INEL 3105 Lecture #20

Note Title 11/18/2009

Cap 10: Análisis de potencia

Régimen permanente cosenoseidal

potencia instantánea $= p(t) = v(t)\, i(t)$

$v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = I_m \cos(\omega t + \theta_i)$; $p(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$

identidad trigonométrica.

$\cos\phi_1 \cos\phi_2 = \frac{1}{2}\cos(\phi_1 + \phi_2) + \frac{1}{2}\cos(\phi_1 - \phi_2)$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

valor promedio

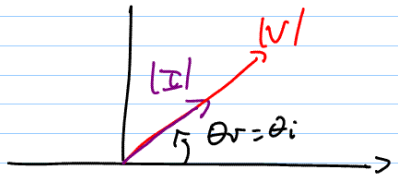
$$\text{potencia promedio} = P = \frac{\int_0^T p(t)\,dt}{\int_0^T dt}$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$\int_0^T \cos(\ldots)\,dt = 0$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Casos: ① Resistencia:



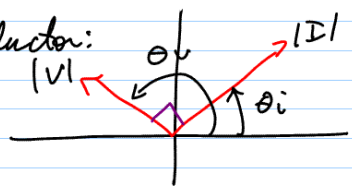
$\hat{I} = I_m \angle\theta_i$

$\hat{V} = V_m \angle\theta_v$

$V_m = R I_m$; $\theta_v = \theta_i$

$$P = \frac{1}{2} V_m I_m \cos 0^\circ = \frac{1}{2} V_m I_m = \frac{1}{2} R I_m^2 = \frac{1}{2}\frac{V_m^2}{R}$$

② Inductor:



$\hat{V} = Z_L \hat{I}$ $\quad Z_L = \omega L \angle 90^\circ$

$\hat{V} = V_m \angle\theta_v = \omega L \angle 90^\circ\, I_m \angle\theta_i$

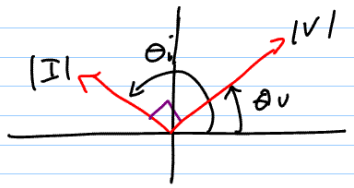
$\hat{I} = I_m \angle\theta_i$

$\theta_v = \theta_i + 90^\circ$

$V_m = \omega L\, I_m$

$P = \frac{1}{2} V_m I_m \cos 90^\circ$; $P = 0$

(3) Condensador:



$\hat{V} = Z_c \hat{I}$ $\quad Z_c = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ$

$\hat{V} = V_m \angle \theta_v$

$\hat{I} = I_m \angle \theta_i$

$\hat{V} = V_m \angle \theta_v = \frac{1}{\omega C} \angle -90^\circ \; I_m \angle \theta_i$

$= \underbrace{\frac{I_m}{\omega C}}_{V_m} \angle (\theta_i - 90^\circ)$ $\rightarrow \theta_v$

$P = \frac{1}{2} V_m I_m \cos(-90^\circ) = 0$

(4) Impedancia (en general)

$\hat{V} = Z \hat{I}$

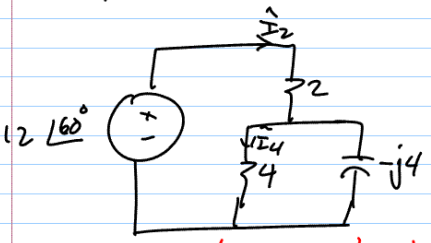
$\hat{V} = V_m \angle \theta_v$

$\hat{I} = I_m \angle \theta_i$

$Z = \frac{\hat{V}}{\hat{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i}$

$Z = |Z| \angle \theta_Z = \frac{V_m}{I_m} \angle \theta_v - \theta_i$ $\qquad P = \frac{1}{2} V_m I_m \cos \theta_Z$

Ejemplo: Determine $P_{2\Omega}$ y $P_{4\Omega}$



$\hat{I}_4 = I_2 \left(\frac{-j4}{4-j4} \right)$

$\frac{4(-j4)}{4-j4}$; $I_2 = \frac{12 \angle 60^\circ}{2 + \frac{-j16}{4-j4}}$ ... $I_2 = \underbrace{2.68}_{I_{2m}} \angle \theta_{i2}$ $\quad \theta_{i2} = 86.5^\circ$

$P_{2\Omega} = \frac{1}{2} I_{2m}^2 R = 7.20\ W$

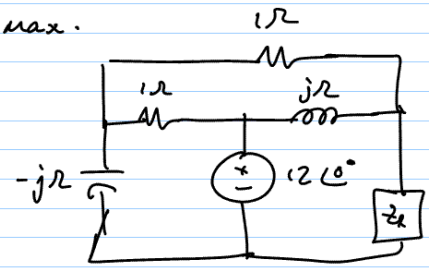
$I_4 = \frac{(.16 + j2.67)(-j4)}{4-j4} = \ldots = I_4 = ?$; $P_{4\Omega} = 7.2\ W$

$P_{demandada} = P_{2\Omega} + P_{4\Omega} = 14.4\,W$

$P_{suplida} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(12)(2.68)\cos(60 - 86.5) \ldots$

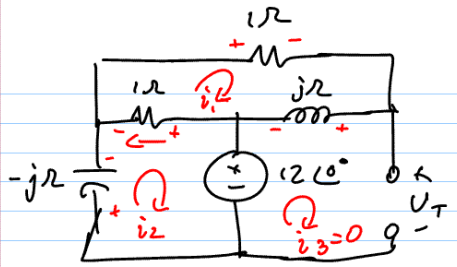
$= 14.39\,W.$

Ejemplo: Evalúe P_{max}.



Sol. Es conveniente reducir el ckto usando el equivalente de Thevenin

Evaluar V_T:



Método de mallas:

Ec. de @ 1: $1(i_1 - i_2) + 1 i_1 = -j i_2$

KVL @ 2: $(-j) i_2 - 1(i_1 - i_2) + 12\angle 0° = 0$

Re-writing eqs: $i_1(2+j) + i_2(-1) = 0$;

$i_1(-1) + i_2(1-j) = -12$

$i_2 = i_1(2+j) \Rightarrow -i_1 + i_1(2+j)(1-j) = -12$

$-i_1 + i_1(2 - j + 1) = -12$

$-i_1 + i_1(3 - j) = -12$

$i_1(-1 + 3 - j) = -12$; $i_1 = \frac{-12}{2-j}$

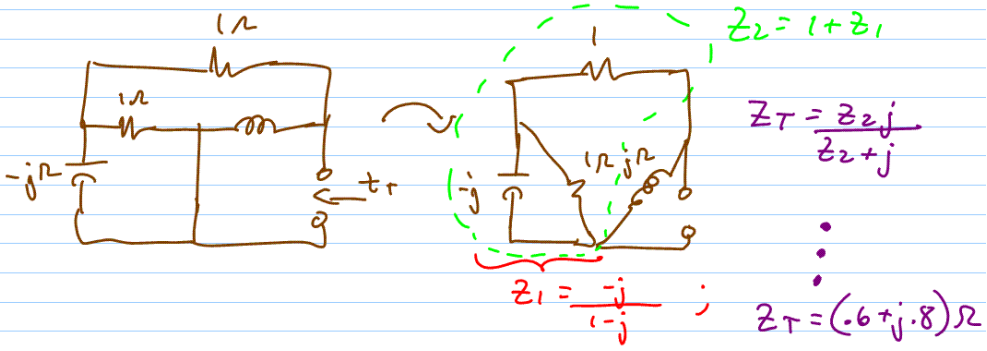
$\ldots \boxed{i_1 = \frac{-24 - j12}{5}}$

$\oint$ e.d.k @ i_3 : $12 + j(i_1 - i_3) - V_1(t) = 0$

$v_T(t) = 12 + j i_1$

$= 12 + j\left(\frac{-12}{2-j}\right) = \ldots \quad \hat{V}_T = \frac{24}{5}(3-j)$

② Eval Z_T:

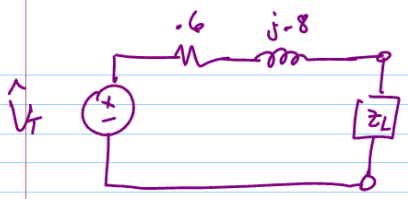


$Z_T = \frac{Z_2 j}{Z_2 + j}$

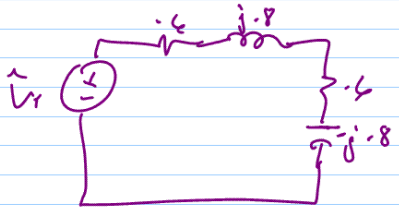
$Z_T = (.6 + j.8)\Omega$

For getting $P_{L\max}$: $Z_L = Z_T^* = (.6 - j.8)\Omega$

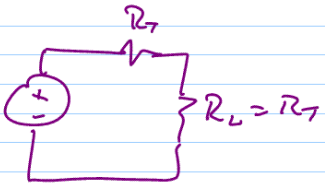
$R_L = .6 \quad X_L = -j.8$



$Z_L = .6 - j.8$



$\Rightarrow$



$P_{L\max} = \frac{1}{8}\frac{|V_T|^2}{R_T}$

$P_{L\max} = \frac{1}{8}\frac{\left(\frac{24}{5}\right)^2 \cdot 10}{.6} = \ldots 48\,W$