Restoration of a Degraded Image

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Abstract

A process of image restoration is described in this paper. A restoration concept based on trial & error method was recommended by Gonzalez [Gonzalez 2001]. An attempt is made in this work to implement this concept toward obtaining a restored image. In this concept the degraded image itself is used to develop the degradation model with the aid of a 5th order Butterworth filter at varying \( \omega_0 \) cut-off frequency, a trial & error method. The FFT of the original image will be approximately recovered by multiplying the inverse FFT of the degradation model by the FFT of the degraded image. Finally taking the inverse FFT of the results will produce an approximate recovery of the original image. In this work, a test of the algorithms necessary for the restoration is accomplished, namely, the Mean Scan filter, the FFT, and the Low Pass Butterworth filter. The final work of the restoration is still in progress.

Introduction

Restoration is a process that attempts to reconstruct or recover an image that has been degraded by a degradation phenomenon known a priori. A degraded image is artificially produced in this research using a Mean Masking filter to blur the image. Then, the degradation model is calculated by low-pass filtering the blurred image. With the degraded image and the degradation model at hand, a restored image can be obtained. As it can be seen, in the restoration process there are three important elements. This work tries to explain the relationship between the degraded image, the degradation model and the restored image. Furthermore, it will specify the technique used to acquire each one.

1. Image Restoration Theory

The image degradation process can be modeled as:

\[
g(x, y) = h(x, y) * f(x, y), \quad \text{[Eq.1]}\]

where \( g(x,y) \) is the degraded image, \( h(x,y) \) is the degradation model , and \( f(x,y) \) is the original image. The restored image can exactly be obtained from the following formula:

\[
F(u, v) = H(u, v)^{-1} G(u, v) \quad \text{[Eq.2]}\]

where \( F(u,v) \), \( H(u,v) \) and \( G(u,v) \) are Fourier transforms of original image, degradation model, and degraded image, respectively.

The degraded image, \( G(u,v) \), would be exactly the same as the degradation model, \( H(u,v) \), if the image were to be an impulse \( d(x,y) \). In this project, for any general image with most of its pixels close to the center of the image, it is assumed that the degradation...
model is, \( H(u,v) = \hat{G}(u,v) \) where \( \hat{G}(u,v) \) is obtained by applying a Butterworth Low Pass filter to \( \hat{G}(u,v) \). The \( \hat{G}(u,v) \) is obtained by artificially blurring the image \( f(x,y) \). A method to degrade the image, and the Butterworth Low Pass filter are considered next.

2. Artificially Degrading an Image

To obtain a degraded image (or blurred image) a Mean Mask filter was implemented. This process smoothes the image by reducing the edges, which represent the high frequencies.

An average low-pass filter (Mean Mask filter) uses the same concept, except it diminishes the value of the edges instead of simply eliminating them. A mask like the one shown in Figure 1

\[
\begin{bmatrix}
1/9 & 1/9 & 1/9 \\
1/9 & 1/9 & 1/9 \\
1/9 & 1/9 & 1/9 \\
\end{bmatrix}
\]

Figure 1. Mean Mask filter

is scanned over every pixel of the image. Each pixel is replaced with an average of its eight neighbors and itself. Larger masks blur the image more rapidly. Figure 2 shows the blurring effect of the Mean Mask filter.

3. FFT Applied to the Image Restoration

The two dimensional Discrete Fourier Transform on a function \( f(x,y) \) is defined as:

\[
F(u,v) = \frac{1}{MN} \sum_{x=0}^{M} \sum_{y=0}^{N} f(x,y) e^{-j2\pi(ux/M + vy/N)}
\]

[Eq.3]

and the two dimensional Inverse Discrete Fourier Transform as

\[
f(x,y) = \sum_{u=0}^{M} \sum_{v=0}^{N} F(u,v) e^{j2\pi (ux/M + vy/N)}
\]

[Eq.4]

The two-dimensional Fast Fourier Transform (FFT) simply involves a number of one-dimensional Fourier transforms. More precisely, a two dimensional transform is achieved by first transforming each row, replacing each row with its transform and then transforming each column, replacing each column with its transform. Thus a two-dimensional transform of a 1K by 1K image requires 2K one dimension transforms. This follows directly from the definition of the discrete Fourier transform of a discrete system. The transform pairs that are commonly derived in one dimension can also be derived for the two dimensional situation.

In order to perform FFT instead of the much slower DFT (Discrete Fourier Transfer), the image must be transformed so that the width and height are an integer power of two.

The Fast Fourier Transform was tested applying the procedure shown in Figure 3 to the image.

![Figure 3. FFT Block Diagram](image-url)
The following results were obtained: Figure 4 shows the original image; Figure 5 shows the original image after passing the FFT / IFFT on it. As expected, both images are the same.

![Figure 4. Original Image](image)

![Figure 5. Image after FFT/ IFFT](image)

**Figure 4. Original Image**

**Figure 5. Image after FFT/ IFFT**

4. **Two Dimensional Butterworth Filter**

[Mitra 2001]

A low-pass Butterworth filter can be designed to help in the recuperation of the degradation model. Instead of eliminating the high frequencies above a cut off frequency, like the ideal low pass filter, it keeps some of the high frequency information which we need to keep. Figure 6 shows the behavior of a low-pass Butterworth filter of order one.

![Figure 6. Lowpass Butterworth Filter](image)

5. **Results**

The image restoration techniques are oriented toward assuming a degradation model. In this project no degradation model was assumed. The trial & error results of restoration are pending.

6. **Future Work**

The results of trial and error method will be compared with other methods.

7. **References**


Eq.5 shows the general equation for the \( n \)th order discrete Butterworth low-pass filter.

\[
H(u,v) = \frac{1}{1 + \left(\frac{u^2 + v^2}{\omega_0^2}\right)^n}
\]  [Eq.5]

The \((u,v)\) represents the spatial position on the image, \( \omega_0 \) is the cutoff frequency, or in the spatial sense, the radius at which the values thereafter would decrease, and \( n \) is the order of the filter. The greater the value of \( n \), the closer the filter will behave to an ideal filter. Figure 7 shows how a five-order filter at \( \omega_0 = 200 \) affects the image.

![Figure 7. 5th Order Butterworth Filter at \( \omega_0=200 \) applied to the image](image)