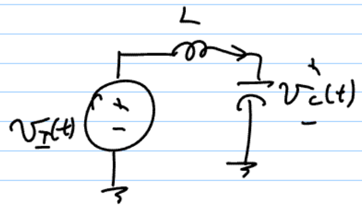
INEL 3105 Lecture #13

Note Title — 10/23/2009



$$i_C = C\frac{dv_C(t)}{dt} = \frac{1}{L}\int_{-\infty}^{t'} (v_I(t) - v_C(t))\,dt$$

differentiate:

$$\frac{1}{L}(v_I(t) - v_C(t)) = C\frac{d^2 v_C(t)}{dt}$$

$$LC\frac{d^2 v_C(t)}{dt^2} + v_C(t) = v_I(t)$$

1st step: Find particular solution v_P

2nd step: Find comp. sol. v_C

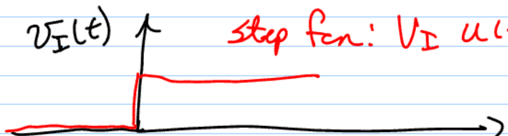
3rd step: Find $v_{TOTAL} = v_P + v_C$

4th step: Solve for constants

Initial conditions: Assume $v_C(0) = 0$; $i_L(0) = 0$

Assume input — Zero-state-response (ZSR)

$v_I(t)$ — step fcn: $V_I\,u(t)$



1st step: particular solution:

$$LC\frac{d^2 v_P}{dt^2} + v_P(t) = v_I(t) \qquad \boxed{V_P = V_I}\ \checkmark$$

2nd step: v_C

$$LC\frac{d^2 v_C(t)}{dt^2} + v_C(t) = 0$$

4 steps: A: assume a solution of the form $v_C(t) = Ae^{st}$

Substituting the solution: $LC\,As^2e^{st} + Ae^{st} = 0$

B : Solve to find $\boxed{s^2 = \frac{-1}{LC}}$ characteristic equation

$s = \pm j\sqrt{\frac{1}{LC}}$; $j = \sqrt{-1}$

using $\omega_0 = \sqrt{\frac{1}{LC}}$

Roots of homog. eq.

C $s = \pm j\omega_0$

Parentesis

$\left[\sqrt{\frac{1}{LC}}\right] = \frac{1}{secs} = Hz.$

D Write $v_c = A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}$

3rd step $v = v_p + v_c$

$v = V_I + A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}$; $i = C\frac{dv}{dt} = C A_1 j\omega_0 e^{j\omega_0 t} + C A_2(-j\omega_0) e^{-j\omega_0 t}$

$v(0) = V_I + A_1 + A_2$

$i(0) = 0 = \cancel{C} A_1 \cancel{j\omega_0} \cancel{e^{j\omega_0 t}}^{1} + \cancel{C} A_2 (-\cancel{j\omega_0}) \cancel{e^{-j\omega_0 t}}^{1}$ $\boxed{A_1 = A_2}$

cancel

substituting in

$A_1 = \frac{-V_I}{2} = A_2$ so,

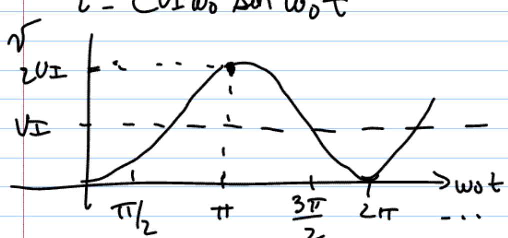
$v = V_I - \frac{V_I}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$

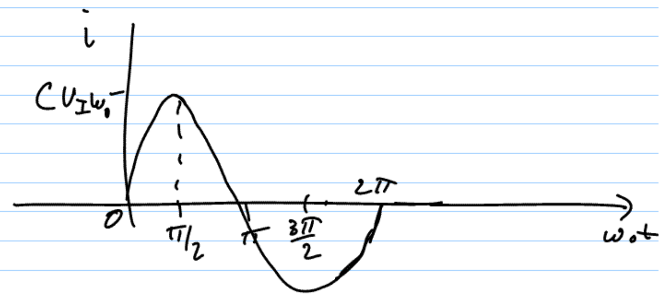
Remember: $e^{jx} = \cos x + j\sin x$ Euler's relation

$\boxed{\frac{e^{jx} + e^{-jx}}{2} = \cos x}$

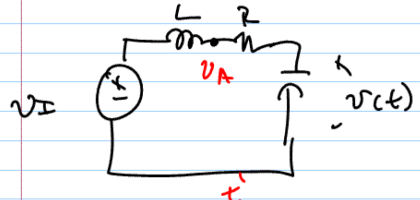
$v = V_I - V_I \cos \omega_0 t$

$i = C V_I \omega_0 \sin \omega_0 t$





Now... lets add a resistance...



$v_L = L\frac{di(t)}{dt}$; $\frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt = i$

$i_C = C\frac{dv_C(t)}{dt}$

$$\frac{v_A - v}{R} = \frac{1}{L}\int_{-\infty}^{t}(v_I - v_A)\,dt = i_C = C\frac{dv(t)}{dt}$$

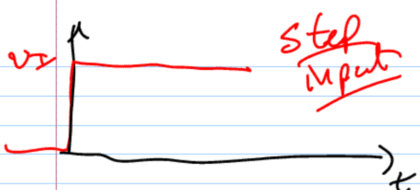
$$\therefore \frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}v_I$$

único término diferente al análisis anterior... si R=0 obtenemos mismo resultado anterior --

Lets assume

$v(0)=0$; $i(0)=0$

ZSR.



Follow SAME Mathematical procedure as before:

1. Find particular solution
2. Find homog. sol. (4 steps)
3. Find total sol.

(1) Find a particular Sol.

$$\frac{d^2v_p}{dt^2} + \frac{R}{L}\frac{dv_p}{dt} + \frac{1}{LC}v_p = \frac{1}{LC}v_I$$

One sol. $\boxed{v_p = v_I}$

(2) Find homogeneous sol. $\frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{v_C}{LC} = 0$

4 steps.

(A) Assume a sol: $v_C = Ae^{st}$ find A, s.

Substitute ; $\cancel{A}s^2\cancel{e^{st}} + \frac{R}{L}\cancel{A}s\cancel{e^{st}} + \frac{1}{LC}\cancel{A}\cancel{e^{st}} = 0$

(B) Find charact. eq. $\boxed{s^2 + \frac{R}{L}s + \frac{1}{LC} = 0}$ Charact. Sol.

assume $\frac{R}{L} = 2\alpha$; $\frac{1}{LC} = \omega_0^2$

(C) Find roots S_1, S_2.

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

(D) General solution:

$$v_c(t) = A_1 e^{(-\alpha + \sqrt{\alpha^2 - \omega_0^2})} + A_2 e^{(-\alpha - \sqrt{\alpha^2 - \omega_0^2})}$$

(3) Total solution

$$v(t) = V_I + A_1 e^{(-\alpha + \sqrt{\alpha^2 - \omega_0^2})t} + A_2 e^{(-\alpha - \sqrt{\alpha^2 - \omega_0^2})t}$$

Use initial conds: $v(0) = 0$; $i(0) = 0$

$v(0) = 0$; $0 = V_I + A_1 + A_2$ ✓

$i(0) = 0$; $i = C\frac{dv(t)}{dt}$; $i(t) = CA_1(-\alpha + \sqrt{\alpha^2 - \omega_0^2})e^{(-\alpha + \sqrt{\alpha^2 - \omega_0^2})t} + CA_2(-\alpha - \sqrt{\alpha^2 - \omega_0^2})e^{(-\alpha - \sqrt{\alpha^2 - \omega_0^2})t}$

so, ✓ $\boxed{0 = A_1(-\alpha + \sqrt{\alpha^2 - \omega_0^2}) + A_2(-\alpha - \sqrt{\alpha^2 - \omega_0^2})}$

2 eqs. 2 variables ...