefining the temporal origin to be centered on range \( r_{0} \), i.e., \( t' = t - 2r_{0}/c \), and the data obtained from the inverse transform of the spectra produced using the FFT will have an output range origin defined by \( x' = x - r_{0} \). The basic system model in Equation (2) then becomes

\[
e_{m}(t', u) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} ff(x', y) \\
\times p_{m}\left[ \left( t' + \frac{2r_{0}}{c}\right)^{2} - \frac{2}{c} \sqrt{(x' + r_{0})^{2} + (y - u)^{2}} \right] \, dx \, dy.
\]

\[
(25)
\]

This model has the 2D spectrum

\[
EE'_{m}(\omega, k_{y}) = P_{m}(\omega) \cdot \exp [j(\sqrt{4k^{2} - k_{u}^{2}} - 2k) \cdot r_{0}] \cdot FF'_{m}(k_{x}, k_{y}) \quad (26)
\]

The placement of the primes in the above equations is explained as follows: The shifted data are defined by

\[
ff(x', y) = ff(x - r_{0}, y). \quad (27)
\]

The spectrum of these shifted data is then related to the original data via

\[
FF'(k_{x}, k_{y}) = FF(k_{x}, k_{y}) \cdot \exp (jkr_{0}). \quad (28)
\]

So, modification of the spatial variable \( x \) to \( x' \) modifies the function definition of the spectral data from \( FF \) to \( FF' \). Similar effects occur for the redefined functions of \( ee \) and \( ss \) and their respective spectra.

Note that Equation (23) produces a modulated estimate of the windowed image spectrum owing to the definition of the Stolt mapping in Equation (24); it is common practice to demodulate the spectral estimate to \( k_{b} \), baseband by simply redefining the Stolt map as

\[
k_{b} = \sqrt{4k^{2} - k_{u}^{2}} - 2k_{b} \quad (29)
\]

This does not change the implementation of the algorithm; it simply redefines the axis against which the \( k_{w} \)-wavenumber data are plotted. The wavenumber samples shown in Figure 4(c) are almost identical to those produced via the wavenumber algorithm [49]. The figure shows the \( k_{b} \) wavenumbers plotted with a baseband definition. The target at 30.5 m is the cause of the dominant sinusoidal nature of the real part of the spectrum. It has the lowest frequency in the image as it is the closest target to the reference range \( r_{0} \); i.e., to produce this spectrum, it is necessary for the FFT to consider the image center, \( r_{0} \), as the origin. The other three targets contribute to the high-frequency information observed in Figure 4(c). The image estimate produced from the inverse FFT of the baseband spectral estimate produced via the wavenumber algorithm produces the function \( ff(x', y) \), a shifted version of the final image estimate (the center of this image corresponds to range \( r_{0} \) on the x axis). As with the range/Doppler algorithm, the target impulse responses are identical.

As an alternative to matched filtering, and given very high signal-to-noise ratios, a higher-resolution image may be generated by the inverse filter reconstruction given as:

\[
\hat{FF}'_{m}(k_{x}, k_{y}) = \left\{ \exp [j(\sqrt{4k^{2} - k_{u}^{2}} - 2k) \cdot r_{0}] \\
\times \frac{P_{m}(\omega)A^{*}(k_{y})}{|P_{m}(\omega)A(k_{y})|^{2}} \cdot EE'_{m}(\omega, k_{y}) \right\} \quad (29)
\]

With the inverse filter approach, it should be remembered that
inversion in regions where either \( P_m(\omega) \) or \( A(k_u) \) is below the noise floor is unwarranted. Perhaps a more useful expression for the inverse filtering algorithm would be a Weiner inverse filter, viz.:

\[
\hat{F}F'_m(k_u, k_v) = s \left\{ \exp [j(\sqrt{4k_u^2 - k_z^2} - 2k) \cdot r_0] \right. \\
\times \frac{P_m^*(\omega) A^*(k_u)}{|P_m(\omega) A(k_u)|^2 + \sigma^2} \cdot EE'_m(\omega, k_u) \right\}
\]

(30)

where \( \sigma \) is an RMS measure of the noise in the data.

Note that when \( EE'_m(\omega, k_u) \), as given by Equation (26) (modified to include the real aperture effects), is plugged into Equation (30), we get

\[
\hat{F}F'_m(k_u, k_v) = FF'(k_u, k_v)
\]

(31)

but only for regions of the spatial wavenumber domain where \( \delta \{ P_m(\omega) A(k_u) \} \) is above the noise floor. This is really no more than an equivalent statement that the image is indeed diffraction (and bandwidth) limited.

In practice, neither the matched filter nor the inverse filter reconstruction techniques are used, as they generally produce unacceptably high side lobes in the final image. What is more often done is to filter the result of the Stolt mapping operation on the Fourier-transformed data with some arbitrary function \( WW(k_u, k_v) \) which may be any number of 2D low side-lobe window functions. The windowed version of the wavenumber image reconstruction algorithm used is most elegantly summarized as

\[
\hat{F}F'_m(k_u, k_v) = WW(k_u, k_v) \cdot \delta \left\{ \exp [j(\sqrt{4k_u^2 - k_z^2} - 2k) \cdot r_0] \right. \\
\times \frac{P_m^*(\omega) A^*(k_u)}{|P_m(\omega) A(k_u)|^2 + \sigma^2} \cdot \delta (k_u, k_v) \right\}
\]

(32)

where the pulse spectrum, \( P_m(\omega) \), is most often chosen to be flat across the pass-band and the window function deconvolutes the aperture effects over the processed \( k_u \)-spatial bandwidth before producing the final weighted spectral estimate.

Figure 5 shows the wavenumber data collected by the imaging system. Although the data collected are a rectangular block in \((\omega, k_u)\) space, in the \((k_u, k_v)\) wavenumber space of the image, the collected data correspond to a curved surface (the curvature of this surface is overemphasized in the figure). In Figure 5, the collected \((\omega, k_u)\) data are represented by heavy dots at radii \(2k_u\), height \(k_v\). To generate the image estimate from these data the inverse FFT requires samples on a rectangular grid. To obtain samples on a rectangular grid requires the use of a precise interpolator. The combination of the redefinition of the coordinate system from \((\omega, k_u)\) to \((k_u, k_v)\) given in Equation (24) and the use of an interpolator to obtain samples on a rectangular grid is the essence of the Stolt map.

What are the disadvantages of the wavenumber algorithm? Despite the apparent elegance of the mathematics, the wavenumber algorithm suffers more or less a similar disadvantage compared to that of the range/Doppler algorithm in that there is a need to interpolate from one 2D sampling grid to another: specifically, the Stolt mapping \( \delta (\cdot) \). To ensure that this does not inject errors into the reconstruction process, an interpolation kernel with a large number of terms is needed, thus losing any gains that might have accrued from the block processing of the data. What is needed is some way of replacing the interpolation process by multiplication in some other domain. This lead to the invention (in two places almost simultaneously \([25,26]\)) of what has become known as the chirp-scaling algorithm.

D. Chirp-Scaling Algorithm. The rather clever trick that is the substance of this algorithm is to recognize that the equivalent of the \(t\) and \(k_u\)-dependent time advance in the range/Doppler algorithm implicit in \( \delta (\cdot) \) can be accomplished by a phase multiplication of the uncompressed range/Doppler domain with the proviso that the transmitted pulse must be linear FM (alternatively, the raw data can be manipulated to have an LFM structure \([49]\)). Thus, rather than starting with the baseband compressed echoes \( ss_b(t, u) \), the chirp-scaling algorithm starts with the chirped echoes \( ee_b(t, u) \). There is a slight storage overhead incurred, as \( ee_b(t, u) \), which exists for \( t \in [0, \tau_{\text{rep}} + \tau_r] \), is larger that \( ss_b(t, u) \), which exists for \( t \in [0, \tau_{\text{rep}}] \), by the length of the pulse \( \tau_r \).

The algorithm (which is far from obvious) starts by moving the uncompressed baseband echo data into the range/Doppler domain, so

\[
eE_b(t, k_u) = \hat{\delta}_t \{ eE_b(t, u) \}
\]

(33)

The chirp rate in each LFM echo in \( eE_b(t, k_u) \) is now modified with respect to \( k_u \) so that the phase centers of the uncompressed echoes follow a common range curvature, usually that of \( r_0 \). It should be noted that the envelopes of the uncompressed echoes still follow their original range curvature; however, as we pulse-compress with the phase-only part of \( P_m^*(\omega) \), there is no loss of signal strength caused by this difference between the phase and envelope centers. We calculate a new spatial wavenumber/temporal frequency domain \( MM'_b(\omega, k_u) \), where

\[
MM'_b(\omega, k_u) = \hat{\delta}_t \{ eE_b(t, k_u) \cdot \phi(t, k_u) \}
\]

(34)

where the chirp-scaling multiplier is given by \([49]\)

\[
\phi(t, k_u) = \exp \{ j\pi k_u \cdot C(k_u) \cdot [t - t_0(k_u)] \}
\]
where $K_s$ is the LFM chirp rate in Hertz per second, the curvature factor $C(k_r)$ is as defined in Equation (21), and the wavenumber-dependent reference time $t_0(k_r)$ is calculated for the reference range $r_0$ as

$$t_0(k_r) = \frac{2}{c} r_0 [1 + C(k_r)].$$

If secondary range compression (SRC) is required for broad-beamwidth, broad-bandwidth, or highly squinted systems, the LFM chirp rate $K_s$ must be replaced by $K_s(k_r)$, a $k_r$-modified chirp rate, calculated quite simply in the following way [49]:

$$K_{src}(k_r) = \frac{8\pi r_0}{c^2} \frac{k_r^2}{(4k_0^2 - k_r^2)^{3/2}}$$

$$K_s(k_r) = \frac{1}{1/K_s - K_{src}(k_r)}$$

The notation in Equation (34) requires clarification. To avoid confusion between modulated functional notation and baseband notation, we denote the frequency ordinate of a baseband function and. If the original signal has a bandwidth of $B_c$ (Hz) by $\omega_r \in [-\pi B_c, \pi B_c]$ and the modulated frequencies by $\omega \in \omega_r + \omega_b$. These frequencies then have the respective baseband and modulated wavenumbers $k_b = \omega_b/c$ and $k = \omega/c$. The distinction between the baseband and modulated system models is critical when determining the appropriate phase functions in the various reconstruction algorithms and when making a comparison between these algorithms. The prime in Equation (34) exists owing to the use of the FFT as explained in Section V.C.

Now everything in $MM'_s(\omega_r, k_r)$ is in a point spread--invariant form, so we can do both bulk range correction (removing the $r_0$ range curvature of the phase centers) and pulse compression by two 2D phase multiplications followed by a 1D Fourier transform back into the range Doppler domain for some touching up of the phase to remove phase residuals injected by the prior steps.

$$\hat{\tilde{F}}_s(x', k_r) = \mathcal{F}^{-1}_{k_r} \{ W(k_r, k_r) \cdot \mathcal{F}\{ MM'_s(\omega_r, k_r) \cdot \Theta(\omega_r, k_r) \} \}$$

$$\times \psi(x, k_r)$$

where the phase-only pulse-compression function is given by

$$\Theta(\omega_r, k_r)$$

$$= \exp \left\{ j \frac{\omega_r^2}{4\pi K_s(k_r)[1 + C(k_r)]} \right\} \exp \left\{ j2k_r r_0 C(k_r) \right\}$$

(Note the appearance of the baseband terms in the phase, not modulated terms.) In Equation (36), pulse compression [including SRC, since we are using the modified form of the chirp rate $K_s(k_r)$] is performed by the first term, and bulk range curvature correction is performed by the second term. The mapping $\mathcal{F}\{ \cdot \}$ is a simple rescaling given by $k_c(\omega_r, k_r) = 2k_b$ and $k_c(t, k_r) = k_r$, and $W(k_r, k_r)$ is a suitably scaled 2D window function (usually made from two 1D window functions). The along-track signal is then compressed and a phase-residual term removed by

$$\psi(x, k_r) = \exp \left\{ j(4k_0^2 - k_r^2 - 2k_0) \cdot x \right\}$$

$$\times \exp \left\{ -j \frac{4\pi}{c} K_s(k_r)[1 + C(k_r)][x - r_0]^2 \right\}$$

The resulting signal in Equation (35) is then inverse-transformed in $k_r$ to yield the image estimate $\tilde{f}_f(x', y)$.

In summary, then, the chirp-scaling algorithm is far from obvious, but it has one significant advantage. There is no grid-to-grid interpolation. As a consequence, it is somewhat faster in comparison to any other reconstruction process, and for that alone, it is likely to become the inversion algorithm of choice for many years.

VI. RANGE AND AZIMUTH SAMPLING FOR STRIP-MAP SYSTEMS

If the transmitted waveform has a maximum frequency of $f_{max}$, then sampling theory tells us we need to sample this real waveform at least as frequently as $2f_{max}$ to sample the signal without aliasing. This may be an option for a sonar waveform, but radar carrier frequencies are usually in the GHz region, so it is usual to complex demodulate (using $I$ and $Q$ channels) down to baseband. If the original signal has a bandwidth of $B_c$, the appropriate sampling rate is $B_c$ or higher in each of the $I$ and $Q$ channels giving an overall sampling rate of $2B_c$ with the complex baseband covering $\pm B_c/2$. (A useful rule of thumb has the overall sampling rate at 2.65 $B_c$, which nicely accounts for realistic filter roll-offs, etc.) This is known as the “fast time” or range-sampling requirement.

As the platform moves down the track which defines the aperture, each pulse echo samples the echo field. Using a real aperture of extent $D$, it is common practice to sample at spacings of $D/2$ to achieve azimuth resolution of $D/2$ [53]. Unfortunately, this $D/2$ sampling spacing is still insufficient to avoid artifacts in azimuth, and these can be seen in some SAR images of high-contrast features surrounded by low-contrast backgrounds such as bridges over water [3, pp. 299–300]. It was shown by Hawkins [49] that it is necessary to sample at aperture spacings of $D/4$ to reduce the artifacts of aliasing to a level where they are imperceptible in an image with $D/2$ azimuth resolution. In this article, we refer to sampling at $D/4$ as appropriate sampling. Any finer sample spacing is considered oversampling and any coarser sample spacing is considered undersampling. Unfortunately, undersampling is the more common situation for any sonar system. How its effects can be minimized and under what conditions are the subjects of the next section.

VII. IMAGE RECOVERY WITH UNDERSampled APERTURES: DIGITAL SPOTLIGHTING IN STRIP-MAP SYSTEMS

Up till now, we have tacitly implied in the mathematics that $e_{esn}(t, u)$ and $s_{sn}(t, u)$ (or their baseband versions) are known continuously along both axes. Although this could conceivably be true along the delay time axis $t$ (for instance, we could have used analogue delay lines to produce $s_{sn}(t, u)$ from $e_{esn}(t, u)$), it is not true and never can be true along the aperture axis $u$. We can make the sampling nature of the $u$ axis more specific by stating

$$s_{sn}(t, u) = 0 \quad u \neq m\Delta_u$$

(37)
where \( m \) is an integer [the argument in this section applies equally to \( e_n(t, u) \) or the baseband versions]. Provided \( \Delta_x \) is small enough (i.e., \( \Delta_x = \delta y_{ab}/2 \)), the sampling along the aperture \( u \) is adequate to meet the spatial sampling criterion (actually, this sample spacing still aliases the along-track signal; however, the aliased energy is that energy that resides in the side lobes of the aperture pattern. If adequate shading of the aperture is employed, then this alias level is acceptable). Thus, \( s_{m}(t, k_u) \) and \( SS_m(\omega, k_u) \) are aliased in the spatial direction \( u \), and we can follow any one of the usual algorithmic paths to calculate \( FF(k_u, k_u) \) and so reconstruct an estimate of the diffraction-limited \( \Phi \).

However, when we travel too fast \( \Delta_x > D/4 = \delta y_{ab}/2 \), the spatial sampling criterion is not met (i.e., energy from the aperture main lobe begins to alias) and spatial undersampling in the \( u \) direction occurs. Although \( ss_m(t, u) = 0 \) for \( u \neq m\Delta_x \) is still a correct statement, \( s_{m}(t, k_u) \) and \( SS_m(\omega, k_u) \) are aliased in the \( k_u \) direction, and so \( ff(x, y) \) is corrupt by grating-lobe artifacts if we use any of the normal reconstruction algorithms.

Unfortunately, in most SAS and some SAR systems, there is a commercial or operational necessity to travel much faster than the maximum speed allowed to meet the spatial sampling criterion. In this case, the aperture is undersampled and we lose valuable information. Unless that information can be recovered using some other a priori information, there is little that can be done. However, the a priori information can often impose quite powerful constraints. Let us presuppose that an SAS is being used in a mine-hunting or a harbor clearance environment where it is known that objects of interest are limited to 4 m in extent where Note now that any one of the usual algorithmic paths to calculate \( FF(k_u, k_u) \) and so reconstruct an estimate of the diffraction-limited \( \Phi \) is small area of terrain is of interest, we can use spotlight SAR to produce a smaller area. This is an advantage if signal-to-noise is a problem.

As we know approximately where the target of interest is located, we can center the aperture to be synthesized about the position of closest approach. (We can think of this process as the equivalent of arranging the target of interest to lie close to the boresight of the synthesized aperture.) Consider now a single temporal frequency \( \omega \) in the \( (\omega, u) \) domain and recall that \( u \) is really a discrete set of samples \( m\Delta_x \). Now, \( Ss_m(\omega, u) \) has an approximately linear spatial frequency dependence in \( u \), and so at some value of \( u \), this linear FM waveform is spatially undersampled. (As noted by Rolt, this is quite noticeable in a sampled system where \( \Delta_x \) equals \( \delta y_{ab} \); however, the undersampling only effects the larger wavenumbers where the amplitudes are small, and so makes only a small contribution to \( ff(x, y) \). To reduce the effects of aliasing to the level, that they are virtually undetectable, \( \Delta_x = D/4 = \delta y_{ab}/2 \) [49].) For the small extended target on boresight, the linear spatial frequency dependence of \( u \) is approximately known and may be removed to produce a spatially compressed signal \( Ss_m^*(\omega, u) \) given by

\[
Ss_m^*(\omega, u) = Ss_m(\omega, u) \cdot \exp(j2\delta y_{ab}/4 + u^2)
\]  

where \( D/4 \) is the sampling rate we should have used in the first place. It then follows that we can calculate \( SS_m^*(\omega, u) \) from \( SS_m^*(\omega, k_u) \). Finally, the decompressed signal that we would have recorded at a higher spatial sampling rate can be calculated by reinserting the linear FM waveform we originally removed from \( Ss_m^*(\omega, u) \) in Equation (38) to obtain

\[
SS_m^*(\omega, u) = SS_m^*(\omega, u) \cdot \exp(-j2\delta y_{ab}/4 + u^2)
\]  

Note now that \( SS_m^*(\omega, u) \) is spatially upsampled versions of \( Ss_m^*(\omega, u) \) and \( ss_m(t, u) \) without the effects of the original undersampling, and we can proceed to use \( SS_m^*(\omega, k_u) \) and \( ss_m(t, u) \) in the reconstruction algorithms as if they have no aliased spatial frequencies at all and had been adequately sampled in the original \( (t, u) \) domain, i.e., if the sampling rate was indeed \( D/4 = \delta y_{ab}/2 \). In effect,

\[
Ss_m^*(\omega, u) = Ss_m(\omega, u)
\]

both sampled at the appropriate sampling rate of \( D/4 \), but of course, only for the small area of radius \( x = r_0, y = 0 \). Since the object of interest is so restricted in extent, the exact transfer function algorithm centered about the same point \( (x = r_0, y = 0) \) would probably be the best reconstruction procedure.

### VIII. SPOTLIGHT SAR IMAGING ALGORITHMS

If it is known even before the pulses are transmitted that only a small area of terrain is of interest, we can use spotlight SAR to image just that part of the object terrain [20,54]. The system geometry is shown in Figure 1(b). (To our knowledge, predetection spotlight SA techniques have been restricted to SAR and no operating SAS uses anything other than strip-map imaging.) In spotlight SAR, a large, steerable, real aperture (effective length \( D \)) that is much greater in extent than the \( D \) we used for strip-map SAR is slewed so that its footprint always stays over the same area of terrain. Since much longer real apertures are used, we get smaller beamwidths, smaller footprints, and so correspondingly much higher power densities irradiating the target area. This is an advantage if signal-to-noise is a problem.

Elementary application of the resolution criteria stated earlier
would have it that the azimuth resolution of a single-look spotlight SAR should be \( D_s/2 \), but that supposition would be incorrect. In the spotlight SAR, the azimuth resolution is proportional to the total slew angle covered, not half of the physical extent of the real aperture used. Mathematically, the azimuth resolution of the spotlight SAR \( \delta Y_{3,ab} \) is independent of \( D_s \) and given by the wavelength-dependent quantity

\[
\delta Y_{3,ab} = \frac{\lambda_s/2}{\theta_{slew}}
\]

where \( \theta_{slew} \) is the total angular change of the boresight, which is very much larger than the real beamwidth.

How we go about computing the diffraction-limited image depends on whether we use the plane wave approximation or the tomographic approximation.

**A. Plane-Wave Approximation and Its Reconstruction Algorithm.** For the plane-wave approximation to be valid, the raw image of the wavefront of the object should be negligible [19,20]. Assuming it is, then the compressed echoes in the pseudo-Fourier space given by the temporally Fourier transformed data \( S_s b(\omega_b, u) \) maps directly into the wavenumber space of the image without the need for a spatial Fourier transform. With the plane-wave assumption, the temporal spectrum of the detected echoes relates to the scene reflectivity via (further details of this derivation are found in [49]):

\[
S_s b(\omega_b, u) = \left| P_b(\omega_b) \right|^2 \exp(-j2k r_0^2 + u^2) \times \int \int \hat{f}f(x', y) \exp(-j2k \cos \theta_b x') - j2k \sin \theta_b y) dx' dy
\]

\[
= \left| P_b(\omega_b) \right|^2 \exp(-j2k r_0^2 + u^2) \cdot FF'(k_x, k_y)
\]

where \( \theta_b = \tan^{-1}(-u/r_0) \) is the aspect angle from the imaging platform position to the scene center at range \( r_0 \), the phase function represents the delay to the scene center (note the use of the modulated wavenumbers there), and \( x' = x - r_0 \) is the redefined \( x \) axis necessary for implementation of the algorithm via the FFT. The pulse-compressed spectral data map into the image wavenumber space via a polar transformation. The image spectral estimate obtained from these polar wavenumber data is

\[
\hat{FF}'(k_x, k_y) = WW(k_x, k_y) \cdot \hat{p}^{-1}\{S_s b(\omega_b, u)\}
\]

where the polar reformatting and coordinate transformation \( \hat{p}^{-1}\{\cdot\} \) is given by

\[
k_x(\omega, u) = 2k \cos \theta_b = \frac{2k r_0}{\sqrt{u^2 + r_0^2}}
\]

\[
k_y(\omega, u) = 2k \sin \theta_b = \frac{-2ku}{\sqrt{u^2 + r_0^2}}
\]

or if the data are to be shifted to \( k_b \) baseband,

\[
k_b(\omega, u) = \frac{2kr_0}{\sqrt{u^2 + r_0^2}} - 2k_0
\]

The magnitude final diffraction limited image \( |\hat{f}f(x', y)| \) is not affected by the choice of modulated or baseband wavenumber \( k_s \).

**B. Tomographic Approximation and Its Reconstruction Algorithm.** If the transmitted waveform is a narrow bandwidth LFM chirp which lasts for much longer than the transit time across the footprint, then the incoming echoes can be demodulated on the fly by a suitably delayed replica of the transmitted pulse, where this delay time \( t_0 \) tracks the center of the footprint (i.e., \( t_0 = 2R_0/c = 2r_0^2 + u^2/c \)). The output of the demodulator is now described by a baseband function

\[
S_s b(\omega_b, u) = \int \int \hat{f}f(x', y) \exp(-j2k \cos \theta_b x' - j2k \sin \theta_b y) dx' dy
\]

\[\omega_b \in [-\pi B_c, +\pi B_c] \]

which bears a striking resemblance to the expression describing an offset version of computed axial tomography [18].

The differences between this expression and that of the plane-wave approximation are that the limit to the extent of the domain in the baseband \( \omega_b \) direction must be made explicit and the phase multiplication arising from the time delay between transmission and detection is automatically compensated for within the demodulation process. As the replica is a delayed version of the chirp which is a linear ramp in frequency, this demodulation from the raw echo directly to baseband \( S_s b(\omega_b, u) \) is called deramp processing [20].

The reconstruction of the baseband-shifted offset Fourier domain from the baseband version \( S_s b(\omega_b, u) \) as recorded is now very simple.

\[
\hat{FF}'(k_x, k_y) = WW(k_x, k_y) \cdot \hat{p}^{-1}\{S_s b(\omega_b, u)\}
\]

To a very large degree, the simplicity of the reconstruction process has made the tomographic approximation very popular, but it does breakdown quite rapidly when the plane wave and/or the tomographic approximations no longer apply [19] (see [49] for the reconstruction algorithm to use in this case).

However, simplicity of the reconstruction algorithm is not the only advantage of spotlight SAR using beam steering and deramp processing. Another is that both the temporal \( (t) \) and spatial \( (u) \) sampling requirements are very much lower than for the stripmap SAR with equivalent range and azimuth resolution. If a spotlight system has a transmitted chirp bandwidth of \( B_c \) and the received echoes are just quadrature-demodulated to complex baseband (i.e., without deramp processing), it would normally be sampled at \( f_r = B_c \) to retain all the information. By using on-the-fly deramp processing before the sampling, this can be reduced to

\[
f_r = \frac{f_p}{\tau_r} B_c
\]
Figure 6. The 2D collection surface of the wavenumber data. The heavy dots indicate the locations of the raw polar samples along radii \(2k\) at angle \(\theta_1\). The underlying rectangular grid shows the format of the samples after the polar to Cartesian mapping (interpolation). The spatial bandwidths \(B_k\) and \(B_c\) outline the rectangular section of the wavenumber data that is extracted, windowed, and inverse Fourier-transformed to produce the image estimate. Note that this extracted rectangular area is much a larger proportion of the polar data in a realistic system.

where \(\tau_u\) is the transit time across the patch, and for spotlight SAR very much shorter than \(\tau_c\), the chirp time. It is quite feasible to have a 400-MHz bandwidth chirped-echo deramped and quadrature sampled by the A/D at only 1 MHz.

Recall, too, that for adequate spatial sampling, we need to send out a pulse every time we moved \(D/4\). If the real aperture is bigger, such as it is with spotlight systems, we can lower the pulse repetition frequency and cover a specific synthetic aperture with fewer samples. Instead of having to deal with, say, 10,000 pulse echoes to reconstruct a strip-map image, it may only need a few hundred pulse echoes to be processed for a spotlight image reconstruction.

For real-time and strategic SAR systems, the lower temporal and spatial sampling rates reduce the computational and data communications burden significantly and perhaps account for some of spotlight SAR’s popularity, especially for satellite systems.

Figure 6 shows the wavenumber data collected by the spotlight imaging system. This time, the rectangular block in \((\omega, k_x)\) space maps into the \((k_x, k_y)\) wavenumber space of the image as a curved surface of polar samples (again, the curvature of this surface is overemphasized in the figure). In Figure 6, the collected \((\omega, k_x)\) data are represented by heavy dots at radii \(2k\), angle \(\theta_1 = \tan^{-1}(-u/r_o)\). To generate the image estimate from these data via the inverse FFT also requires the use of an interpolator to produce samples on a rectangular grid. Though the use of an interpolator dominates the processing time in the spotlight processor, this drawback is offset by the fact that the plane-wave assumption has reduced the processing in other areas significantly. Thus, the tomographic reconstruction algorithm is still very efficient.

IX. MULTIPLE-LOOK SYNTHETIC APERTURE IMAGING

As stated earlier, the range resolution is dependent solely on transmitted bandwidth \((\delta f_{\text{trans}} = c/2B_c)\) and the azimuth resolution solely on antenna extent \((\delta f_{\text{ant}} = D/2)\). In fact, this is only true if a complete synthetic aperture’s worth of pulse echoes are processed coherently to the full extent of the bandwidth. It may even be beneficial to compromise on the range or azimuth resolution so as to improve the appearance of the image. Imaging using radar or sonar requires coherent integration of amplitudes, and just like any coherent imaging process, it suffers from speckle, where the statistics predict that the amplitude of any point (pixel) in the image has a high probability of being zero. This speckle can be reduced by averaging the intensities of several statistically independent images of the same terrain [55].

Multiple-look imaging can be approached in a general way by recognizing that in a way similar to a hologram, any portion of the offset Fourier domain can reconstruct an estimate of the image, albeit at a lower resolution than that reconstructed from the full domain. Thus, we can take as much of the offset Fourier domain \(FF(k_x, k_y)\) as we have recorded, partition it into three or four equal areas, reconstruct three or four lower-resolution images from each partition, and finally add all these together noncoherently to produce an image with much reduced speckle. How the offset Fourier domain is partitioned depends on whether it is a radar or sonar and whether it is a spotlight or strip-map system.

Because of the large ratio of the velocity of propagation to platform speed, SAR data are commonly well sampled in azimuth with small real apertures giving good azimuth resolution, but are forced to use a narrow bandwidth or large depression angles resulting in poor ground-range resolution. With a much smaller ratio between the propagation velocity to platform speed, SAS frequently has poorer azimuth resolution than range resolution. To see how the inequalities of single-look resolution are used to advantage, we take two examples. Seasat SAR has a bandwidth of 20 MHz and an antenna length of 10.7 m, so the resolution of a single-look image is about 7.5 m in slant range (about 25 m ground range at the depression angle used by Seasat) by 5.3 m in azimuth. So, in an effort to combat speckle, Seasat uses a four-look partition in azimuth and achieves 25 x 25-m ground-plane resolution in the image that has the appearance of an aerial photograph with little apparent speckle. For sonar, the Kiwi-SAS has a 20-kHz bandwidth and a 0.30-m transducer length. This gives a single-look resolution of 0.05 m in range and 0.15 m in azimuth, so it makes sense to consider a three-look partition in range wavenumber, with each partition having the full \(k_x\) bandwidth and a third of the \(k_y\) bandwidth to produce a three-look image with \(0.15 \times 0.15\)-m resolution.

Spotlight SARs usually partition the offset Fourier domain in both range and azimuth wavenumbers [20, pp. 112–121], but since there is a close relationship between aperture and Doppler wavenumber, they can also partition the actual synthetic aperture itself. That is, the full aperture of length \(2Y_o\) [Fig. 1(b)] is partitioned into four subapertures, each effectively covering a band of spatial Doppler wavenumbers. (This subaperture technique is also used to search for highly specular reflectors covered by natural foliage [56].)

However the offset Fourier domain is divided, it is clear that multiple-look processing is a useful technique that improves the visual quality of the final image.

X. CONCLUSIONS

Although much of the early SAR literature developed system models in terms of what appears to be temporal Doppler effects, the effect exploited is a geometrically induced modulation and should be less ambiguously referred to as a spatial Doppler modulation. The actual temporal Doppler effects that do occur within
pulses (and alter the modulation of the return echoes) are small enough that in all but extreme cases, they can be ignored. The ability to ignore true temporal Doppler effects allows the modeling of SA systems using a quasinonstatic model where the platform is assumed to be stationary for the duration of one pulse repetition period. This is frequently called the stop-and-hop scenario.

There are a number of reconstruction algorithms in common use. All of them take the detected echoes with their phase histories and invert them to estimate the image to the resolution limit imposed by the transmitted bandwidth and the extents of the real apertures used to transmit the signal and receive the echoes. This estimate is known as the diffraction-limited image. All of the algorithms can be put into a common notation and framework so that their specific attributes can be compared.

Of the common algorithms, the exact, the range/Doppler, and the wavenumber algorithms all require some form of interpolation between one 2D sampling grid and another. For narrow-beamwidth systems where the range migration is less than a range resolution cell, the range/Doppler algorithm can dispense with the interpolation step and so is relatively efficient under those circumstances.

Neither the exact transfer function nor the chirp-scaling algorithms need interpolation, but the former is a point variant process which needs to be recomputed for every image point. Thus, only the chirp-scaling algorithm is truly a block array process that needs no interpolation. It is also inherently phase preserving and can run backward as well as forward. Thus, the chirp-scaling algorithm can take an image and compute the raw uncompressed echoes which can then be used by other algorithms to test their efficacy.

Spotlight SAR uses quite a different reconstruction process, mostly because certain approximations inherent in the system geometry mean that the plane-wave approximation or the tomographic approximation can be used in the inversion from detected echoes to an estimate of the diffraction-limited image. A major advantage of spotlight SAR (to our knowledge, spotlight SAS has not been tried) is that the sampling requirements on the detected and demodulated echoes are much reduced over a strip-map SAR with the same resolving power. This is a significant advantage in strategic SAR and satellite SAR, where near real-time images are needed or a bandwidth-limited data link used to download the stored baseband echoes to a ground terminal.

Multilook processing is an essential adjunct to high-quality imagery, and it can involve multiple looks in spatial Doppler or temporal frequency, or, for spotlight SAR, even multiple looks in subapertures. In general, any partition of the offset Fourier domain can be the basis of a multiple-look low-speckle image. Strip-map SAR mostly uses multiple looks in Doppler wavenumber $k_d$, while strip-map SAS mostly uses multiple looks in frequency $\omega$ or range wavenumber $k_r$ (due mainly to the difference in the speed of propagation between sound in water and electromagnetic in the atmosphere), while spotlight SAR often uses a combination of both.

For most strip-map SAS systems, the almost inevitable undersampling along the aperture results in aliasing in the offset Fourier domain and in an image corrupted by artifacts. These artifacts can be removed almost completely, provided the object is of limited extent using a technique called postdetection digital spotlighting. As an object of limited extent is a common scenario for mine countermeasures, we anticipated that digital spotlighting will become a major part of the processing in a mine-hunting sonar (Reference [50] details such an application).

REFERENCES


