INEL 3105 lecture #19.

Note Title 11/10/2009

(3) Condensadores: $i_c(t) = C\frac{dv_c(t)}{dt}$

$i_c(t) = I_m e^{j(\omega t+\theta_i)}$

$v_c(t) = V_m e^{j(\omega t+\theta_v)}$

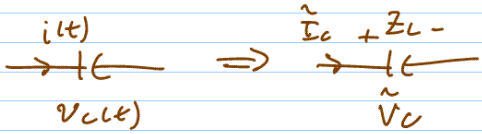
$I_m e^{j\omega t} e^{j\theta_i} = C\frac{d}{dt}\left[V_m e^{j\omega t} e^{j\theta_v}\right] = C V_m e^{j\theta_v} j\omega e^{j\omega t}$

$I_m e^{j\theta_i} = j\omega C V_m e^{j\theta_v}$

$I_m e^{j\theta_i}$: fasor de i_c ; $V_m e^{j\theta_v}$: fasor de v_c

$\hat{I}_c = j\omega C \hat{V}_c \Rightarrow \boxed{Z_c = \frac{1}{j\omega C}}$ IMPEDANCIA CAPACITIVA.

$\hat{V}_c = Z_c \hat{I}_c$

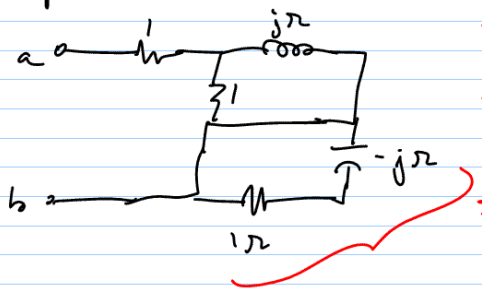


$I_m e^{j\theta_i} = j\omega C V_m e^{j\theta_v}$

$= \omega C V_m [1 e^{j\pi/2}] e^{j\theta_v} = \underbrace{\omega C V_m}_{I_m = \omega C V_m} e^{j(\theta_v + \pi/2)}$

$\theta_i = \theta_v + \pi/2$

Ejemplo: Evaluar V_{ab}



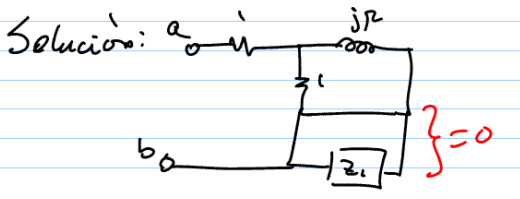
$Z_L = j\omega L$; $\omega L = 1$

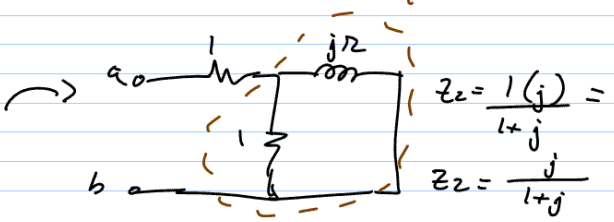
$Z_C = -j\left(\frac{1}{\omega C}\right)$; $(\omega C)^{-1} = 1$

$Z_1 = 1 - j$

Paralelo

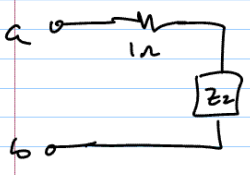
Solución:





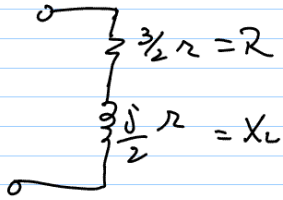
$Z_2 = \frac{1(j)}{1+j} = \frac{j}{1+j}$

$$Z_2 = \frac{j}{1+j}\left(\frac{1-j}{1-j}\right) = \frac{j-j^2}{1-j^2} = \frac{1+j}{2}$$

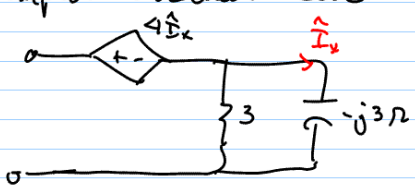


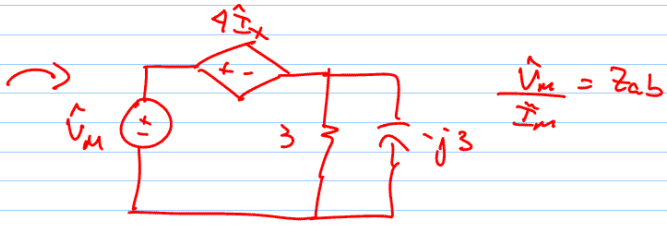
$$Z_{ab} = Z_2 + 1 = \frac{1}{2} + \frac{j}{2} + 1 = \frac{3}{2} + \frac{j}{2} = Z_{ab}$$

Circuito equivalente:



Ejemplo: Evaluar Z_{ab}





KCL: $\hat{I}_M - \hat{I}_3 - I_x = 0$; $\hat{I}_3 = \frac{\hat{V}_3}{3\Omega}$; $\hat{I}_x = \frac{V_3}{-3j}$

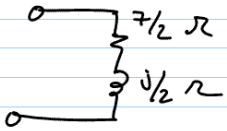
auxiliar: KVL: $\hat{V}_M - 4\hat{I}_x - \hat{V}_3 = 0$; $I_m = \frac{\hat{V}_3}{3} + \frac{V_3}{-3j}$;

$$V_3 = \frac{\hat{I}_m}{\frac{1}{3} - \frac{1}{3j}}$$

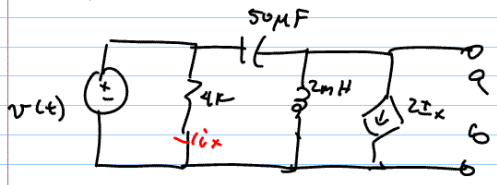
$$V_M = 4\left(\frac{\hat{V}_3}{-3j}\right) + \hat{V}_3 = \hat{V}_3\left(1 - \frac{4}{3j}\right) \; ; \; \hat{V}_3 = \frac{\hat{V}_M}{1 - \frac{4}{3j}}$$

$$\frac{\hat{V}_M}{\hat{I}_M} = Z_{ab} = \frac{\left(1 - \frac{4}{3j}\right)}{\left(\frac{1}{3} - \frac{1}{3j}\right)} = \frac{3 - 4/j}{1 - \frac{1}{j}} = \ldots = \frac{7}{2} + j/2$$

Circuito equivalente:



Ejemplo: Evaluar V_{ab}:

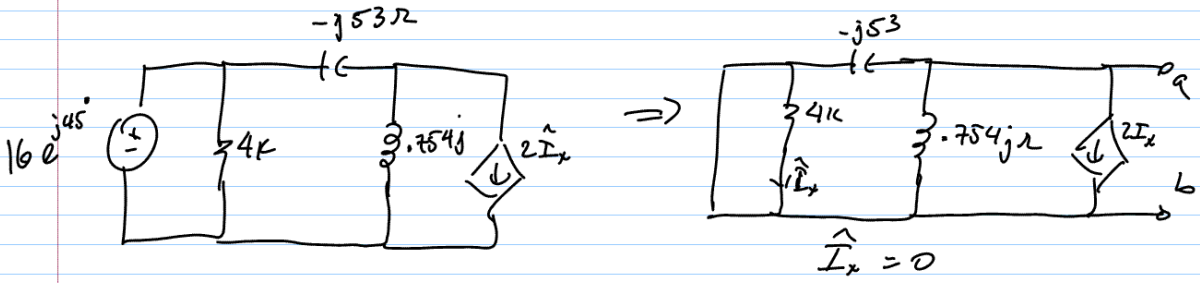


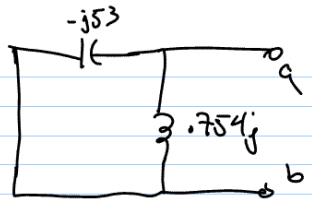
$v(t) = 16\cos(377t + 45°)\,V$

$f = ?$ $\quad \omega = 377 = 2\pi f$; $f = 60\,Hz$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{377(50\mu F)} = -j\,53\,\Omega$$

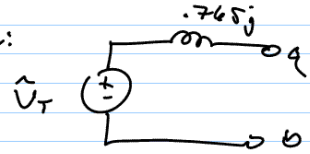
$$Z_L = j\omega L = j(377)(2\times10^{-3}) = .754j$$



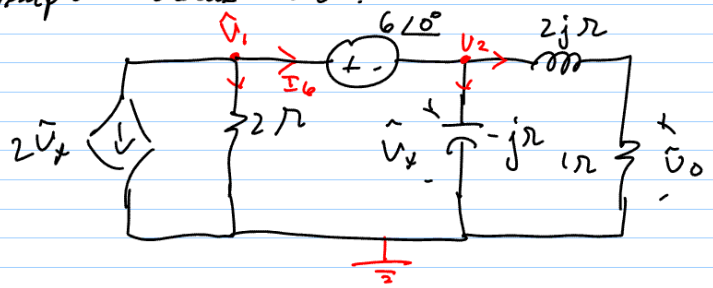


$$Z_{ab} = \frac{-53j\,(.754j)}{-53j + .754j}\,\Omega = j\cdot.765\,\Omega$$

Circuito equivalente:



Ejemplo: Evalúe $\hat{V}_o$.



ANALISIS POR NUDOS:

Solución: KCL @ $\hat{V}_1$: $2\hat{V}_x + \frac{\hat{V}_1}{2} + I_6 = 0$

KCL @ $\hat{V}_2$: $I_6 - \frac{\hat{V}_2}{-j\Omega} - \frac{\hat{V}_2}{1+j2\Omega} = 0$

además: $\hat{V}_x = V_2$; $\hat{V}_1 - \hat{V}_2 = 6\angle 0° = 6e^{j0°}$

$\hat{V}_0 = \frac{V_2}{1+2j} \cdot 1\Omega$

Going back to KCL @ $\hat{V}_1$: $2\hat{V}_2 + \frac{\hat{V}_1}{2} + \frac{\hat{V}_2}{-j} + \frac{\hat{V}_2}{1+2j} = 0$

re-writing: $\hat{V}_2\left(2 + \frac{1}{-j} + \frac{1}{1+2j}\right) = -\frac{1}{2}\left(\hat{V}_2 + 6\right)$

$\hat{V}_2\left[2 + j + \frac{1}{1+2j}\right] = \frac{-1}{2}\left(\hat{V}_2 + 6\right)$

$\hat{V}_2\left[2 + j + \frac{1}{1+2j}\right] = -3$; $\hat{V}_2 = \frac{-3}{\frac{5}{2} + j + \frac{1}{2+j}} = \frac{-3}{\frac{5}{2} + j + \frac{1-2j}{5}} = \frac{-3}{2.7 + .6j}$

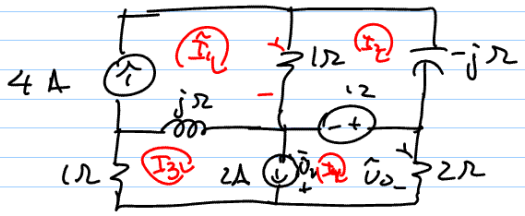
$\hat{V}_0 = \frac{\frac{-3}{2.7 + .6j}}{1+2j} = \frac{-3}{(1+2j)(2.7+.6j)} = \frac{-3}{(2.7 + .6j + 5.4j - 1.2)}$

$\hat{V}_0 = \frac{-3}{1.5 + 6j} = \frac{-3(1.5 - 6j)}{2.25 + 36} = \frac{-4.5 + 18j}{38.25}$;

$\hat{V}_0 = -.11 + .47j = .48\angle 104°$ V

Otro ejemplo:

Evalúe $\hat{V}_0$:



Método: Mallas. $\hat{I}_1 = 4\,A$

KVL @ $\hat{I}_2$; $1\Omega(\hat{I}_1 - \hat{I}_2) - \hat{I}_2(-j\Omega) - 12 = 0$

$\hat{I}_1(1\Omega) + \hat{I}_2(-1+j\Omega) = 12$; $\hat{I}_2 = \frac{12-4}{-1+j\Omega} = \frac{8}{-1+j\Omega}$

$\hat{I}_2 = \frac{8}{-1+j\Omega}\left(\frac{-1-j\Omega}{-1-j\Omega}\right) = \frac{-8-8j}{1+1} = \frac{-8-8j}{2} = -4-4j = \hat{I}_2$

KVL @ I_3 ; $1\Omega(I_3) + 0 - \hat{V}_2 = -j(\hat{I}_3 - 4A)$

Suma de voltajes ~~KVL~~ ~~@~~ ~~I_3~~ ~~;~~ ~~$\sum \vec{E} \cdot \vec{dl}$~~

KVL @ I_4: $\hat{I}_4(2) - 12 + \hat{V}_2 = 0$; auxiliares: $\hat{I}_3 - \hat{I}_4 = 2\,A$, $\hat{V}_0 = 2\hat{I}_4$

$\hat{V}_2 = 12 - 2\hat{I}_4$

Reemplazo $\hat{I}_3(1+j) - j4 = 12 - 2\hat{I}_4$

$(2+\hat{I}_4)(1+j) - 4j + 2\hat{I}_4 = 12$

$(2+2j) + I_4(1+j+2) = 12 + j4$

$I_4 = \frac{10+j2}{3+j} = \frac{(10+j2)(3-j)}{10} = \frac{30+6j-10j+2}{10}$

$= \frac{32-4j}{10} = 3.2 - .4j = 3.22\angle -7.12^\circ\ A$

$\hat{V}_0 = 6.44\angle -7.12^\circ\ V$