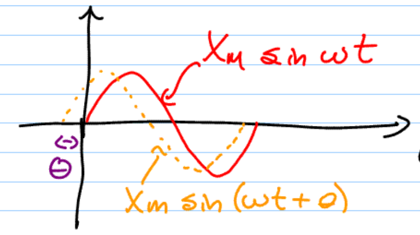
INEL 3105 Lecture #17

Note Title 11/4/2009

Fasores:



Θ = phase angle

We say that $X_m \sin \omega t$ lags $X_m \sin(\omega t + \theta)$ by θ radians.

If $x_1(t) = X_{m1} \sin(\omega t + \theta)$
$x_2(t) = X_{m2} \sin(\omega t + \phi)$

We say X_1 leads X_2 by $\theta - \phi$ radians or X_2 lags X_1 by $\theta - \phi$ radians ; if $\theta = \phi \Rightarrow$ señales en fase.

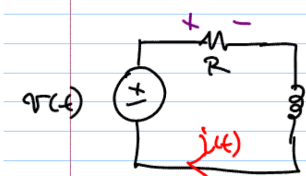
if $\theta \neq \phi \Rightarrow$ señales fuera de fase.

$$x(t) = X_m \sin(\omega t + \pi/2) = X_m \sin(\omega t + 90^\circ)$$

identidades:

$$\cos \omega t = \sin(\omega t + \pi/2)$$
$$\sin \omega t = \cos(\omega t - \pi/2)$$
$$-\cos \omega t = \cos(\omega t \pm 180^\circ)$$
$$-\sin \omega t = \sin(\omega t \pm 180^\circ)$$
$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$
$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

Ejemplo:



Evaluar $i(t)$ en régimen permanente

$$v(t) = V_m \cos \omega t$$

Solución: $\oint \vec{E} \cdot d\vec{l} = -L \frac{di(t)}{dt}$; $R\,i(t) + 0 - v(t) = -L \frac{di(t)}{dt}$

$$L \frac{di(t)}{dt} + R\,i(t) = v(t) = V_m \cos \omega t$$

La solución a esta ecuación diferencial tiene la forma:

$$i(t) = A_1 \cos \omega t + A_2 \sin \omega t$$

$$V_M \cos \omega t = L \frac{d}{dt}\left[A_1 \cos \omega t + A_2 \sin \omega t\right] + R\left[A_1 \cos \omega t + A_2 \sin \omega t\right]$$

$$V_M \cos \omega t = -L A_1 \omega \sin \omega t + L A_2 \omega \cos \omega t + R A_1 \cos \omega t + R A_2 \sin \omega t$$

$$\underbrace{(L\omega A_2 + R A_1)}_{V_M} \cos \omega t + \underbrace{(R A_2 - L A_1 \omega)}_{=0} \sin \omega t = V_M \cos \omega t$$

$$A_2 = \frac{L\omega}{R} A_1$$

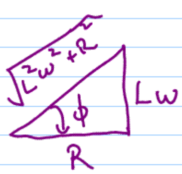
$$L\omega\left(\frac{L\omega A_1}{R}\right) + R A_1 = V_M$$

$$A_1 = \frac{V_M}{R + \frac{L^2\omega^2}{R}} = \boxed{\frac{R V_M}{R^2 + L^2\omega^2} = A_1} \; ; \quad A_2 = \frac{L\omega}{R} \frac{R V_M}{R^2 + L^2\omega^2} ;$$

$$\boxed{A_2 = \frac{L\omega}{R^2 + L^2\omega^2} V_M}$$

$$\therefore \quad i(t) = A_1 \cos \omega t + A_2 \sin \omega t$$

$$i(t) = \frac{R}{R^2 + L^2\omega^2} V_M \cos \omega t + \frac{L\omega}{R^2 + L^2\omega^2} V_M \sin \omega t$$



$$\cos \phi = \frac{R}{\sqrt{\omega^2 L^2 + R^2}} \; ; \quad \sin \phi = \frac{L\omega}{\sqrt{\omega^2 L^2 + R^2}}$$

$$\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{L\omega}{R}$$

Re-writing $i(t)$:

$$i(t) = \frac{V_M}{\sqrt{R^2 + L^2\omega^2}} \cdot \underbrace{\frac{R}{\sqrt{R^2 + L^2\omega^2}}}_{\cos \phi} \cos \omega t + \frac{V_M}{\sqrt{R^2 + L^2\omega^2}} \underbrace{\frac{L\omega}{\sqrt{R^2 + L^2\omega^2}}}_{\sin \phi} \sin \omega t$$

$$i(t) = \frac{V_M}{\sqrt{R^2+L^2\omega^2}} \cos\phi \cos\omega t + \frac{V_M}{\sqrt{R^2+L^2\omega^2}} \sin\phi \sin\omega t$$

$$= \frac{V_M}{\sqrt{R^2+L^2\omega^2}} \underbrace{\left[\cos\phi \cos\omega t + \sin\phi \sin\omega t\right]}_{\cos \omega t - \phi}$$

$$i(t) = \frac{V_M}{\sqrt{R^2+L^2\omega^2}} (\cos\omega t - \phi) \quad ; \quad \phi = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

$$i(t) = \frac{V_M}{\sqrt{R^2+L^2\omega^2}} \cos\left[\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right]$$

$$\boxed{i(t) = I_M \cos(\omega t - \phi) \quad ; \quad \phi = \tan^{-1}\left(\frac{L\omega}{R}\right) \; ; \; I_M = \frac{V_M}{\sqrt{R^2+L^2\omega^2}}}$$