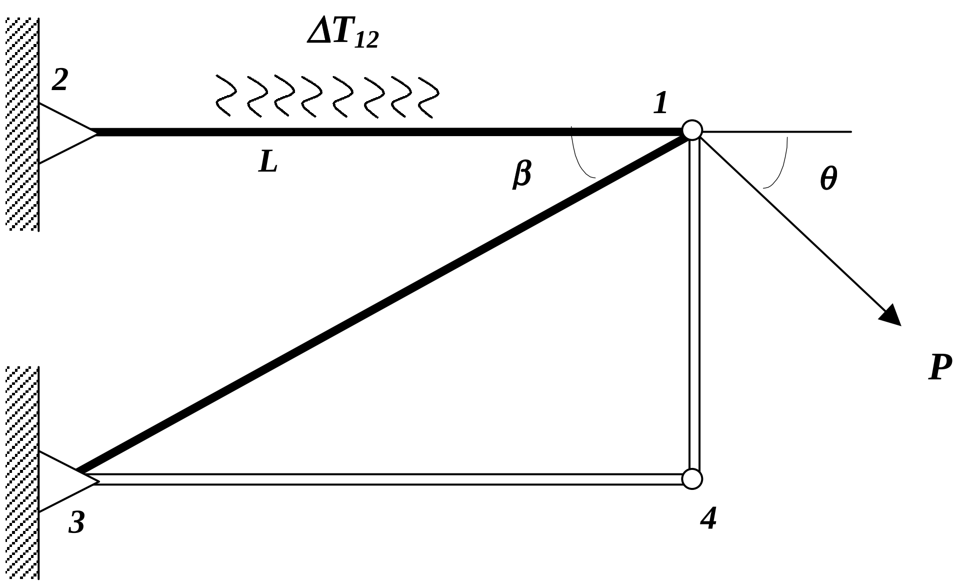
The structure shown below has two rigid bars and two elastic bars connected by smooth pins. The elastic bars 2-1 and 3-1 have the same Modulus of Elasticity E. The bar 3-1 has twice the area of bar 2-1, which has an area of A. Bar 2-1 is subject to a thermal change of ΔT_{12}. Determine:

1. Horizontal and vertical displacements at node 1 by using:
   (a) Principle of Virtual Work,
   (b) Castigliano's First Theorem.
2. Member forces
3. Magnitude and direction of the reaction force at node 3



The following is known:

$$
\begin{aligned}
E &= 10 \times 10^6 psi \\
A &= 1.0 in^2 \\
L &= 100 in \\
\beta &= 30^\circ \\
\theta &= 45^\circ \\
\Delta T_{12} &= -100^\circ F \\
\alpha &= 13 \times 10^{-6} \frac{1}{^\circ F} \\
P &= 10000 lbs
\end{aligned}
$$

GENERAL INFORMATION

Consider using the following table in solving problem with truss bar structures:

Bars	α_{ij}	$\cos\alpha_{ij}$	$\sin\alpha_{ij}$	L_{ij}	$E_{ij}A_{ij}$	$k_{ij} = \frac{E_{ij}\,A_{ij}}{L_{ij}}$	$N^t_{ij} = (E\,A\,\alpha\,\Delta T)_{ij}$	$u_{ij} = q_{2j-1} - q_{2i-1}$	$v_{ij} = q_{2j} - q_{2i}$
2-1									
3-1									
3-4									
4-1									

I. DEFLECTIONS USING PRINCIPLE OF VIRTUAL WORK

Bars	$s_{ij} = u_{ij}\cos\alpha_{ij} + v_{ij}\sin\alpha_{ij}$	δs_{ij}	$N_{ij} = k_{ij}\,s_{ij} - N^t_{ij}$
2-1			
3-1			

Bars	$N_{ij}\delta s_{ij}$
2-1	$13000\delta q_1 + 100000 q_1\delta q_1$
3-1	$129904 q_1\delta q_1 + 75000 q_2\delta q_1 + 75000 q_1\delta q_2 + 43301.3 q_2\delta q_2$

$$\delta W_{int} = \sum N_{ij}\, \delta\, s_{ij} = 13000\, \delta q_1 + 229904\, q_1\, \delta q_1 + 75000\, q_2\, \delta q_1 + 75000\, q_1\, \delta q_2 + 43301.3\, q_2\, \delta q_2$$

$$\delta W_{ext} = P \cos\theta \delta q_1 - P \sin\theta \delta q_2 = 7071.07 \delta q_1 - 7071.07 \delta q_2$$

$$\begin{aligned}\delta W_{ext} - \delta W_{int} &= 0 \\ &= -5928.93\, \delta q_1 - 229904\, q_1\, \delta q_1 - 75000\, q_2\, \delta q_1 - 7071.07\, \delta q_2 - 75000\, q_1\, \delta q_2 - 43301.3\, q_2\, \delta q_2 \\ &= -(5928.93 + 229904\, q_1 + 75000\, q_2)\, \delta q_1 - (7071.07 + 75000\, q_1 + 43301.3\, q_2)\, \delta q_2 = 0\end{aligned}$$

II. DEFLECTIONS USING CASTIGLIANO'S FIRST THEOREM

Bars	$s_{ij} = u_{ij} \cos\alpha_{ij} + v_{ij} \sin\alpha_{ij}$	$\frac{k_{ij}\, s_{ij}^2}{2}$	$N_{ij}^t s_{ij}$
2-1			
3-1			

Bars	$U_{ij} = \frac{1}{2} k_{ij}\, s_{ij}^2 - N_{ij}^t s_{ij}$
2-1	
3-1	

$$U = \sum U_{ij} = 13000\, q_1 + 50000\, {q_1}^2 + 50000\, \sqrt{3} \left(\frac{\sqrt{3}\, q_1}{2} + \frac{q_2}{2} \right)^2$$

$$Q_1 = 10000 \cos 45^\circ = 7071.07\, lbs$$

$$Q_2 = -10000 \sin 45^\circ = -7071.07\, lbs$$

III. FORCE MEMBERS

N_{ij} is the internal force acting on each bar. Therefore,

$$N_{21} = 13000 + 100000\,q_1 = 13000 + 100000\,(0.0631852) = 19318.5\,lbs$$

$$N_{31} = 150000\,q_1 + 86602.5\,q_2 = 150000\,(0.0631852) + 86602.5\,(-0.272739) = -14142.1\,lbs$$

IV. REACTION FORCE

Note that Q_5 denotes the horizontal component of the reaction at node 3, positive right, and Q_6 denotes the vertical component of the reaction at node 3, positive up. The axial force in bar 3-1 acts on the free body diagram of node 3, and note that the force is assumed tensile in the equations below.

$Q_5 + N_{31}\cos\alpha_{31} = 0$
$Q_5 + (-14142.1\,lbs)\cos 30° = 0$
$Q_5 = 12247.4\,lbs$

$Q_6 + N_{31}\sin\alpha_{31} = 0$
$Q_6 + (-14142.1\,lbs)\sin 30° = 0$
$Q_6 = 7071.07\,lbs$

Therefore,
$\overrightarrow{R_3} = 12247.4\boldsymbol{i} + 7071.07\boldsymbol{j}\ lbs$