Chapter 11: AC Steady State Power

Exercises:

Ex. 11.3-1

First calculate i(t)
Phasor circuit. →

$$Z = j3 + \frac{4(-j2)}{4-j2} = 0.8 + j1.4 = 1.6\angle 60.3^\circ\ \Omega$$

$$\therefore I = \frac{V}{Z} = \frac{7\angle 0^\circ}{1.6\angle 60.3^\circ} = 4.38\angle -60.3^\circ\ \text{A}$$

So $i(t) = 4.38\cos(10t - 60.3^\circ)$ A

Now $p(t) = v(t)\cdot i(t) = (7\cos 10t)(4.38\cos(10t - 60.3^\circ)) = \frac{(7)(4.38)}{2}\left[\cos(60.3^\circ) + \cos(20t - 60.3^\circ)\right] = \underline{7.6 + 15.3\cos(20t - 60.3^\circ)\ \text{W}}$

Now $V_L = I\cdot Z_L = (4.38\angle -60.3^\circ)(j3) = 13.12\angle 29.69^\circ$ V

$\therefore v_L(t) = 13.12\cos(10t + 29.69^\circ)$V

So $P_c(t) = v_L(t)\cdot i(t) = \left[(13.12\cos(10t + 29.69^\circ)(4.38\cos(10t - 60.3^\circ)\right] = \frac{57.47}{2}\left[\cos(29.69^\circ + 60.3^\circ) + \cos(20t + 29.69^\circ - 60.3^\circ)\right]$

$\underline{p_c(t) = 28.7\cos(20t - 30.6^\circ)\ \text{W}}$

Ex. 11.3-2

a)

So $i_R(t) = \frac{v(t)}{R} = \frac{V_m}{R}\cos(\omega t + \theta)$ A

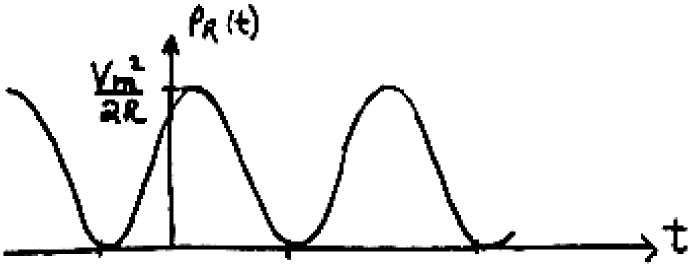
The power into the resistor is $P_R(t) = v(t)\cdot i(t) = V_m\cos(\omega t + \theta)\cdot\frac{V_m}{R}\cos(\omega t + \theta)$

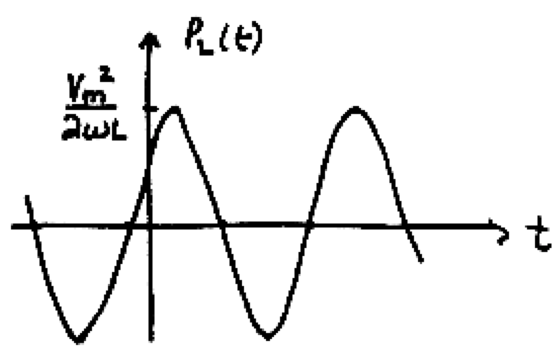
$$= \frac{V_m^2}{R}\cos^2(\omega t + \theta) = \underline{\frac{V_m^2}{2R} + \frac{V_m^2}{2R}\cos(2\omega t + \theta)}$$

(b) When the element is an inductor, the current will lag the voltage by 90°

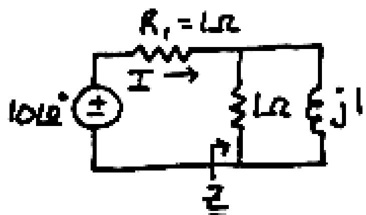
$Z_L = j\omega L = \omega L\angle 90^\circ$ so $I = \frac{V}{Z} = \frac{V_m\angle\theta}{\omega L\angle 90^\circ} = \frac{V_m}{\omega L}\angle\theta - 90^\circ$

So $P_L(t) = i(t)\cdot v(t) = \frac{V_m}{\omega L}\cos(\omega t + \theta - 90^\circ)\cdot V_m\cos(\omega t + \theta) = \underline{\frac{V_m^2}{2\omega L}\cos\left[2\omega t + 2\theta - 90^\circ\right]\ \text{W}}$





Ex. 11.3-3



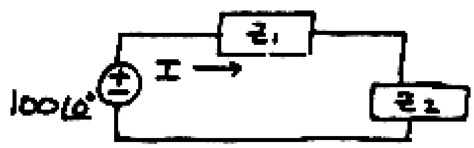
$$Z = \frac{j}{1+j} = \frac{1}{2}(1+j)$$

$$\therefore \; I = \frac{10\angle 0^\circ}{1+\frac{1}{2}(1+j)} = \frac{20}{\sqrt{10}}\angle -18.4^\circ$$

(a) $P_{source} = \frac{1}{2}|I||V|\cos\theta = \frac{1}{2}(10)\left(\frac{20}{\sqrt{10}}\right)\cos\left(-18.4^\circ\right) = \underline{30.0\,W}$

(b) $P_{R_1} = \frac{1}{2} I^2_{max} R_1 = \frac{1}{2}\left(\frac{20}{\sqrt{10}}\right)^2 (1) = \underline{20\,W}$

Ex. 11.3-4



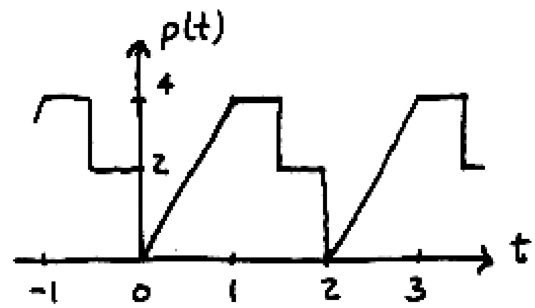
$$I = \frac{100\angle 0^\circ}{Z_1+Z_2} = \frac{100\angle 0^\circ}{6-j8+2+j16} = \frac{100\angle 0^\circ}{8\sqrt{2}\,\angle 45^\circ}$$
$$= 8.84\angle -45^\circ$$

Now if $Z_1 = R_1 + jX_1$, $Z_2 = R_2 + jX_2$

then $P_{Z1} = P_{R1} = \frac{1}{2} I^2_{max} R_1 = \frac{1}{2}(8.84)^2 (6) = \underline{234\text{ W}}$

$P_{Z2} = P_{R2} = \frac{1}{2} I^2_{max} R_2 = \frac{1}{2}(8.84)^2 (2) = \underline{78.1\text{ W}}$

Ex. 11.3-5



$\underline{0 < t < 1}$ $P(t) = v(t)\cdot i(t) = 2t\,(2) = 4t$ W

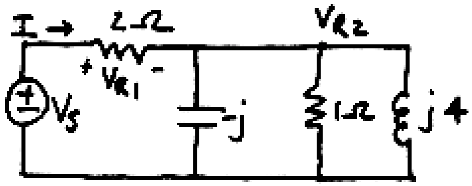
$\underline{0 < t < 1.5}$ $P(t) = (2)\,(2) = 4$ W

$\underline{1.5 < t < 2}$ $P(t) = (1)\,(2) = 2$ W

$$P_{ave} = \frac{1}{T}\int_0^T P(t)dt = \frac{1}{2}\left[\int_0^1 4t\,dt + \int_1^{1.5} 4\,dt + \int_{1.5}^2 2\,dt\right]$$
$$= \frac{1}{2}\left[2+2+1\right] = \underline{2.5\text{ W}}$$

Ex. 11.3-6

Phasor circuit



$V_S = 5\angle 0^\circ$

$$Z = 2 + \left(\frac{1}{-j} + 1 + \frac{1}{j4}\right) = 2.64 - j0.48\ \Omega$$

$$= 2.68\ \angle -10.3^\circ\ \Omega$$

$$I = \frac{V}{Z} = \frac{5\angle 0^\circ}{2.68\angle -10.3^\circ} = 1.87\angle 10.3^\circ\ \text{A}$$

$$\therefore P_{\substack{ave \\ source}} = \frac{V_m\ I_m}{2}\cos\ \theta = \frac{(5)(1.87)}{2}\cos\left(-10.3^\circ\right) = \underline{4.6\ \text{W}}$$

Now $P_R = P_{R1} = P_{R2}$ where $P_{R1} = 1/2\ I_m^2\ R_1 = 1/2(1.87)^2\ (2) = \underline{3.5\ \text{W}}$

$P_{R2} = 1/2\ V_{mR2}^2/R$ where $V_{R2} = V_S - V_{R1} = 1.48\angle -26.9^\circ$

so $P_{R_2} = \frac{1}{2}(1.48)^2 = 1.1$ W

Now with $P_C = P_L = 0$

so $\underline{P_R = 1.1 + 3.5 = 4.6\ \text{W}}$

Ex. 11.4-1

$$I_{eff} = \sqrt{1/T\int_0^t i^2(t)\ dt} = \sqrt{\frac{1}{3}\left[\int_0^2 (10)^2\,dt + \int_2^3 (5)^2\,dt\right]} = \underline{8.66}$$

Ex. 11.4-2

(a) $i(t) = 2\cos 3t \Rightarrow I_{eff} = \frac{I_{max}}{\sqrt{2}} = 2/\sqrt{2} = \underline{\sqrt{2}}$

(b) $i(t) = \cos\left(3t - 90^\circ\right) + \cos\left(3t + 60^\circ\right)$

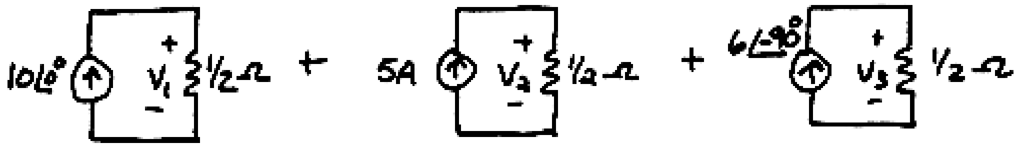
$\therefore I = 1\angle -90^\circ + 1\angle 60^\circ = -j + 1/2 + j\ \sqrt{3}/2 = .518\angle -15^\circ$

So $i(t) = .518\cos\left(3t - 15^\circ\right) \Rightarrow \underline{I_{eff} = \frac{.518}{\sqrt{2}} = .366}$

(c) $I_{eff}^{\ 2} = \left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2 \Rightarrow \underline{I_{eff} = 2.55}$

Use superposition

Ex. 11.4-3



$$\left.\begin{aligned} V_1 &= 5\angle 0^\circ \text{ V} \\ V_2 &= 2.5 \text{ V (DC)} \\ V_3 &= 3\angle{-90}^\circ \text{ V} \end{aligned}\right\}$$

now since V_1 & V_3 have the same frequencies, can add them

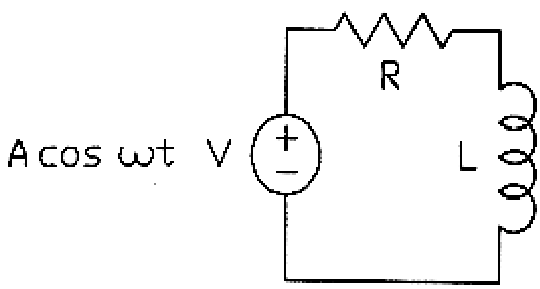
$$V_1 + V_3 = 5\angle 0^\circ + 3\angle -90^\circ = 5 - j3 = 5.83\angle -31.0^\circ$$

$$\text{So } v_R(t) = v_1 + v_2 + v_3 = 2.5 + 5.83 \cos(100t - 31.0^\circ) \text{ V}$$

$$V_{R_{eff}}^2 = (2.5)^2 + \left(\frac{5.83}{\sqrt{2}}\right)^2 = 23.24$$

$$\therefore \underline{V_{R_{eff}} = 4.82 \text{ V}}$$

Ex. 11.5-1



Mathcad analysis

Enter the parameters of the voltage source: A: = 12 ω:=2

Enter the values of R, L, and C: R:= 10 L:= 4 C:=0.1

The impedance seen by the voltage source is: Z:= R+j·ω·L

The mesh current is: $I := \frac{A}{Z}$

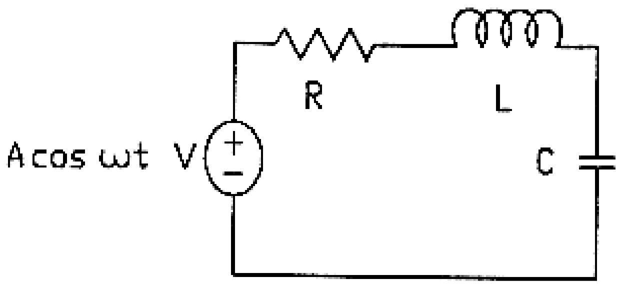
The complex power delivered by the source is: $Sv := \frac{\bar{I}\cdot(I\cdot Z)}{2}$ Sv = 4.39 + 3.512i

The complex power delivered to the resistor is: $Sr := \frac{\bar{I}\cdot(I\cdot R)}{2}$ Sr = 4.39

The complex power delivered to the inductor is: $S1 := \frac{\bar{I}\cdot(I\cdot j\cdot\omega\cdot L)}{2}$ S1 = 3.512i

Verify Sv = Sr + Sl: Sr +Sl = 4.39+3.512i Sv = 4.39 + 3.512i

Ex. 11.5-2



<u>Mathcad analysis</u>

Enter the parameters of the voltage source: $A := 12$ $\omega := 2$

Enter the values of R, L, and C: $R := 10$ $L := 4$ $C := 0.1$

The impedance seen by the voltage source is: $Z := R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C}$

The mesh current is : $I := \frac{A}{Z}$

The complex power delivered by the source is : $Sv := \frac{\bar{I} \cdot (I \cdot Z)}{2}$ $Sv = 6.606 + 1.982i$

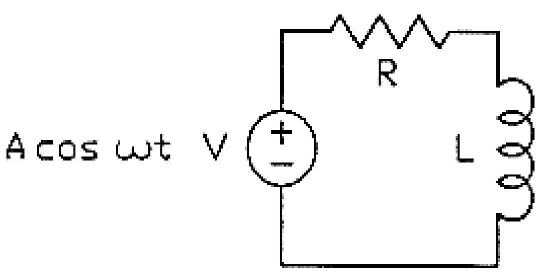
The complex power delivered to the resistor is : $Sr := \frac{\bar{I} \cdot (I \cdot R)}{2}$ $Sr = 6.606$

The complex power delivered to the inductor is : $S1 := \frac{\bar{I} \cdot (I \cdot j \cdot \omega \cdot L)}{2}$ $S1 = 5.284i$

The complex power delivered to the capacitor is : $Sc := \frac{\bar{I}\left(I \cdot \frac{1}{j \cdot \omega \cdot C}\right)}{2}$ $Sc = -3.303i$

Verify Sv = Sr + Sl + Sc : $Sr + Sl + Sc = 6.606 + 1.982i$ $Sv = 6.606 + 1.982i$

Ex. 11.5-3



<u>Mathcad analysis</u>

Enter the parameters of the voltage source: $A := 12$ $\omega := 2$

Enter the average and reactive power delivered to the RL circuit: $P := 8$ $Q := 6$

The complex power delivered to the RL circuit is: $S := P + j \cdot Q$

The impedance seen by the voltage source is: $Z := \frac{A^2}{2 \cdot \bar{S}}$

Calculate the required values of R and L $R := \mathrm{Re}(Z)$ $L := \frac{\mathrm{Im}(Z)}{\omega}$ $R = 5.76$ $L = 2.16$

The mesh current is : $I := \frac{A}{Z}$

The complex power delivered by the source is : $Sv := \frac{\bar{I}\cdot(I\cdot Z)}{2}$ $Sv = 8 + 6i$

The complex power delivered to the resistor is : $Sr := \frac{\bar{I}\cdot(I\cdot R)}{2}$ $Sr = 8$

The complex power delivered to the inductor is : $Sl := \frac{\bar{I}\cdot(I\cdot j\cdot\omega\cdot L)}{2}$ $Sl = 6i$

Verify Sv = Sr + Sl : Sr + Sl = 8 + 6i Sv = 8+ 6i

Ex. 11.6-1

$$PF = \cos(\angle Z) = \cos\left[\tan^{-1}\left(\frac{\omega L}{R}\right)\right] = \cos\left[\tan^{-1}\frac{(377)(5)}{100}\right] = \underline{0.053}$$

Ex. 11.6-2

$$PF = \cos(\angle Z) = \cos\left[\tan^{-1}\left(\frac{Z_I}{Z_R}\right)\right] = \cos\left[\tan^{-1}\left(\frac{80}{50}\right)\right] = 0.53$$

<u>Assume lagging</u>

$$\Rightarrow X_1 = \frac{(50)^2+(80)^2}{50\tan(\cos^{-1}1)-80} = -111.25\ \Omega$$

$$\underline{\therefore Z_1 = -j\ 111.25\ \Omega}$$

Ex. 11.6-3

$$P_T = 30+86 = 116$$

$$\therefore S_T = P_T + j\,Q_T = 116 + j51 = 126.7\ \angle 23.7^\circ$$

$$\Rightarrow \underline{PF_{plant} = \cos 23.7^\circ = 0.915}$$

Ex. 11.6-4

$$P = VI\cos\theta$$

$$I = \frac{P}{V\cos\theta} = \frac{4000}{(110)(.82)} = 44.3\ A$$

$$Z = \frac{V}{I}\angle\cos^{-1}(.82) = 2.48\ \angle 34.9^\circ = 2.03 + j\ 1.42 = R + jX$$

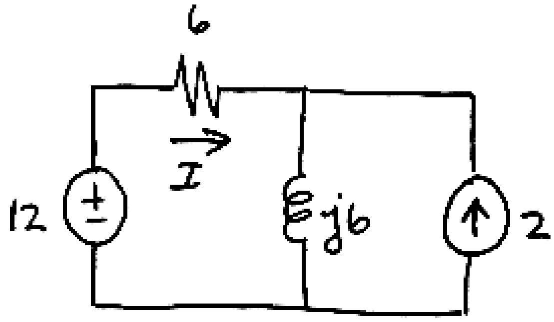
To correct power factor to 0.95 have

$$X_1 = \frac{R^2+X^2}{R\tan(\cos^{-1}pfc)-X} = \frac{(2.03)^2+(1.42)^2}{(2.03)\tan(18.19^\circ)-1.42} = -\ 8.16\ \Omega$$

$$C = \frac{-1}{\omega X_1} = \underline{325\ \mu F}$$

Ex. 11.7-1

a)

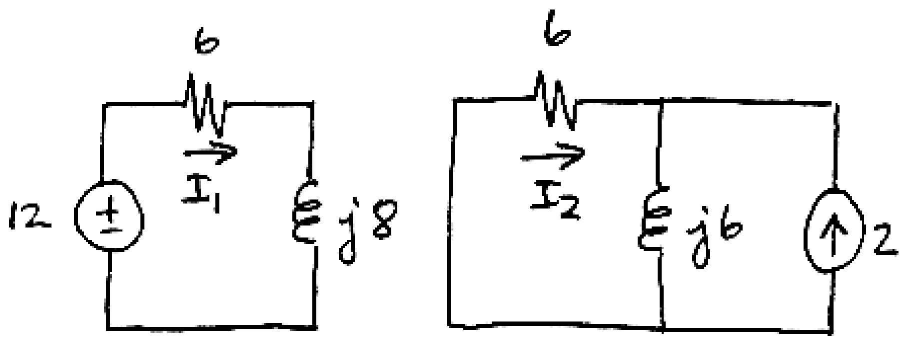


$$I = \frac{12}{6+j6} - \frac{j6}{6+j6}2 = 2e^{-j90}$$

$$i(t) = 2\cos(3t - 90^\circ)\text{ A}$$

$$P = \left(\frac{2}{\sqrt{2}}\right)^2 6 = \underline{12\text{ W}}$$

b)



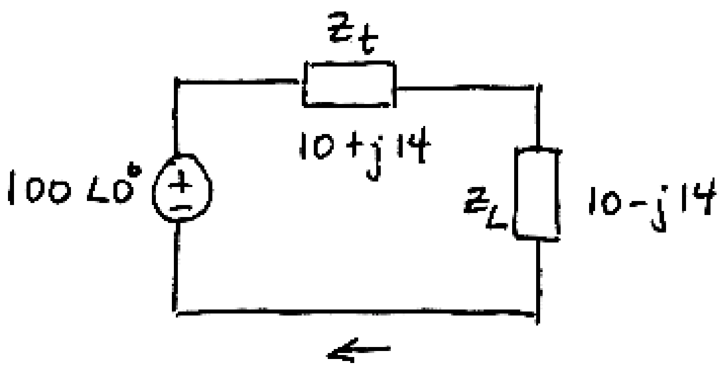
$$I_1 = \frac{12}{6+j8} = 1.2e^{-j53.13} \qquad I_2 = -\frac{j6}{6+j6}2 = 1.414e^{-j135}$$

$$i_1(t) = 1.2\cos(4t - 53.13^\circ) \qquad i_2(t) = 1.4\cos(3t - 135^\circ)$$

$$i(t) = i_1(t) + i_2(t) = 1.2\cos(4t - 53^\circ) + 1.4\cos(3t - 135^\circ)$$

$$P = P_1 + P_2 = \left(\frac{1.2}{\sqrt{2}}\right)^2 6 + \left(\frac{1.414}{\sqrt{2}}\right)^2 6 = \underline{10.32\text{ W}}$$

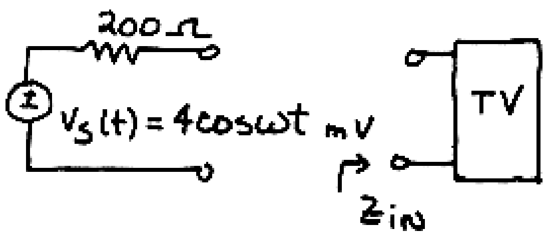
Ex. 11.8-1 For maximum power, transfer $Z_L = Z_t^* = \underline{10 - j14}$



$$I = \frac{100}{(10+j14)(10-j14)} = 5$$

$$P_L = \left(\frac{5}{\sqrt{2}}\right)^2 \text{Re}(10 - j14) = \underline{125\text{ W}}$$

Ex. 11.8-2



If the station transmits a signal in 52 MHZ, then $\omega = 2\pi f = 104\pi\times10^6$ rad/sec

So the received signal is

$v_s(t) = 4\cos(104\pi\times10^6 t)$ mV

a) If receiver has input impedance $Z_{in} = 300\ \Omega$

$$V_{in} = \frac{Z_{in}}{R+Z_{in}}\cdot V_s = \frac{300}{200+300}\times4\times10^{-3} = 2.4\text{ mV}$$

$$P = \frac{1}{2}V_m^2\left(\frac{1}{R_L}\right) = \left(\frac{2.4\times10^{-3}}{2(300)}\right)^2 = \underline{9.6\text{ nW}}$$

b) If two receivers are in parallel

$$Z_{in} = \frac{(300)(300)}{300+300} = 150\ \Omega$$

$$V_{in} = \frac{Z_{in}}{R+Z_{in}}V_S = \frac{150}{200+150}\ (4\times10^{-3}) = 1.71\times10^{-3}\text{ V}$$

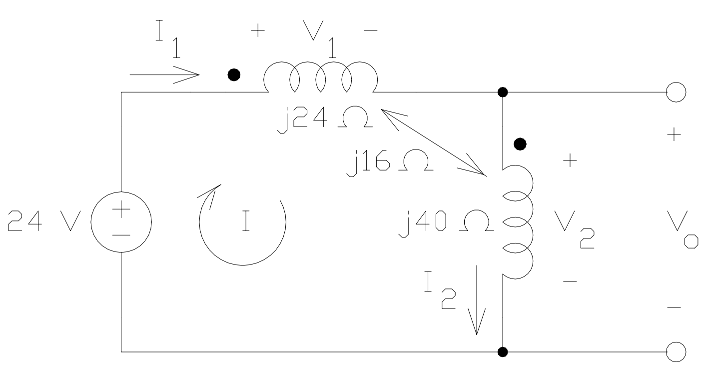
$$\text{total } P = \frac{V_{in}^{\ 2}}{2}\left(\frac{1}{Z_{in}}\right) = \frac{(1.71\times10^{-3})^2}{2(150)} = \underline{9.7\text{nW}} \quad \text{or} \quad 4.85\text{nW to each set}$$

c) Need $Z_{in} = R$ for max. power $\therefore$ need another R_a in || with Z_{in}

$$\Rightarrow \frac{R_a(300)}{R_a+300} = 200 \Rightarrow R_a = 600\ \Omega$$

$$\text{So } P_{max} = \frac{V_m^2}{2Z_{in}} = \frac{(2\times10^{-3})^2}{2(200)} = \underline{10\text{nW}} \qquad \text{or} \qquad 5\text{nW to each set}$$

Ex 11.9-1



Coil voltages:

$$\mathbf{V}_1 = j\,24\,\mathbf{I}_1 + j16\,\mathbf{I}_2 = j\,40\,\mathbf{I}$$

$$\mathbf{V}_2 = j16\,\mathbf{I}_1 + j\,40\,\mathbf{I}_2 = j\,56\,\mathbf{I}$$

Mesh equation:

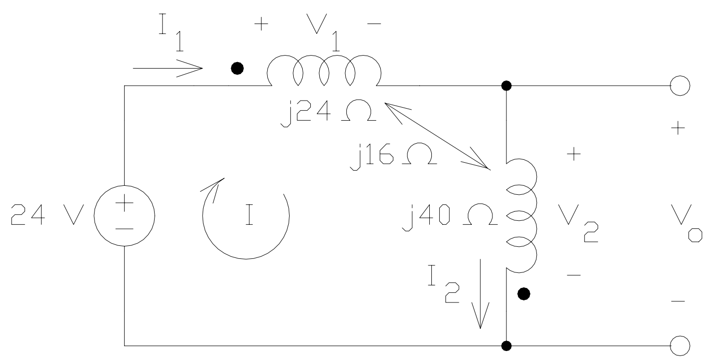
$$24 = \mathbf{V}_1 + \mathbf{V}_2 = j\,40\,\mathbf{I} + j\,56\,\mathbf{I} = j\,96\,\mathbf{I}$$

$$\mathbf{I} = \frac{24}{j96} = -j\frac{1}{4}$$

$$\mathbf{V}_o = \mathbf{V}_2 = (j56)\left(-j\frac{1}{4}\right) = 14$$

$$v_o = 14\cos 4t\ \text{ V}$$

Ex 11.9-2



Coil voltages:

$$\mathbf{V}_1 = j\,24\,\mathbf{I}_1 - j16\,\mathbf{I}_2 = j\,8\,\mathbf{I}$$

$$\mathbf{V}_2 = -j16\,\mathbf{I}_1 + j\,40\,\mathbf{I}_2 = j\,24\,\mathbf{I}$$

Mesh equation:

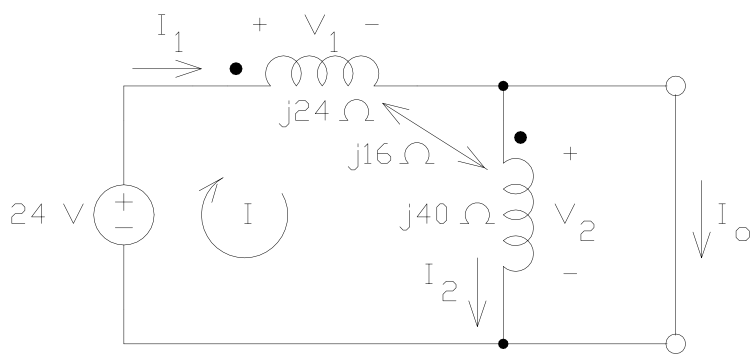
$$24 = \mathbf{V}_1 + \mathbf{V}_2 = j\,8\,\mathbf{I} + j\,24\,\mathbf{I} = j\,32\,\mathbf{I}$$

$$\mathbf{I} = \frac{24}{j32} = -j\frac{3}{4}$$

$$\mathbf{V}_o = \mathbf{V}_2 = (j24)\left(-j\frac{3}{4}\right) = 18$$

$v_o = 18 \cos 4t$ V

Ex 11.9-3



$$0 = \mathbf{V}_2 = j16\,\mathbf{I}_1 + j\,40\,\mathbf{I}_2$$

$$\Rightarrow \mathbf{I}_1 = -\frac{40}{16}\mathbf{I}_2 = -2.5\;\mathbf{I}_2$$

$$\mathbf{V}_s = \mathbf{V}_1 = j\,24\,\mathbf{I}_1 + j16\,\mathbf{I}_2$$

$$= j\,(24(-2.5)+16)\,\mathbf{I}_2$$

$$= -j44\,\mathbf{I}_2$$

$$\mathbf{I}_2 = \frac{24}{-j44} = j\frac{6}{11}$$

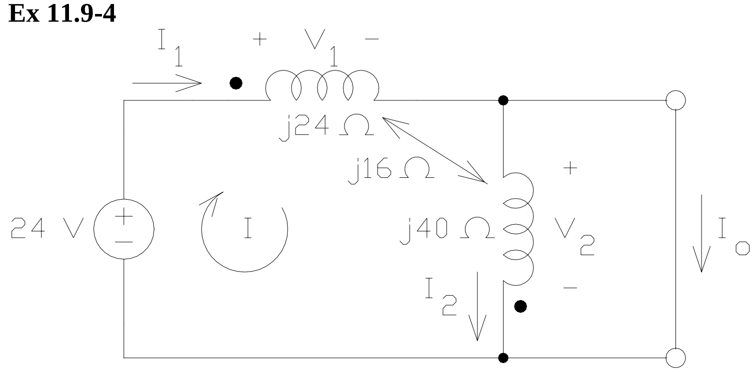
$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = (-2.5-1)\,\mathbf{I}_2$$

$$= -3.5\,\mathbf{I}_2$$

$$= -3.5\left(j\frac{6}{11}\right) = -j1.909$$

$i_o = 1.909 \cos (4t - 90°)$ A

Ex 11.9-4



$$0 = \mathbf{V}_2 = -j16\,\mathbf{I}_1 + j\,40\,\mathbf{I}_2$$

$$\Rightarrow \mathbf{I}_1 = \frac{40}{16}\mathbf{I}_2 = 2.5\ \mathbf{I}_2$$

$$\mathbf{V}_s = \mathbf{V}_1 = j\,24\,\mathbf{I}_1 - j16\,\mathbf{I}_2$$

$$= j\,(24(2.5) - 16)\,\mathbf{I}_2$$

$$= j44\,\mathbf{I}_2$$

$$\mathbf{I}_2 = \frac{24}{j44} = -j\frac{6}{11}$$

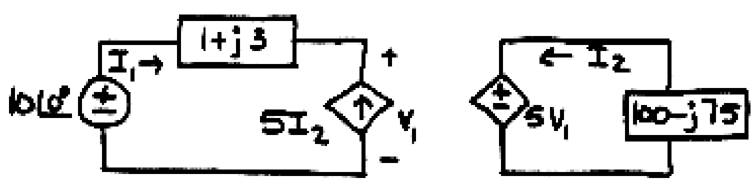
$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = (2.5 - 1)\,\mathbf{I}_2$$

$$= 1.5\,\mathbf{I}_2$$

$$= 1.5\left(-j\frac{6}{11}\right) = -j\,0.818$$

i_o = 0.818 cos (4t - 90°) A

Ex. 11.10-1



(model of ideal transformer)

KVL left ckt: $(1+\text{j}3)\text{I}_1 + \text{V}_1 = 10$ (1)

KVL right ckt: $\text{I}_2 = {-5\text{V}_1}/{100-\text{j}75}$ (2)

also : $\text{I}_1 = -5\text{I}_2$ (3)

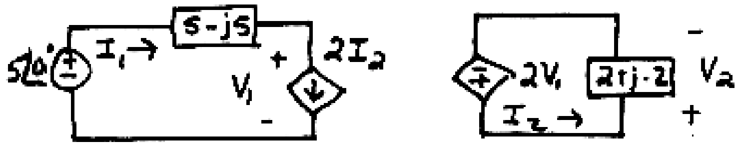
(2) into (3) $\Rightarrow \text{I}_1 = \dfrac{25\text{V}_1}{100-\text{j}75} = \dfrac{\text{V}_1}{4-\text{j}3}$

$\therefore$plugging into (1) $\Rightarrow (1+\text{j}3)\left(\dfrac{\text{V}_1}{4-\text{j}3}\right) + \text{V}_1 = 10$

$$\Rightarrow \underline{\text{V}_1 = 10\angle -36.9^\circ}$$

$$\therefore\ \underline{\text{I}_1 = \frac{\text{V}_1}{4-\text{j}3} = 2\angle 0^\circ}$$

Ex. 11.10-2



$2V_1 = V_2 \Rightarrow V_1 = \frac{1}{2} V_2$

also $I_1 = 2I_2$

So $Z_1 = \dfrac{V_1}{I_1} = \dfrac{V_2/2}{2I_2} = \dfrac{1}{4} Z_2 = \dfrac{2.01}{4}\angle 5.7^\circ = 0.5\angle 5.7^\circ\ \Omega$

$Z_{in} = 5 - j5 + \dfrac{1}{4}(2 + j0.2) = 5.5 - j4.95$

$I_1 = \dfrac{5\angle 0^\circ}{Z_{in}} = \dfrac{5\angle 0^\circ}{7.4\angle -42^\circ} = 0.68\angle 42^\circ\ \text{A}$

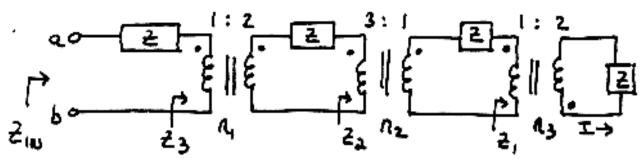
$V_1 = I_1 \cdot Z_1 = (.5\angle 5.7^\circ)(.68\angle 42^\circ) = 0.34\angle 47.7^\circ\ \text{V}$

So $\underline{i_1(t) = 0.68\cos(10t + 42^\circ)\text{A}}$ & $\underline{v_1(t) = 0.34\cos(10t + 47.7^\circ)\ \text{V}}$

$v_2(t) = nv_1(t) = \underline{0.68\cos(10t + 47.7^\circ)\ \text{V}}$

$i_2(t) = \dfrac{i_1(t)}{n} = \underline{0.34\cos(10t + 42^\circ)\ \text{A}}$

Ex. 11.10-3



$Z_1 = \dfrac{Z}{n_3^2} = \dfrac{Z}{4}$

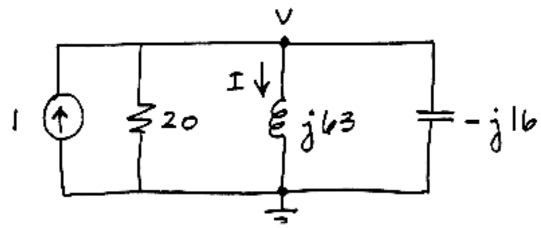
$Z_3 = \dfrac{1}{n_1^2}(Z + Z_2)$

$Z_2 = \dfrac{1}{n_1^2}\left(Z + \dfrac{Z}{n_3^2}\right) = 9\left(Z + \dfrac{Z}{4}\right)$

Now $Z_{ab} = Z_{IN} = Z + Z_3 = Z + \dfrac{1}{4}\left(Z + 9\left(Z + \dfrac{Z}{4}\right)\right) = \underline{4.0625\ Z}$

Problems
Section 11-3: Instantaneous Power and Average Power

P. 11.3-1



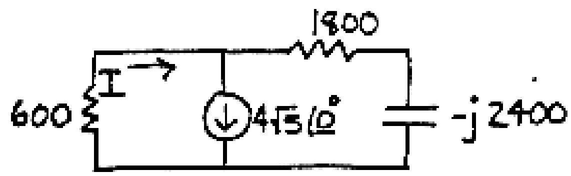
$$1=\frac{V}{20}+\frac{V}{j63}+\frac{V}{-j16} \Rightarrow V=14.6\angle-43^\circ$$

$$I=\frac{V}{j63}=0.23\angle-133^\circ$$

$$p(t)=i(t)v(t)=0.23(\cos(2\pi\cdot10^3t-133^\circ))\times14.6\cos(2\pi\cdot10^3t-43^\circ)$$

$$=3.36\cos(2\pi\cdot10^3t-133^\circ)\cos(2\pi\cdot10^3t-43^\circ)=1.68(\cos(90^\circ)+\cos(4\pi\cdot10^3t-176^\circ))=1.68\cos(4\pi\cdot10^3t-176^\circ)$$

P. 11.3-2

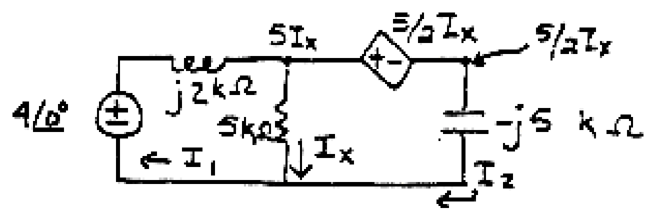


Using current divider $I=4\sqrt{5}\left[\frac{1800-j2400}{1800-j2400+600}\right]=5\sqrt{5/2}\angle-8.1^\circ$ mA

$\therefore P_{600\Omega}=1/2\,|I|^2 600=300(25)(5/2)=\underline{1.875\times10^4\,\mu W=18.75\,mW}$

$P_{source}=\frac{1}{2}|V||I|\cos\theta=\frac{1}{2}(600)\left(5\sqrt{5/2}\right)\left(4\sqrt{5}\right)\cos(-8.1^\circ)=\underline{2.1\times10^4\,\mu W=21\,mW}$

P. 11.3-3



KCL at $5I_X$: $\frac{5I_X-4}{j2}+I_X+\frac{\frac{5}{2}I_X}{-j5}=0$

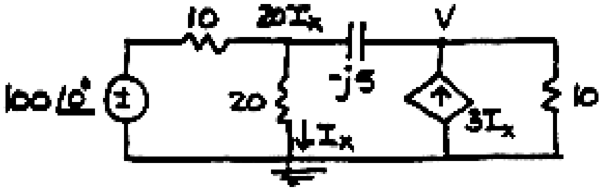
$$\Rightarrow I_X=\frac{2}{\sqrt{5}}\angle-26.6^\circ$$

$\therefore I_2=\frac{5}{2}\,\frac{I_X}{-j5}=\frac{1}{\sqrt{5}}\angle63.4^\circ,\ I_1=I_X+I_2=1\text{mA}$

(a) $P_{\text{indep source}}=\frac{1}{2}(4)\left(I_{1_{max}}\right)\cos(0^\circ)=\frac{1}{2}(4)(1)=\underline{2\text{mW}}$

(b) $P_{\text{dep. source}}=\frac{1}{2}\left(\frac{5}{2}I_{X_{max}}\right)\left(I_{2_{max}}\right)\cos(90^\circ)=\underline{0}$

P. 11.3-4



KCL at $20I_X$: $\frac{20I_X - 100}{10} + I_X + \frac{20I_X - V}{-j5} = 0 \Rightarrow \underline{I_X(20-j15)-V=-j50}$ (1)

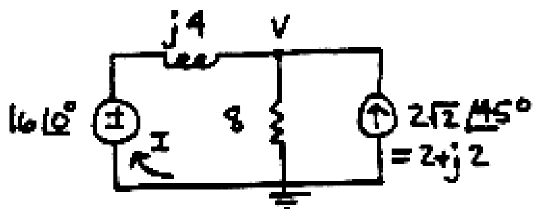
KCL at V : $\frac{V-20I_X}{-j5} - 3I_X + \frac{V}{10} = 0 \Rightarrow \underline{I_X(-40+j30)+V(-2-j)=0}$

Using Cramer's rule

$$I_X = \frac{j50(2-j)}{(40-j30)-(20-j15)(2-j)} = \frac{50\sqrt{5}\angle 63.4^\circ}{25\angle 53.1^\circ} = 2\sqrt{5}\angle 10.3^\circ \text{ A}$$

$$\therefore \underline{P_{AVE} = \tfrac{1}{2}|I_X|^2(20) = 10\left(2\sqrt{5}\right)^2 = 200\text{w}}$$

P. 11.3-5



KCL at V: $\frac{(V-16)}{j4} + \frac{V}{8} - (2+j2) = 0$

$$\Rightarrow V = 16\sqrt{2/5}\ \angle 18.4^\circ$$

$$I = (16-V)/j4 = \sqrt{3.2}\ \angle -116.6^\circ$$

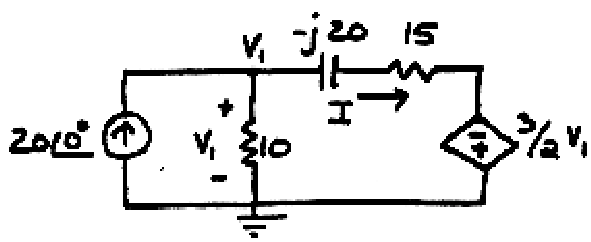
$$\therefore P_{AVE\ 8\Omega} = \tfrac{1}{2}\frac{|V|^2}{8} = \tfrac{1}{2}\frac{\left(16\sqrt{2/5}\right)^2}{8} = \underline{6.4 \text{ W absorbed}}$$

$$P_{\text{AVE source current}} = -\tfrac{1}{2}|V|\left(2\sqrt{2}\right)\cos\theta = -\tfrac{1}{2}\left(16\sqrt{2/5}\right)\left(2\sqrt{2}\right)\cos\left(26.6^\circ\right) = \underline{-12.8 \text{ W absorbed}}$$

$$P_{\text{AVE inductor}} = 0$$

$$P_{\text{AVE Voltage source}} = -1/2(16)|I|\cos\theta = -1/2(16)(\sqrt{3.2})\cos(-116.6^\circ) \underline{= 6.4 \text{ W absorbed}}$$

P11.3-6



$$\text{KCL at } V_1:\ -20+\frac{V_1}{10}+\frac{V_1+(3/2)V_1}{15-j20}=0$$

$$\Rightarrow\ V_1=50\sqrt{5}\ \angle-26.6^\circ$$

$$\Rightarrow\ I=\frac{V_1+3/2\,V_1}{15-j20}=\frac{5/2V_1}{25\angle-53.10^\circ}=5\sqrt{5}\ \angle 26.6^\circ$$

$$P_{AVE\ 10\Omega}=\frac{1}{2}\frac{|V_1|^2}{10}=\frac{1}{2}\frac{(50\sqrt{5})^2}{10}=\underline{625\text{ W absorbed}}$$

$$P_{AVE\ \text{current source}}=-\frac{1}{2}|V|(20)\cos\theta=-\frac{1}{2}(50\sqrt{5})(20)\cos\ (-26.6^\circ)$$

$$=\underline{\ -1000\text{ W absorbed}}$$

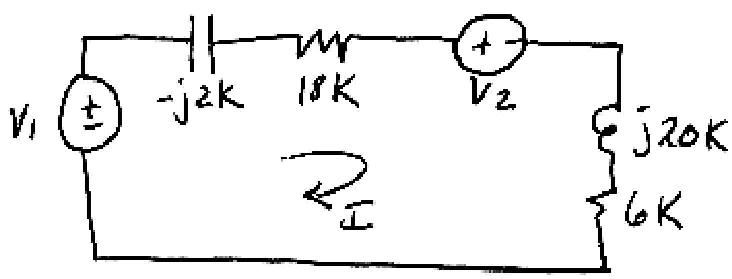
$$P_{AVE\ 15\Omega}=\ \tfrac{1}{2}|I|^2(15)=\ -\tfrac{1}{2}\left(5\sqrt{5}\right)^2\ (15)=\underline{\ 937.5\text{ W absorbed}}$$

$$P_{AVE\ \text{voltage source}}=\ -\frac{1}{2}|I|\left|\frac{3}{2}V_1\right|\cos\theta=\ -\tfrac{1}{2}\left(5\sqrt{5}\right)\left(75\sqrt{5}\right)\cos\left(-53.1^\circ\right)$$

$$=\ \underline{-562.5\text{ W absorbed}}$$

P_{AVE} capacitor $=\ \underline{0W}$

P11.3-7



$V_1\ =4\angle 60^\circ$ V

$V_2=\ 8\angle 0^\circ$ V

$\omega\ =\ 5\times 10^6$ rad/sec

KVL : $-4\angle 60^\circ - j2\ I\ +\ 18\ I\ +\ 8\angle 0^\circ + j\ 20I\ +6I=0$ Note: I in mA

Solving yields $I=0.231\ \angle-67^\circ$ mA

$$\text{Then } P_{ave\ 6K\Omega}=\ \frac{1}{2}|I|^2\,R\ =\ \left(\frac{0.231\times10^{-3}}{2}\right)^2\left(6\times10^3\right)=\ \underline{160\mu W}\text{ absorbed}$$

$$P_{ave\ V_1}=\ \frac{1}{2}|V|\ |I|\cos\theta\ =\ \frac{1}{2}(4)\left(0.231\times10^{-3}\right)\cos\left(-67-60^\circ\right)=\frac{277\mu\text{ W}}{\text{delivered}}$$

P11.3-8

$$Z = \frac{200\,(j200)}{200\,(1+j)} = \frac{200\ \angle 90^\circ}{\sqrt{2}\ \angle 45^\circ} = \frac{200}{\sqrt{2}}\angle 45^\circ$$

$$I = \frac{120\angle 0^\circ}{\frac{200}{\sqrt{2}}\angle 45^\circ} = 0.85\ \angle -45^\circ, \quad I_R = \left(\frac{200}{200+j200}\right) I = 0.6\angle 0^\circ\ \text{A}$$

$$P = I^2 R = (0.6)^2 (200) = 72\ \text{W} \qquad \underline{w = (72)\ (1) = 72\ \text{J}}$$

Section 11-4: Effective Value of a Periodic Waveform

P11.4-1

a) $i = 2 - 4\cos 2t$

(Treat as two sources of differing frequency)

$$\text{2A source :} \quad I_{eff} = \lim_{T\to\infty} \sqrt{\frac{1}{T}\int_o^T (2)^2\, dt} = 2$$

$$4\ \cos\ 2t\ \text{source : } I_{eff} = 4/\sqrt{2}$$

$$\therefore\ I_{eff}{}^2{}_{Total} = (2)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 = 12$$

$$\Rightarrow\ \underline{I_{rms} = I_{eff} = \sqrt{12} = 2\sqrt{3}\text{A}}$$

(b) $i = 3\cos(\pi t - 90^\circ) + \sqrt{2}\ \cos \pi\, t$

$$I = 3\angle -90^\circ + \sqrt{2}\ \angle 0^\circ = \sqrt{2} - j3 = 3.32\angle -64.8^\circ$$

$$\Rightarrow\ \underline{I_{rms} = \frac{3.32}{\sqrt{2}} = 2.35\ \text{A}}$$

(c) $i = 2\cos 2t + 4\sqrt{2}\ \cos\left(2t + 45^\circ\right) + 12\cos\left(2\ t - 90^\circ\right)$

$$I = 2\angle 0^\circ + 4\sqrt{2}\ \angle 45^\circ + 12\angle -90^\circ = 2 + 4 + j4 - j12 = 10\angle -53.1^\circ$$

$$\Rightarrow\ \underline{I_{rms} = 10/\sqrt{2} = 5\sqrt{2}\ \text{A}}$$

P11.4-2

(a)

$$f(t)=\begin{cases}\sqrt{3/5} & 0<t<4\\ -\sqrt{3/5}(t-5) & 4<t<5\\ 0 & 5<t<9\\ \sqrt{3/5}(t-9) & 9<t<10\end{cases}$$

$$f_{rms}^{\ 2}=\frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{10}\left[\int_0^4\left(\frac{3}{5}\right)dt+\int_4^5 \frac{3}{5}(t-5)^2\,dt+\int_9^{10}\frac{3}{5}(t-9)^2dt\right]=\frac{1}{10}\left(\frac{14}{5}\right)\Rightarrow \underline{f_{rms}=\frac{\sqrt{7}}{5}}$$

(b)

$$\imath(t)=\begin{cases}\sqrt{3/2}-\sqrt{3/6}\,t & 0<t<6\\ -3\sqrt{3/2}+\sqrt{3/6}\,t & 6<t<12\end{cases}$$

$$f_{rms}=\sqrt{\frac{1}{12}\left[\int_0^6\left(\sqrt{3/2}-\sqrt{3/6}\,t\right)^2dt+\int_6^{12}\left(-3\sqrt{3/6}+\sqrt{3/6}\,t\right)^2dt\right]}=\underline{1/2}$$

(c) $f(t)=\begin{cases}\sin t & 0<t<\pi\\ 0 & \pi<t<2\pi\end{cases}$

$$f_{rms}=\sqrt{\frac{1}{2n}\left[\int_0^n \sin^2 t\,dt+0\right]}=\underline{\sqrt{1/4}=1/2}$$

P11.4-3 period = T = 4

$$\therefore V_{ave}=\frac{(1)(1)+(3)(2)+(0)(1)}{4}=\underline{1.75\text{V}}$$

$$V_{rms}=\left[\frac{(1)^2(1)+(3)^2(2)+(0)^2(1)}{4}\right]^{1/2}=\underline{2.18\text{V}}$$

P11.4-4

a) $v(t)=1+\cos\left(\frac{2\pi}{T}t\right)$

$$V_{eff}^{\ 2}=V_{DC}^{\ 2}+V_{AC}^{\ 2}$$

$$V_{DC}^{\ 2}=\left(\frac{1}{T}\int_0^T 1dt\right)=\left.\frac{t}{T}\right|_0^T=\left(\frac{T}{T}-0\right)=1$$

$$V_{AC}=\frac{1}{\sqrt{2}}\quad\therefore\quad V_{eff}=\sqrt{1^2+\left(\frac{1}{\sqrt{2}}\right)^2}=\underline{1.225\text{V}}$$

b) $\omega = \frac{2\pi}{T}$, $I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$

$$I_{rms}^2 = \frac{1}{T}\int_0^{T/2}(A \sin \omega t)^2 dt = \frac{A^2}{T}\int_0^{T/2}\frac{1}{2}(1-\cos 2\omega t)dt$$

$$= \frac{A^2}{2T}\left[\int_0^{T/2}dt - \int_0^{T/2}\cos 2\omega t\, dt\right] = \frac{A^2}{4}$$

So $I_{rms} = \sqrt{\frac{A2}{4}} = \frac{A}{2}$, where $A = 10$ mA

So $\underline{I_{rms} = 5\text{ mA}}$

P11.4-5

$$V_t = \begin{cases} 90\,t & 0 \le t \le .1 \\ 90(0.2-t) & .1 \le t \le .2 \\ 0 & .2 \le t \le .3 \end{cases}$$

$$V_{rms}^2 = \frac{1}{.3}\left[\int_0^{.1}(90t)^2 dt + \int_{.1}^{.2}\left[90(0.2-t)\right]^2 dt\right]$$

$$= \frac{90^2}{.3}\left[\int_0^{.1}t^2 dt + \int_{.1}^{.2}(0.2-t)^2 dt\right]$$

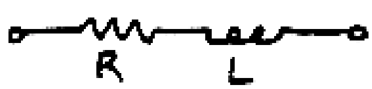
$$= \frac{90^2}{.3}\left[\frac{.001}{3} + \frac{.001}{3}\right] = 18$$

$\therefore V_{rms} = \sqrt{18} = \underline{4.24\text{ V}}$

Section 11-5: Complex Power

P11.5-1

$\Rightarrow X = -\frac{1}{\omega C} = -11.588 \Rightarrow \underline{C = \frac{1}{(11.588)(2\pi)(60)} = 229\mu F}$



$R = \frac{P}{I^2} = \frac{20}{(2)^2} = \underline{5\Omega}$

$\cos\theta = \frac{P}{VI} = \frac{20}{(2)(26)} = .3846 \Rightarrow \theta = 67.4^\circ$

reactive power : $Q = VI\sin\theta = (2)(26)\sin 67.4^\circ = 48$ VAR

but $Q = I^2 X_L$

$\therefore X_L = \frac{Q}{I^2} = \frac{48}{(2)^2} = 12\Omega \Rightarrow \underline{L = X_L/\omega = 12/377 = 31.8\text{ mH}}$

P11.5-2 From Problem P11.5-1:

$$S_{resistor} = 20 + j0 \quad VA$$
$$S_{coil} = 0 + j48 \quad VA$$
$$Z = R + j\omega L = 5 + j12 = 13\angle 67.4^\circ$$

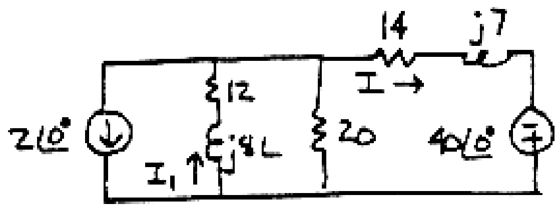
For 26V source : $I = \dfrac{V}{Z} = \dfrac{26\angle 0^\circ}{13\angle 67.4^\circ} = 2\angle -67.4^\circ$

$$\therefore \; \underset{\left(\substack{\text{delivered}\\ \text{by}}\right)}{S_{26v}} = VI^* = (26)\left(2\angle 67.4^\circ\right) = 52\angle 67.4^\circ = 20 + j48 \text{ VA}$$

$\underline{\text{So } S_{26V} = -20 - j48 \text{ VA}}$ delivered

$S_{Total} = S_{resistor} + S_{coil} + S_{26V} = 20 + j48 - 20 - j48 \underline{= 0}$

P11.5-3



If $P_{complex} = \frac{50}{3}\angle 53.1^\circ = \frac{1}{2} I^* \left(40\angle 0^\circ\right)$

$\Rightarrow I^* = \frac{5}{6}\angle 53.1^\circ$

$\Rightarrow I = \frac{5}{6}\angle -53.1^\circ = \frac{1}{2} - j\frac{2}{3}$ A

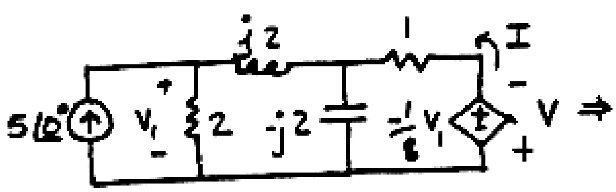
From current divider: $I_1 = (2 + I)\left(\dfrac{20}{32 + j8L}\right) = \dfrac{75 - j20}{48 + j12L}$ (1)

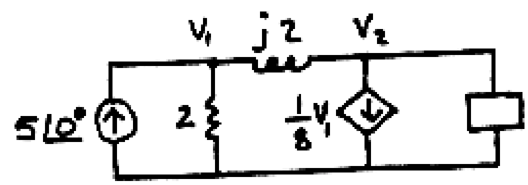
Also from KVL: $I_1(12 + j8L) + I(14 + j7) - 40 = 0$

$$\Rightarrow I_1 = \frac{170 + j35}{72 + j48L} \qquad (2)$$

Equating (1) & (2) (both real & imaginary parts) $\Rightarrow$ $\underline{L = 2H}$

P11.5-4





KCL at V_1 : $-5 + V_1/2 + (V_1-V_2)/j2 = 0 \Rightarrow V_1(1+j) - V_2 = 10$ (1)

KCL at V_2: $(V_2-V_1)/j2 + \frac{1}{8}V_1 + V_2/(.8+j.4) = 0 \quad \Rightarrow V_1(-4+j) + V_2(j8) = 0$ (2)

Using Cramer's rule

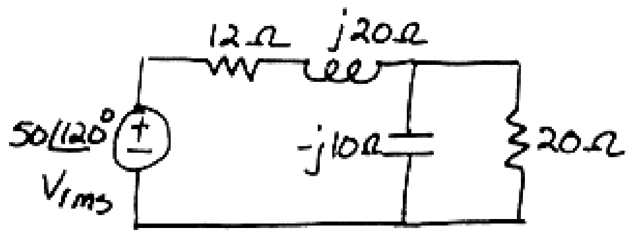
$V_1 = \dfrac{80}{(4-j)-j8(1+j)} = (16/3)\angle 126.9^\circ$

$\therefore I = -\frac{1}{8}V_1 - V_2 = -\frac{1}{8}V_1 - V_1(1+j) + j10 = 2.6\bar{6}\angle 126.9^\circ$

$\therefore$Complex power = S= 1/2 $I^*(-(1/8)V_1)$ = 1/2 $\left(2.\overline{66}\angle -126.9^\circ\right)\left(-(2/3)\angle 36.9^\circ\right) = -j8/9$ VA

Now S=P+jQ=j $\frac{8}{9} \Rightarrow \underline{P=0, Q=\frac{8}{9}\text{VAR}}$

P11.5-5



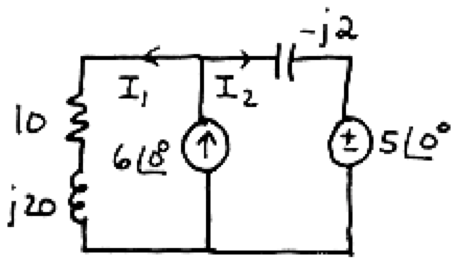
$$Z_{eq} = 16 + j12 = 20\angle 36.87^\circ$$

$$I = \frac{50\angle 120^\circ}{20\angle 36.87^\circ} = 2.5\angle 83.13^\circ \text{ A}$$

$$S = VI^* = \left(50\angle 120^\circ\right)\left(2.5\angle -83.13^\circ\right)$$

$$= 125\ \angle 36.87^\circ = \underline{100 + j75 \text{ VA}}$$

P 11.5-6



$$\text{KVL}: \left(10+j20\right)I_1 = 5\angle 0^\circ - j2I_2 \qquad (1)$$

$$\text{or } \left(10+j20\right)I_1 + j2I_2 = 5\angle 0^\circ$$

$$\text{KCL}: \quad I_1 + I_2 = 6\angle 0^\circ \qquad (2)$$

$\underline{\text{Powers delivered}}$ by sources $\rightarrow S = \frac{1}{2} VI^*$

$$S_{5\angle 0^\circ} = \frac{1}{2}\left(5\angle 0^\circ\right)\left(-I_2^*\right) = 2.5\left(6.41\angle(180-4.47)\right) = -16.0 + j1.1$$

$$S_{6\angle 0^\circ} = \frac{1}{2}\left[5 - j2I_2\right]\left(6\angle 0^\circ\right) = \left[5 - j2\left(6.39 + j.5\right)\right]3 = 18.0 - j38.3$$

$$S_{\substack{\text{Total}\\\text{delivered}}} = S_{5\angle 0^\circ} + S_{6\angle 0^\circ} = \underline{2.0 - j37.2} \text{ VA}$$

$$\text{From (1) \& (2) } \Delta = \begin{vmatrix} 10 + j20 & j2 \\ 1 & 1 \end{vmatrix} = 10 + j18$$

$$I_1 = \frac{1}{\Delta}\begin{vmatrix} 5 & j2 \\ 6 & 1 \end{vmatrix} = \frac{5 - j12}{10 + j18} = 0.63\angle 232^\circ \text{A} = -.39 - j.5$$

$$I_2 = 6 - I_1 = 6 + 3.9 + j.5 = 6.39 + j.5 = 6.41\angle 4.47^\circ$$

$\underline{\text{Powers absorbed}}$

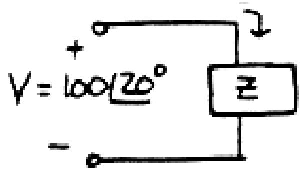
$$S_{10\Omega} = \frac{1}{2}10\left|I_1\right|^2 = \frac{10}{2}(.63)^2 = 2.0$$

$$S_{j20\Omega} = \frac{j20}{2}\left|I_1\right|^2 = j4.0$$

$$S_{-j2\Omega} = \frac{1}{2}(-j2)\left|I_2\right|^2 = -j\ (6.41)^2 = -j41.1$$

$$S_{\substack{\text{Total}\\\text{absorbed}}} = \underline{2.0 - j37.1\text{VA}} \quad \text{to numerical accuracy } S_{del} = S_{abs}$$

P11.5-7



(a) $Z = \frac{V}{I} = \frac{100\angle 20^\circ}{25\angle -10^\circ} = \underline{4\angle 30^\circ\,\Omega}$

(b) $P = \frac{1}{2}|I||V|\cos\theta = \frac{1}{2}(100)(25)\cos 30^\circ = \underline{1082.5\text{W}}$

(c) $Y = \frac{1}{Z} = .25\angle -30^\circ = .2165 - j.125$

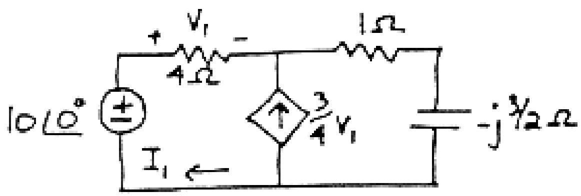
So to cancel phase, need to add $Y_c = j.125$

$= $ admittance of a capacitor

$\therefore\ \omega C = .125$

$\Rightarrow \underline{C = 1.25\text{mF}}$

P 11.5-8



KCL at top node: $-V_1/4 - 3/4\,V_1 + (10 - V_1)/(1 - j\frac{3}{2}) = 0 \ \Rightarrow\ V_1 = 4\angle 36.9^\circ$

$\therefore\ I_1 = V_1/4 = 1\angle 36.9^\circ \ \Rightarrow I_{1rms} = 1/\sqrt{2}\,\angle 36.9^\circ$

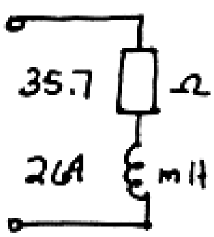
$\therefore$ complex power phasor for source = $S = I^*_{1rms}\ V_{rms} = \left(1/\sqrt{2}\angle -36.9^\circ\right)\left(\frac{10}{\sqrt{2}}\angle 0^\circ\right) = \underline{5\angle -36.9^\circ\ \text{VA}}$

power factor = pf $= \underline{\cos\left(-36.9^\circ\right) = .8 \text{ leading}}$

P11.5-9

1) Inrush conditions $\quad Z\text{ coil} = \frac{VA(\text{inrush})}{I^2(\text{inrush})} = \frac{136.2}{(1.135)^2} = 105.73\Omega$

$R_{coil} = \frac{\text{Watts(inrush)}}{I^2(\text{inrush})} = \frac{46.0}{(1.135)^2} = \underline{35.7\Omega}$

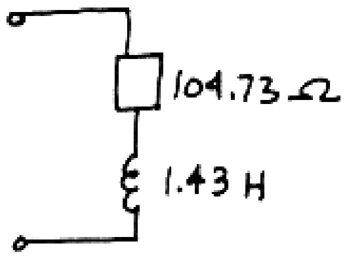


$X_L = \sqrt{Z^2 - R^2} = \sqrt{(105.73)^2 - (35.7)^2} = 99.51\Omega$

$L = \frac{X_L}{2\pi f} = \frac{99.51}{2\pi(60)} = \underline{0.264\text{ H}}$

2) Seal conditions : $Z_{coil} = \frac{\text{Seal VA}}{(\text{Seal I})^2} = \frac{26.22}{(0.2185)^2} = 549.2\Omega$

$R_{coil} = \frac{\text{Seal watts}}{(\text{Seal I})^2} = \frac{5.0}{(0.2185)^2} = \underline{104.73\Omega}$



$$X_L = \sqrt{(549.2)^2 - (104.73)^2} = 539.12\Omega$$

$$L = \frac{539.12}{2\pi 60} = \underline{1.43H}$$

P 11.5-10

R_s = Source resistance = 3Ω

R_L = Lamp equivalent resistance

p_T = Total power consumption

P_L = Lamp power consumption

η = Power transfer efficiency

$$P_T = \left(\frac{V_s}{R_L + R_S}\right)^2 (R_L + R_S) = \underline{\frac{v_s^2}{R_L + R_S}},\ P_L = \underline{\left(\frac{v_s}{R_L + R_S}\right)^2 \cdot R_L}$$

$$\underline{\eta = \frac{P_L}{P_T} = \frac{R_L}{R_L + R_S}}$$

For incandescent : $P_T = \dfrac{120^2}{192 + 3} = \underline{73.85\ W}$; $P_L = \left(\dfrac{120}{195}\right)^2 (192) = \underline{72.71\ W}$

$$z = \frac{72.71}{73.85} = \underline{0.98}$$

For compact flourescent: $P_T = \dfrac{(120)^2}{800 + 3} = 17.93\ W$; $P_L = \dfrac{(120)^2}{(803)^2} \cdot 800 = \underline{17.87\ W}$

$$z = \frac{17.87}{17.97} = \underline{0.99}$$

Although less power is delivered to the flourescent lamp, it has higher illumination and power transfer efficiency.

Section 11.6: Power Factor

P11.6-1

For heating : $P_{AV} = 30kW$

For motor : $\theta = \cos^{-1}(.6) = 53.1^\circ$, $VI_{rms} = 150\ kVA$

$$\Rightarrow\ P_{AV} = 150 \cos 53.1^\circ = 90kW$$

$$Q = 150 \sin 53.1^\circ = 120kVAR$$

$\therefore$ total $P_{av} = 30 + 90 = 120\ kW$

so for the plant: $P_{AV} = 120\ kW$

$$Q = 0 + 120 = 120\ kVAR$$

$$\Rightarrow\ S = 120 + j120 = 170\angle 45^\circ$$

$$\therefore\ \underline{pf = \cos 45^\circ = .707}$$

Then the current in the plant is $\underline{I = \dfrac{(VI)}{4kV} = \dfrac{170kVA}{4kV} = 42.5A}$

P 11.6-2

Load 1: $P_1 = VI\cos\theta = (12\text{kVA})(.7) = 8.4\text{kW}$

$Q_1 = VI\sin\left(\cos^{-1}(.7)\right) = 12\sin\left(45.6^\circ\right) = 8.57\text{kVAR}$

Load 2: $P_2 = (10\text{kVA})(.8) = 8\text{kW}$

$Q_2 = 10\sin\left(\cos^{-1}(.8)\right) = 10\sin\left(36.9^\circ\right) = 6.0\text{kVAR}$

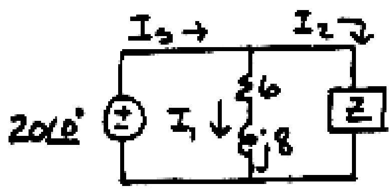
$\therefore$ Total: $S_T = P_T = jQ_T = 8.4 + 8 + j(8.57 + 6.0)$

$= 16.4 + j14.57 = 21.9\angle 41.6^\circ$

So $\underline{pf_T = \cos\left(41.6^\circ\right) = .75}$, $\underline{P_{TAV} = 8.4\text{kW},}$ and

$VI_T = 21.9\text{kVA}$

P11.6-3



$S = \frac{1}{2} V_s I_s^* \Rightarrow I_s^* = \frac{2S}{Vs}$

$= \dfrac{2\left(50\angle\cos^{-1}0.8\right)}{20\angle 0^\circ}$

$= 5\angle 36.9^\circ$

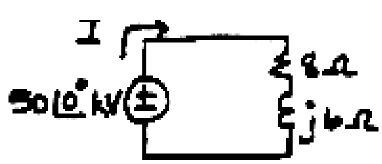
$\therefore\ I_s = 5\angle -36.9^\circ = 4 - j3\text{ A}$

Now $I_1 = \dfrac{V_s}{6 + j8} = \dfrac{20\angle 0^\circ}{10\angle 53.1^\circ} = 2\angle -53.1^\circ = 1.2 - j1.6$

$I_2 = I_s - I_1 = 4 - j3 - 1.2 + j1.6 = 2.8 - j1.4 = 3.13\angle -26.6^\circ$

$\therefore\ Z = \dfrac{V_s}{I_z} = \dfrac{20\angle 0^\circ}{3.13\angle -26.6^\circ} = \underline{6.39\angle 26.6^\circ\ \Omega}$

P11.6-4



$f = 60\text{ Hz}$

$I = \frac{50\angle 0^\circ}{8 + j6} = 5\angle -36.9^\circ\text{ KA}$

(a) $P_{complex} = S = 2VI^* = 2\left(50\angle 0^\circ\right)\left(5\angle 36.9^\circ\right) = \underline{500\angle 36.9^\circ\text{ MVA}}$

(b) Power factor = $pf = \cos\left(36.9^\circ\right) = \underline{.8\text{ lagging}}$

(c) Using Equation 11-25

$$X = \frac{8^2 + 6^2}{-8\tan\left(\cos{-}1(.95)\right) - 6} = -11.588$$

P 11.6-5 Using all rms values

(a) $P = |I|^2 R = |V|^2 / R \Rightarrow |V|^2 = P \cdot R = (500)(20)$

$|V| = 100\text{W}$

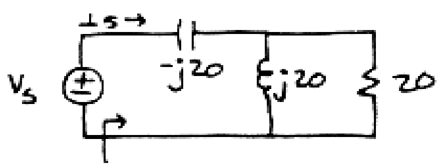
$\therefore\ \underline{V_{rms} = 100\angle 0^\circ\ \text{V}}$

(b) $I_s = I + I_L$ = where $I = V/20 = 100\angle 0^\circ / 20 = 5\ \text{A}$

$I_L = V/j20 = 100\angle 0^\circ / 20\angle 90^\circ = -j\,5\ \text{A}$

$\therefore \underline{I_s = 5 - j5 = 5\sqrt{2}\angle -45^\circ\ \text{A}}$

(c)



$Z_s = -j20 + \dfrac{(20)(j20)}{20 + j20} = 10\sqrt{2}\angle -45^\circ\,\Omega$

$\therefore$ circuit power factor = $\underline{\cos(-45^\circ) = 1/\sqrt{2}\ \text{leading}}$

(d) Because no average power gets dissipated in the capacitor or inductor, then

$P_{\underset{\text{source}}{AVE}} = P_{\underset{20\Omega}{AVE}} = 500\ \text{W}$

$\Rightarrow |V_s||I_s|\cos\theta = 500\ \text{W}$

$\Rightarrow |V_s| = \dfrac{500}{|I_s|\cos\theta} = \dfrac{500}{(5\sqrt{2})\left(1/\sqrt{2}\right)} = \underline{100\ \text{V}}$

P11.6-6

$V = 100\angle 160^\circ\ \text{V}$

$I = 2\angle 190^\circ\ \text{A} = -1.97 - j0.348$

$P_1 = 23.2\text{W},\ Q_1 = 50\ \text{VARs}$

$\underline{S}_1 = P_1 + jQ_1 = 23.2 + j50 = 55.12\angle 65.1^\circ\ \ \text{VA}$

$pf_1 = \cos 65.1^\circ = .422\ \text{lag}$

Now $\underline{I_1}^* = \dfrac{S_1}{V_s} = \dfrac{55.12\angle 65.1^\circ}{100\angle 160^\circ} = 0.551\angle -94.9^\circ$, so $I_1 = 0.551\angle 94.9$

$\underline{I}_2 = \underline{I} - \underline{I}_1 = -1.97 - j.348 + .047 - j.549 = 2.12\angle -155^\circ\ \text{A}$

$\underline{S}_2 = VI_2^* = (100\angle 160^\circ)(2.12\angle 155^\circ) = 212\angle -45^\circ = 150 - j150$

$pf_2 = \cos(-45^\circ) = .707\ \text{leading}$

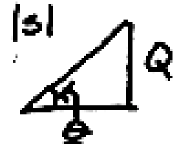
$\underline{S} = \underline{S}_1 + \underline{S}_2 = (23.2 + j50) + (150 - j150) = 173.2 - j100 = 200\angle -30^\circ$

So total pf = $\cos(-30^\circ) = \underline{.866\ \text{leading}}$

P11.6-7

$\underline{S} = P + jQ,\ I = 1,\ V = \sqrt{2}(120), \cos\theta = .6$

(a) $P = \frac{1}{2} VI\cos\theta = \frac{1}{2}(120)\ (1)\ \sqrt{2}\ (.6) = 50.9\ \text{W}$



$Q = \frac{P\sin\theta}{\cos\theta} = 67.8\ \text{VAR}$ $\qquad \sin\theta = .8$

so $\theta = 53.1^\circ$

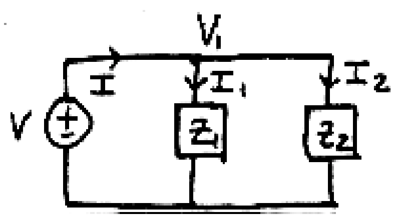
$\underline{S = 50.9 + j67.8}$

(b) With $Z = R + j67.8$

$P = \frac{I^2 R}{2}$ or $R = \frac{2P}{I^2} = \frac{2(50.9)}{1^2} = \underline{101.8\Omega}$

$Q = \frac{1}{2}\omega L I^2$ or $L = \frac{2Q}{\omega I^2} = \frac{2(67.8)}{377(1^2)} = \underline{0.36\ \text{H}}$

P11.6-8



$P_1 = V_1 I_1 \cos\theta_1 = 4800(.85) = 4080\ \text{W}$

$\therefore\ P_T = P_1 + P_2 = 4080 + 4000 = 8.08\ \text{kW}$

$Q_1 = V_1 I_1 \sin\theta_1 = 4800\ \sin 31.8^\circ = 2529\ \text{VAR}$

$Q = VI \sin\left(\cos^{-1}.75\right) = 8080 \sin 41.4^\circ = 5343\ \text{VAR}$

So $Q_2 = Q - Q_1 = 5343 - 2529 = 2814\ \text{VAR}$

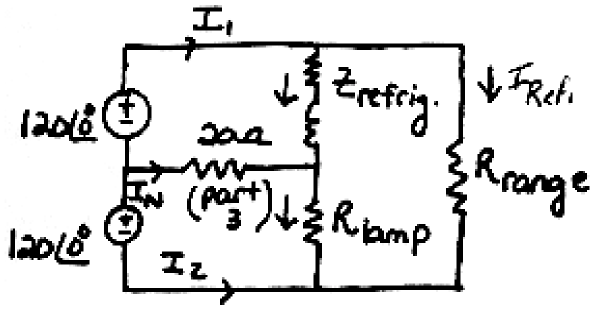
Now $P_2 = V_2 I_2 \cos\theta_2$, $Q_2 = V_2 I_2 \sin\theta_2$

$\therefore\ \frac{Q_2}{P_2} = \tan\theta_2 = \frac{2814}{4000} = 0.704 \Rightarrow \theta_2 = 35.1^\circ$

Apparent power load 2 $= V_2 I_2 = \frac{P_2}{\cos\theta_2} = \frac{4000}{0.82} = \underline{4878\ \text{W}}$

Power factor $= \cos\theta^2 = \underline{0.82}$

P 11.6-9



$$|Z_{refrig}| = \frac{120V}{8.5A} = 14.12\Omega$$

$$Z_{refrig} = 14.12\angle 45^\circ = 10 + j10\Omega$$

$$R_{lamp} = V^2/P = \frac{(120)^2}{100} = 144\Omega$$

$$R_{range} = \frac{(240)^2}{12{,}000} = 4.8\Omega$$

1) Now $I_{refrig} = \dfrac{120\angle 0^\circ}{10+j10} = 8.5\angle -45^\circ \text{ A}$, $I_{lamp} = \dfrac{120\angle 0^\circ}{144} = 0.83\angle 0^\circ \text{ A}$

$$I_{range} = \frac{240\angle 0^\circ}{4.8} = 50\angle 0^\circ \text{ A}$$

From KCL: $I_1 = I_{refrig} + I_{range} = 56 - j6 = \underline{56.3\angle -6.1^\circ \text{ A}}$

$$I_2 = -I_{lamp} - I_{range} = \underline{50.83\angle 180^\circ} \text{ A}$$

$$I_N = -I_1 - I_2 = \underline{7.92\angle -49^\circ \text{ A}}$$

2) $P_{refrig} = I_{refrig}{}^2 R_{refrig} = \underline{722.5\text{W}}$

$Q_{refrig} = I_{refrig}{}^2 = \underline{722.5 \text{ VAR}}$

Now $|S| = |V||I| = (120)(8.5) = 1020 \text{ VA}$

$$S = 1020\angle 45^\circ = 722 + j722 \text{ VA}$$

$$P_{lamp} = 100\text{W},\ Q_{lamp} = 0$$

$$P_{TOT} = 722 + 100 + 12{,}000 = \underline{12.82\text{kW}}$$

$$Q_{TOT} = 722 + 0 + 0 = \underline{722\text{VAR}}$$

$\Rightarrow$ $S = 12{,}822 + j722 = \underline{12.84\text{kVAR}\angle 3.2^\circ}$

pf = $\cos(3.2^\circ) = \underline{0.998}$

Continued

3) Mesh equations: $\begin{bmatrix} 30+j10 & -20 & -10-j10 \\ -20 & 164 & -144 \\ -10-j10 & -144 & 158.8+j10 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 120\angle 0^\circ \\ 120\angle 0^\circ \\ 0 \end{bmatrix}$

yields $I_A = 54.3\angle -1.7^\circ \text{ A}$

$$I_B = 51.3\angle -0.2^\circ \text{ A}$$

$$I_C = 50.0\angle 0^\circ \text{ A}$$

Thus $|V_{lamp}| = R_{lamp}|I_B - I_C| = 144|1.27\angle -8.6^\circ| = \underline{183.2\text{V}}$

Lamp will not last very long!

P 11.6-10

a) $VI = 220(7.6) = \underline{1672\ VA}$

$pf = \frac{P}{VI} = \frac{1317}{1672} = \underline{.788}$

$\theta = \cos^{-1} pf = 38.0^\circ \Rightarrow \underline{Q = VI\sin\theta = 1030VAR}$

b) To restore the pf to 1.0, a capacitor is required to eliminate Q by introducing –Q, then

$$1030 = \frac{V^2}{X_c} = \frac{(220)^2}{X_c} \Rightarrow X_c = 47\Omega$$

$$\therefore \underline{C = 1/\omega X = 1/(377)(47) = 56.5\mu F}$$

c) $P = VI\cos\theta$ where $\theta = 0^\circ$

then $1317 = 220I$

$\therefore$ $\underline{I = 6.0A \text{ for corrected}}$ pf

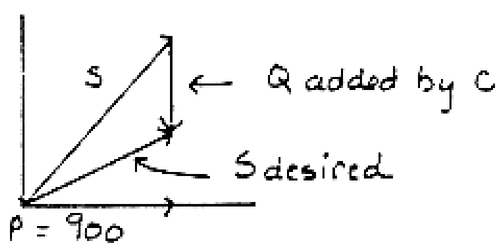
*Note $I = 7.6A$ for uncorrected pf

P11.6-11

First load: $S_1 = P + jQ = P(1 + j\tan(\cos^{-1}(.6))) = 500(1 + j\tan 53.1^\circ) = 500 + j677kVA$

Second load: $S_2 = 400 + j600kVA$

$\therefore S = S_1 + S_2 = 900 + j1277kVA$



$S_{desired} = P + jP\tan(\cos^{-1}(.90))$
$= 900 + j436$

From vector diagram $S_{desired} = S + Q$

$\therefore 900 + j436 = 900 + j1277 + Q \Rightarrow Q = -j841$ VAR

$$\text{So } \frac{|V|^2}{Z^*} = -j841 \Rightarrow Z^* = \frac{|V|^2}{-j841} = \frac{(1000)^2}{-j841} = j1189$$

$$\therefore Z = -j1189 = -j/(377)C$$

So $\underline{C = 1/(1189)(377) = 2.20\mu F}$

P11.6-12

(a) $S=P+jQ=P+jP\tan(\cos^{-1}pf)=1000+j1000\tan(\cos^{-1}.8)=1000+j750$

Let $V_L=100\angle 0^\circ$ rms

Then $I^*=S/V_L=\dfrac{1000+j750}{100\angle 0^\circ}=10+j7.5$

so $I=10-j7.5$

$\therefore Z_L=\dfrac{V_L}{I}=\dfrac{100\angle 0^\circ}{12.5\angle -36.9^\circ}=8\angle 36.9^\circ=6.4+j4.8$

$\Rightarrow V=[6.4+j(200)(.024)+Z_L](I)=\underline{(12.8+j9.6)(10-j7.5)=200\angle 0^\circ\ V}$

(b) Need $Z_L^*=Z_L||Z_{new}=\dfrac{1}{Y_L+Y_{new}}$

$\dfrac{1}{(6.4-j4.8)}=Y_L+Y_{new}$

$\Rightarrow Y_{new}=\dfrac{1}{6.4-j4.8}-\dfrac{1}{6.4+j4.8}=j.15$

So $Z_{new}=-j6.67\Omega$

$\therefore$ need capacitor $\dfrac{1}{\omega C}=6.67\ \Rightarrow\ C=\dfrac{1}{(6.67)(200)}=\underline{0.075\mu F}$

P11.6-13

$S=P+jQ$ and $|S|=\dfrac{P}{pf}=\dfrac{100}{.8}=125kVA$

So $Q=|S|\sin(\cos^{-1}.8)=125\sin(36.9^\circ)=75k\ VAR$

(a) pf of .95 lagging

$P=100kW,\ Q=P\tan(36.9^\circ)=32.9\ kVAR$

$\Rightarrow |S|=(P^2+Q^2)^{1/2}=[(100)^2+(32.9)^2]^{1/2}=105.3kVA$

$\therefore$ <u>released capacity=125–105.3=19.7 kVA</u>

(b) pf of 1.0

Need $Q=0\ \Rightarrow\ |S|=P=100kVA$

$\therefore$ <u>released =125–100=25kVA</u>

(c) Relative capacity required

Part (a): 75–32.9 = <u>42.1 kVAR</u>

Part (b): 75 – 0 = <u>75 kVAR</u>

(d) Corrected pf

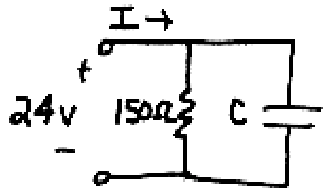
	.95	1.0
released capacity	19.7 kVA	25 kVA
required reactive capacity	42.1 kVAR	75 kVAR
ratio	~ 1/2	1/3

P11.6-14

$I = 0.2$ A

$f = 400$ Hz

pf = .8 leading



$$|Y| = \frac{I}{V} = \frac{0.2}{24} = 8.33 \text{ mS}$$

$$\angle\theta_Y = \cos^{-1}(.8) = 36.9^\circ$$

$$\therefore\ Y = 8.33\angle 36.9^\circ = 6.67 + j5 \text{ mS}$$

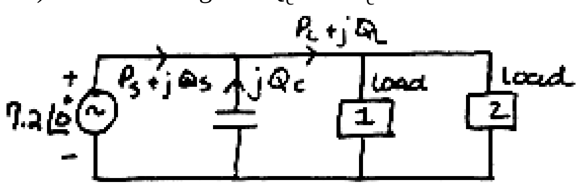
$$\text{so } R = \frac{1}{G} = \frac{1}{6.67\times10^{-3}} = 150\Omega \ \Rightarrow \text{ checks}$$

$$B = \omega C \ \text{ or } \ C = \frac{5\times10^{-3}}{2\pi(400)} = \underline{1.99\mu F}$$

P11.6-15

This example demonstrates that loads can be specified either by kW or kVA. The procedure is as follows:

1) Combine loads and determine the overall $P_L + jQ_L$
2) Given that $P_S = P_L$, determine the required Q_S
3) Determine $Q_C = Q_L - Q_S$ (Note: assume $f = 60H_z$)
4) Determine C given Q_c and V_c



$$P_L + jQ_L = (45+45) + j(21.8+20.5)$$
$$= 90 + j42.3 \text{ KVA}$$

$$\therefore P_s = 90 \text{ kW and } Q_s = \frac{90}{0.97}\sin(\cos^{-1}0.97)$$
$$= 22.6 \text{ kVAR}$$

$$\text{so } \underline{Q_c = 42.3 - 22.6 = 19.7 \text{ kVAR}}$$

$$X_c = \frac{|V_c|^2}{Q_c} = \frac{(7.2\times10^3)^2}{19.7\times10^3} = 2626\ \Omega \ \Rightarrow\ C = \frac{1}{377\,(2626)} = \underline{1.01\mu F}$$

P11.6-16

pf	P	Q kVAR	$\lvert S\rvert$ (kVA)	$\lvert I\rvert$	
.6 lag	48 kW	64	80	160	load 1
.96 load	24 kW	-7	25	50	load 2
	72 kW	57	91.8	184	Total

$$\text{Load 1:} |S_1| = \frac{48kW}{.6} = 80kVA,\quad Q_1 = \sqrt{(80)^2 - (48)^2} = 64 \text{ kVAR}$$

$$|I_1| = \frac{80 \text{ kVA}}{500V} = 160A$$

$$\text{Load 2:} |S_2| = \frac{24kVA}{.96} = 25kVA,\ Q_2 = -\sqrt{(25)^2 - (24)^2} = -7 \text{ since lead pf}$$

$$|S| = \sqrt{|S_1|^2 + |S_2|^2} = \sqrt{80^2 + 25^2} = 91.8 \text{ kVA}$$

$$|I| = \frac{91.8 \text{ kVA}}{500} = 184 \qquad \text{total pf} = \frac{P_{TOT}}{|S|_{TOT}} = \frac{72}{91.8} = .784$$

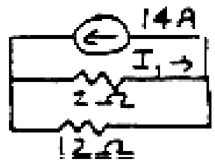
Need correction $Q_c = -57$ kVA

$$C = \frac{-Q_c}{\omega|V|^2} = \frac{57\times10^3}{377(500)^2} = \underline{605\mu F}$$

Section 11-7: The Power Superposition Principle

P11.7-1

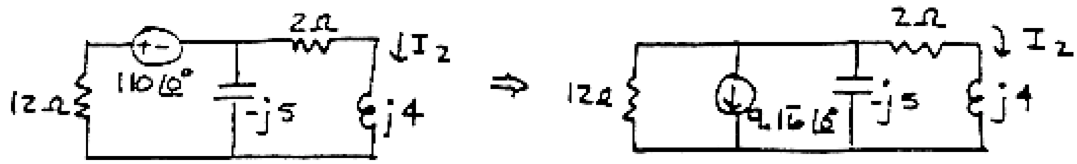
Use superposition since we have two different frequency sources.
First consider dc sources ($\omega = 0$)



$$I_1 = 14\left(\frac{12}{12+2}\right) = 12 \text{ A}$$

$$\therefore P_{1\,AVE} = I_1^2 R = (12)^2(2) = \underline{288 \text{ W}}$$

Consider $\omega = 20$ rad/s source



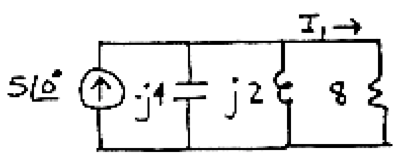
$$\text{Current divider} \Rightarrow I_2 = -9.166\left[\frac{\dfrac{-j60}{(12-j5)}}{\dfrac{-j60}{(12-j5)}+2+j4}\right] = \frac{25}{\sqrt{5}}\angle 116.6^\circ$$

$$\therefore P_{2AVE} = 1/2\, |I_2|^2\ (2) = 1/2(125)(2) = \underline{125 \text{ W}}$$

$$\text{so } P_{AVE} = P_{1\,AVE} + P_{2\,AVE} = 288 + 125 = \underline{413 \text{ W}}$$

P11.7-2

Use superposition since we have two different frequency sources
First consider $\omega = 2000$ rad/s source

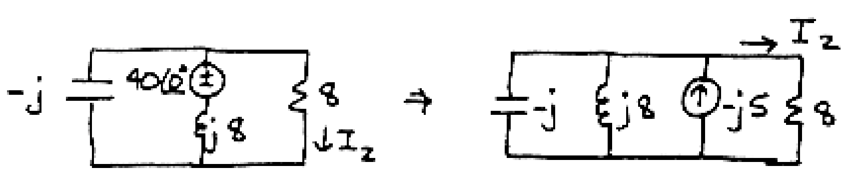


Using current divider

$$I_1 = 5\left[\frac{\dfrac{8}{-j2}}{\dfrac{8}{-j2}+8}\right] = \frac{5}{\sqrt{5}}\ \angle 63.4^\circ$$

$$\therefore\ P_{1\,AVE} = \tfrac{1}{2}\ |I_1|^2 8 = \underline{20 \text{ W}}$$

Next consider $\omega = 8000$ rad/s source



$$I_2 = -j5\left[\frac{\dfrac{j7}{8}}{\dfrac{8}{j7}+8}\right] = \frac{5}{\sqrt{50}}\ \angle -171.9^\circ$$

Current divider yields

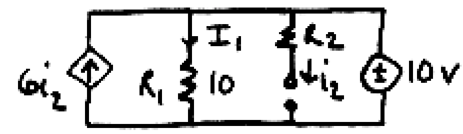
$$\therefore\ P_{2\,AVE} = \tfrac{1}{2}|I_2|^2 8 = \underline{2 \text{ W}}$$

$$\text{So } \underline{P_{AVE} = P_{1\,AV} + P_{2\,AV} = 22 \text{ W}}$$

P11.7-3

Use superposition

$\underline{\omega = 0}$

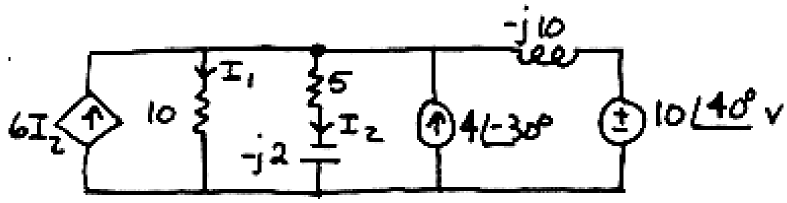


$$I_1 = 10\left(\frac{1}{10}\right) = 1\text{ A} \quad i_2(t) = 0$$

$$\therefore P_{R_1\,AVE} = I_1^2 R_1 = 1^2(10) = 10\text{ W}$$

$$P_{R_2\,AV} = 0\text{ W}$$

$\underline{\omega = 5}$



KCL at top node ($V_a = 10\,I_1$)

$$-6I_2 + I_1 + I_2 - 4\angle{-30^\circ} + \frac{(10I_1 - 10\angle 40^\circ)}{j10} = 0 \qquad (1)$$

KVL $\quad -10I_1 + (5 - j2)I_2 = 0 \qquad (2)$

Solving (1) and (2) yields $I_1 = -0.56\angle{-64.3^\circ}$ A

$$I_2 = -1.04\angle{-42.5^\circ}\text{ A}$$

So $P_{R_1 AV} = \frac{1}{2} I_{1m}^2 R_1 = \frac{1}{2}(.56)^2(10) = 1.57\text{ W}$

$$P_{R_2 AV} = \frac{1}{2} I_{2m}^2 R_2 = \frac{1}{2}(1.04)^2(5) = 2.7\text{ W}$$

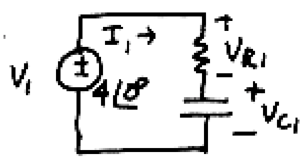
$$\therefore P_{R_{1\,TOT}} = 10\text{ W} + 1.57\text{ W} = \underline{11.57\text{ W}}$$

$$P_{R_{2\,TOT}} = 0 + 2.7\text{ W} = \underline{2.7\text{ W}}$$

P11.7-4

Use superposition

$\underline{\omega = 10}$

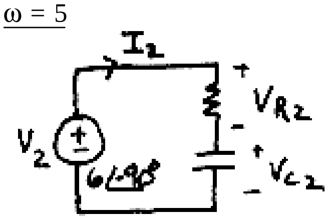


$$I_1 = \frac{V_1}{Z} = \frac{4\angle 0^\circ}{2 - j5} = 0.28 + j0.7\text{ A}$$

$$V_{R_1} = 2I_1 = 2(.28 + j.7) = .56 + j1.4 = 1.51\angle 68.2^\circ$$

$$V_{C_1} = -j5I_1 = 3.77\angle{-21.8^\circ}\text{ V}$$

$\underline{\omega = 5}$



$$I_2 = \frac{V_2}{Z} = \frac{6\angle{-90^\circ}}{2 - j10} = .577 - j0.12\text{ A}$$

$$V_{R_2} = 2I_2 = 2(.577 - j0.12) = 1.15 - j0.24$$

$$= 1.17\angle{-11.8^\circ}\text{ V}$$

$$V_{C_2} = -j10I_2 = 5.9\angle 258.3^\circ \text{ V}$$

So $V_R(t) = 1.51\cos(10t + 68.2^\circ) + 1.17\cos(5t - 11.8^\circ)$ V

$V_C(t) = 3.77\cos(10t - 21.8^\circ) + 5.9\cos(5t - 258.3^\circ)$ V

so $V_{Reff}^2 = \left(\frac{1.51}{\sqrt{2}}\right)^2 + \left(\frac{1.17}{\sqrt{2}}\right)^2 = 1.82 \Rightarrow \underline{V_{Reff} = 1.35 \text{ V}}$

$V_{Ceff}^2 = \left(\frac{3.77}{\sqrt{2}}\right)^2 + \left(\frac{5.9}{\sqrt{2}}\right)^2 = 24.52 \Rightarrow \underline{V_{Ceff} = 4.95 \text{ V}}$

Section 11-8: Maximum Power Transfer Theorem

P11.8-1

$$Z_t = 4000 \| -j2000 = 800 - j1600$$

$$Z_L = Z_t^* = 800 + j1600$$

$$R + j1000L = 800 + j1600 \Rightarrow \begin{cases} R = 800\Omega \\ L = 1.6 \text{ H} \end{cases}$$

P11.8-2

$$Z_t = 25{,}000 \| -j50{,}000 = 20{,}000 - j10{,}000$$

$$Z_L = Z_t^* = 20{,}000 + j10{,}000$$

$$R + j\omega L = 20{,}000 + j10{,}000 \Rightarrow \begin{cases} R = 20 \text{ k}\Omega \\ 100L = 10{,}000 \\ L = 100 \text{ H} \end{cases}$$

The choices yield $|I| = 1.4\text{mA}$ and

$$P = \left(\frac{0.14 \times 10^{-2}}{\sqrt{2}}\right)^2 (20\text{k}) = 19.5\text{mW}$$

Since the maximum power is >12 mW, then yes, we can deliver 12mW to the load.

P11.8-3

$$Z_t = 800 + j1600$$

$$Z_L = \frac{R\left(\frac{-j}{\omega C}\right)}{R - \frac{j}{\omega C}} = \frac{R - j\omega R^2 C}{1 + (\omega RC)^2} = Z_t^* = 800 - j1600$$

Equating the real parts gives

$$800 = \frac{R}{1 + (\omega RC)^2} = \frac{4000}{1 + [(5000)(4000)C]^2}$$

$$\Rightarrow C = 0.1\mu\text{F}$$

P 11.8-4

$$Z_t = 400 + j800$$
$$Z_L = 2000 \parallel -j1000 = 400 - j800$$

Since $Z_L = Z_t^*$ the average power delivered to the load is maximum and cannot be increased by adjusting the value of the capacitance.

The voltage across the 2000Ω resistor is

$$V_R = 5\frac{Z_L}{Z_t + Z_L} = 2.5 - j5 = 5.59e^{-j63.4}$$

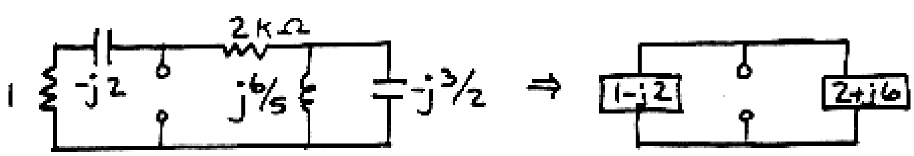
So $$P = \left(\frac{5.59}{\sqrt{2}}\right)^2 \frac{1}{2000} = \underline{7.8 \text{ mW}}$$ is the average power delivered to the 2000Ω resistor.

P11.8-5

Notice that $\mathbf{Z}_t$,not $\mathbf{Z}_L$, is being adjusted .When $\mathbf{Z}_t$ is fixed, then the average power delivered to the load is maximized by choosing $\mathbf{Z}_L = \mathbf{Z}_t$*. In contrast, when $\mathbf{Z}_L$ is fixed, then the average power delivered to the load is maximized by minimizing the real part of $\mathbf{Z}_t$. In this case, choose R = 0. Since no average power is dissipated by capacitors or inductors, all of the average power provided by source is delivered to the load.

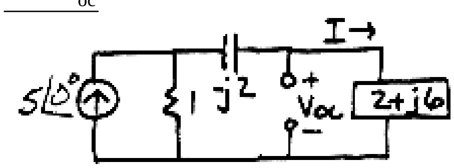
P 11.8-6

Find Z_T (open current source)



$$\therefore Z_T \frac{(1-j2)(2+j6)}{1-j2+2+j6} = \underline{2\sqrt{2}\angle -44.9^\circ \text{ k}\Omega}$$

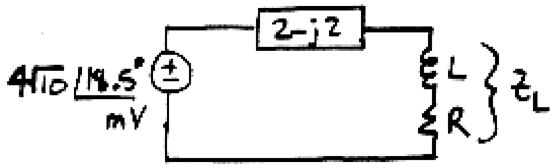
Find V_{oc}



Current divider $\Rightarrow$ $I = 5\left[\frac{1}{1+2+j4}\right] = 1\angle -53.1^\circ$ mA

$$\Rightarrow V_{oc} = (2+j6)(I) = \sqrt{40}\angle 71.6^\circ\left(1\angle 53.1^\circ\right) = \underline{2\sqrt{10}\angle 18.5^\circ \text{ V}}$$

$\therefore$ have



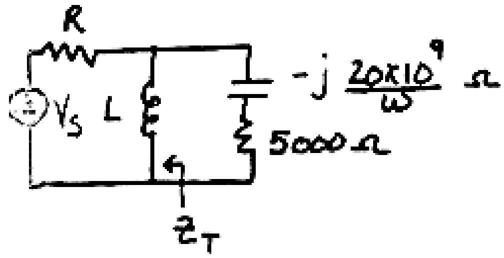
For maximum power transfer $Z_L = Z_T^*$

$\Rightarrow R + j2 \times 10^6 L = 2 + j2 \text{ k}\Omega$

$\Rightarrow R = 2\text{k}\Omega$ and $L = 1\text{mH}$

$$\text{Now } P_{Lmax} = 1/2 \frac{\left|\frac{V_{oc}}{2}\right|^2}{\text{Re}(Z_L)} = 1/2 \frac{\left(1\sqrt{10}\right)^2}{2\text{k}\Omega} = 2.5 \text{ mW}$$

P11.8-7



Here $\omega = 4\times10^6\,\text{s}^{-1}$, $V_s = 10\angle0^\circ$

$$Z_T = \frac{(R)(j\omega L)}{R + j\omega L} = \frac{(R - j\omega L)(R)(j\omega L)}{R^2 + (\omega L)^2}$$

$$= \frac{(\omega L)^2 R}{R^2 + (\omega L)^2} + \frac{\omega L R^2}{R^2 + (\omega L)^2}$$

$$\&\quad Z_L = 5000 - j\frac{20\times10^9}{\omega}$$

So equating $Z_L = Z_T^*$ yields:

$$5000 = \frac{(\omega L)^2 R}{R^2 + (\omega L)^2} \quad (1) \qquad \& \qquad \frac{20\text{x}10^9}{\omega} = \frac{\omega L R^2}{R^2 + (\omega L)^2} \quad (2)$$

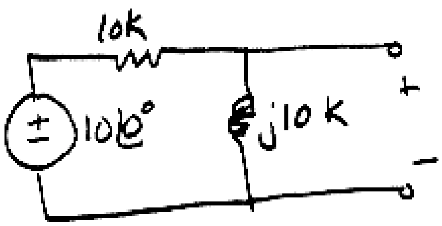
Dividing (2) by (1) yields : $\frac{R}{L} = 4\times10^6$

From (2) we can write $L = \frac{\left[(R/L)^2 + \omega^2\right] 20\,\text{x}\,10^9}{\omega^2 (R/L)^2}$

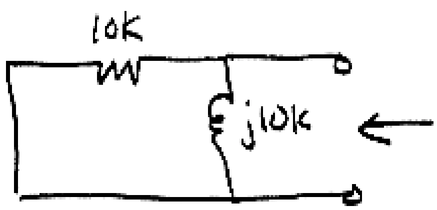
$$L = \frac{\left[16\times10^{12} + 16\times10^{12}\right] 20\times10^9}{\left(16\times10^{12}\right)\left(16\times10^{12}\right)} = 2.5\times10^{-3}\text{H} = \underline{2.5\text{mH}}$$

$$\therefore R = 4\times10^6 L = \underline{10\text{k}\Omega}$$

Thévenin Equivalent



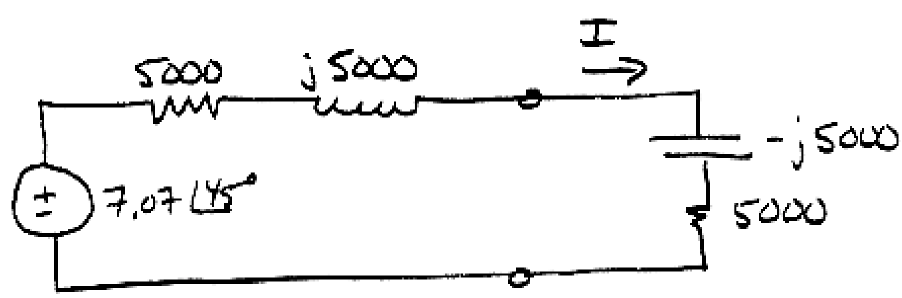
$$V_{oc} = \left(\frac{j10\text{k}\Omega}{10\text{k}\Omega + j10\text{k}\Omega}\right) 10\angle0^\circ = 7.07\angle45^\circ\ \text{V}$$



$$Z_{TH} = \frac{(10\text{k}\Omega)\,(j10\text{k}\Omega)}{10 + j10\text{k}\Omega} = 7070\angle45^\circ$$

$$= 5000 + j5000\Omega$$

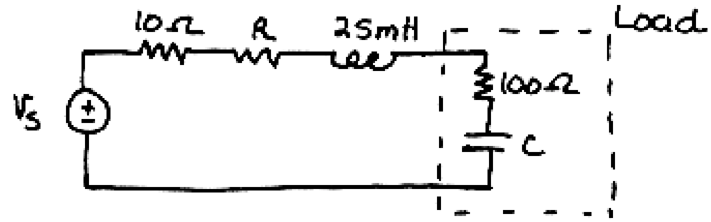
So we have



$$I = \frac{7.07\angle 45^\circ}{(5+j5-j5+5)\ k\Omega} = 0.7\angle 45^\circ\ mA$$

$$P = I^2R = (0.7)^2(5000) = \underline{2.5mW}$$

P11.8-8



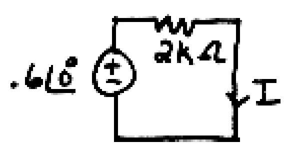
For maximum power transfer, set R = zero, then set $X_L = X_{TH}^*$

$$\Rightarrow 1/\omega C = \omega L \text{ or } C = \frac{1}{\omega^2 L} = \frac{1}{(10^3)^2(.025)} = \underline{40\mu F}$$

$$P_{Lmax} = 1/2\ I^2(100) = 1/2\left(\frac{100}{10+100}\right)^2(100) = \underline{41.3\ W}$$

P11.8-9

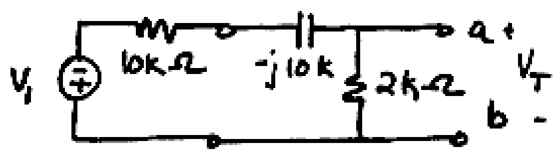
Lefthand circuit



$$I = \frac{.6}{2k\Omega} = .3mA$$

$50I = 15mA,\ V_1 = 150\ V$

Right-side equivalent circuit

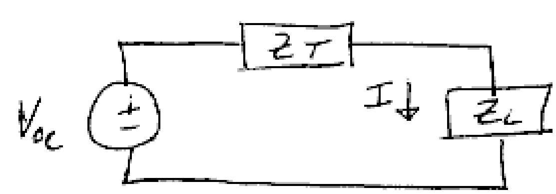


$$V_T = \frac{2000(150)}{12{,}000 - j10{,}000} = 19.2\angle 39.8\ V$$

To get $Z_T = 0$ set $V_s = 0$ then $I = 0$

$$Z_T = \frac{2k\Omega(10k\Omega - j10k\Omega)}{2k\Omega + 10k\Omega - j10k\Omega} = \frac{2(10-j10)}{(12-j10)}\ k\Omega = \underline{1.81\angle -5.2^\circ\ k\Omega}$$

$V_{oc} = 19.21\angle -140^\circ\ V$ So

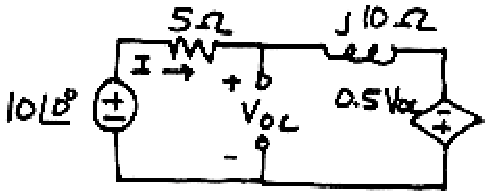


$$|I| = \frac{19.21}{2(1805)} = 5.3mA$$

and $$P_L = \frac{|I|^2}{2}R = \frac{(5.3x10^{-3})^2(1805)}{2} = \underline{25.6\ mW}$$

P11.8-10

a) Find V_{oc}



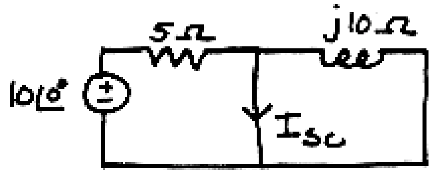
$\omega = 100 \text{ rad/sec}$

KVL: $-10+5I+j10I-0.5V_{oc} = 0$ (1)

also : $I = \frac{10 - V_{oc}}{5}$ (2)

Solving (1) & (2) yields $\underline{V_{oc} = 8\angle 36.9^\circ = 6.4+j4.8 \text{ V}}$

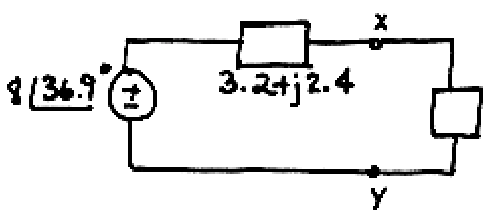
Find I_{sc}



$I_{sc} = \frac{10\angle 0^\circ}{5} = 2\angle 0^\circ \text{ A}$

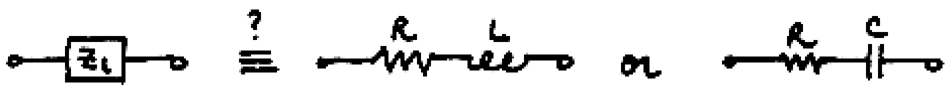
$\therefore Z_{TH} = \frac{V_{oc}}{I_{sc}} = \underline{3.2 + j2.4\Omega}$

So the Thévenin Equivalent circuit is



$Z_L = Z_{TH}^* = \underline{3.2-j2.4\Omega}$ for maximum power transfer

b)

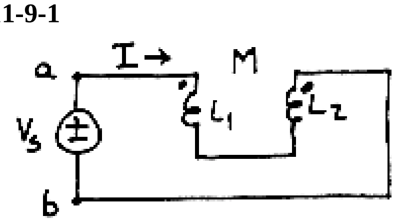


$\Rightarrow$ capacitive load with $\underline{R = 3.2\Omega}$ & $\underline{C = \frac{1}{(100)(2.4)} = 4.17\text{mF}}$

c) $P_{L_1 max} = \frac{|V_{TH}|^2}{8R_L} = \frac{64}{8(3.2)} = \underline{2.5 \text{ W}}$

Section 11-9: Mutual Inductance

P11-9-1

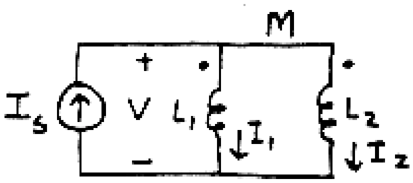


KVL : $V_s + Ij\omega L_1 + Ij\omega M + Ij\omega L_2 - Ij\omega M = 0$

$\Rightarrow j\omega(L_1+L_2-2M) = \frac{V_s}{I}$

$\underline{\therefore L_{ab} = L_1+L_2-2M}$

P11.9-2



KCL at top : $I_1 + I_2 = I_s$ (1)

also : $V = I_1 j\omega L_1 + I_2 j\omega M$ (2)

$V = I_2 j\omega L_2 + I_1 j\omega M$ (3)

Solving for I_1 in (1) and plugging into (2) and (3)

$$\Rightarrow I_2 = \frac{V - j\omega L_1 I_s}{j\omega(M - L_1)} \qquad (4)$$

$$V = I_2 j\omega L_2 + (I_s - I_2) j\omega M \qquad (5)$$

Plugging (4) into (5)

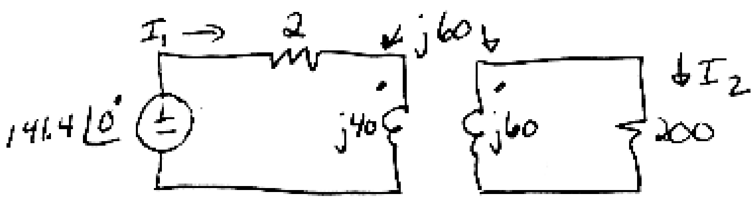
$$V = \frac{(V - j\omega L_1 I_s)\left[j\omega(L_2 - M)\right]}{j\omega(M - L_1)} + j\omega M I_s$$

Solving for the ratio V/I_s

$$\Rightarrow \frac{V}{I_s} = j\omega\left[\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}\right]$$

$$\therefore L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

P11.9-3



$\omega = 100$ rad/sec

loop l : $-141.4\angle 0^\circ + 2I_1 + j40I_1 - j60I_2 = 0$ (1)

loop2 : $200\, I_2 + j60I_2 - j60I_1 = 0 \Rightarrow I_2 = (0.23\angle 51^\circ)I_1$ (2)

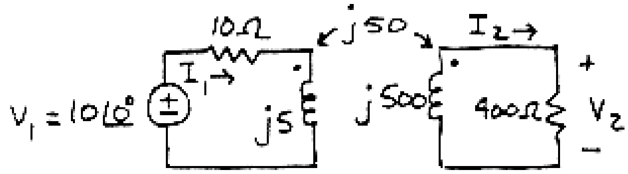
Plugging this into (1) yields $I_1 = 4.17\angle -68^\circ$ A

Plugging this into (2) yields $I_2 = 0.96\angle -17^\circ$ A

So $i_1(t) = 4.2\cos(100t - 68^\circ)$ A

and $i_2(t) = 1.0\cos(100t - 17^\circ)$ A

P11.9-4



KVL I_1: $(10 + j5)I_1 - j50I_2 = 10$

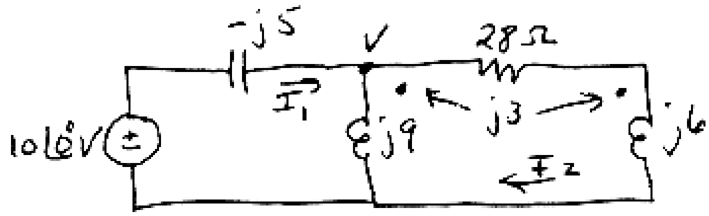
KVL I_2: $-j50I_1 + (400 + j500)I_2 = 1$

Using Cramer's rule, $I_2 = \dfrac{(10+j5)(0)-(-j50)(10)}{(10+j5)\,(400+j500) - (-j50)^2} = 0.062\angle 29.7^\circ$

$$\therefore \frac{V_2}{V_1} = \frac{400 I_2}{10\angle 0^\circ} = 40I_2 = 40\left(0.062\angle 29.7^\circ\right) = 2.5\angle 29.7^\circ$$

P11.9-5

$\omega = 30$ rad/sec



loop1 : $-10\angle 0^\circ - j5I_1 + j9I_1 + j3I_2 = 0$ (1)

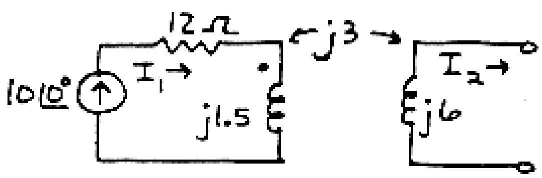
loop2 : $28I_2 + j6I_2 + j3I_1 + j9I_2 - j3I_2 = 0$ (2)

Solving (1)+(2) yields $I_1 = 0.25\angle 161^\circ$ and $I_2 = 2.55\angle -86^\circ$

but $V = j\,9(I_1 - I_2)$ where $I_1 - I_2 = 2.6\angle -81^\circ$, so $V = 23\angle 9^\circ$

or $\underline{v(t) = 23\cos(30t + 9^\circ)\text{ V}}$

P11.9-6



(a) Open - circuit

$I_2 = 0 \Rightarrow I_1 = 10\angle 0^\circ \Rightarrow i_1(0) = 10\text{ A}$

$\therefore \underline{W = 1/2\, L_1 i_1^2(0) = \frac{1}{2}(.3)(10)^2 = 15\text{ J}}$

(b) Short - circuit

KVL right loop : $j\,6\,I_2 - j\,3\,I_1 = 0 \Rightarrow I_1 = 2\,I_2$

$I_1 = 10\angle 0^\circ \Rightarrow I_2 = 5\angle 0^\circ$

$\therefore W = \frac{1}{2} L_1 i_1^2(0) + \frac{1}{2} L_2 i_1^2(0) - M\, i_1(0)\, i_2(0)$

$\underline{W = \frac{1}{2}(.3)(10)^2 + \frac{1}{2}(1.2)(5)^2 - (.6)(10)(5) = 0}$

(c) Connected to 7Ω

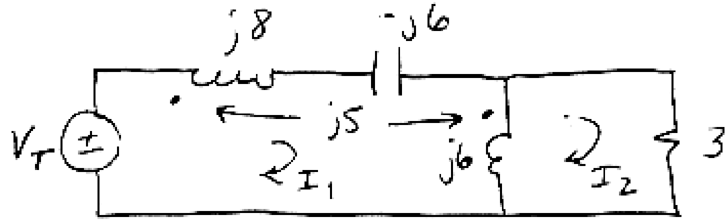
KVL right loop : $(7 + j6)\,I_2 - j\,3\,I_1 = 0 \Rightarrow I_2 = 3.25\angle 49.4^\circ$

so $i_2(t) = 3.25\cos(5t + 49.4)$

$\Rightarrow i_2(0) = 2.12\text{ A}$

$\therefore \underline{W = \frac{1}{2}(.3)(10)^2 + \frac{1}{2}(1.2)(2.12)^2 - (.6)(10)(2.12) = 5.0\text{ J}}$

P11.9-7



$$-V_T + j8I_1 + j5(I_1 - I_2) - j6I_1 + j6\,(I_1 - I_2) + j5I_1 = 0 \quad (1)$$

$$3I_2 + j6(I_2 - I_1) - j5I_1 = 0 \quad (2)$$

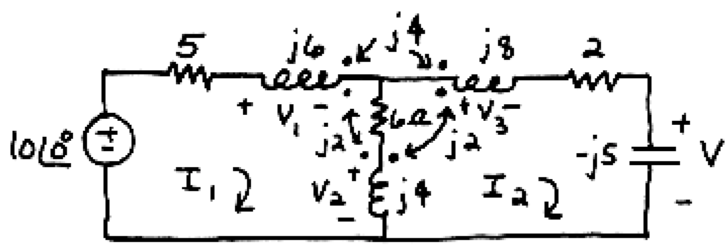
From Eq. (2) : $I_2 = (1.64\,\angle 27^\circ)\,I_1$ (3)

From Eq. (1) : $I_1(j18) + I_2(-j11) = V_T$ (4)

Plugging Eq. (3) into (4) and solving yields

$$Z = \frac{V_T}{I_1} = 8.2 + j^2 = \underline{8.4\,\angle 14^\circ\,\Omega}$$

P11.9-8



$$\text{3 coils :}\begin{cases} V_1 = j6I_1 - j2\,(I_1 - I_2) - j4I_2 = j4I_1 - j2I_2 \\ V_2 = j4(I_1 - I_2) - j2I_1 + j2I_2 = j2I_1 - j2I_2 \\ V_3 = j8I_2 - j4I_1 + j2(I_1 - I_2) = -j2I_1 + j6I_2 \end{cases}$$

mesh I_1 : $5\,I_1 + V_1 + 6\,(I_1 - I_2) + V_2 = 10\angle 0^\circ$

mesh I_2 : $-V_2 + 6\,(I_2 - I_1) + 2\,I_2 + V_3 - j5\,I_2 = 0$

Substituting V_1, V_2, V_3 into mesh equations

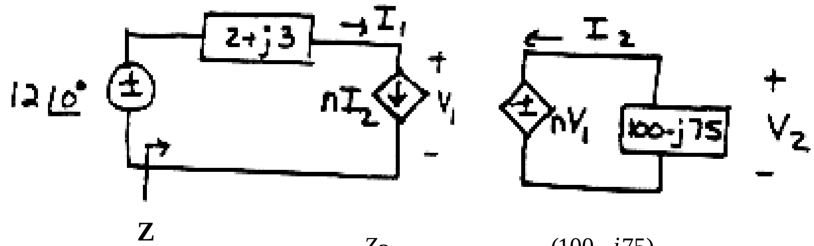
$$I_2 = \frac{\begin{vmatrix} 11 + j6 & 10 \\ -6 - j4 & 0 \end{vmatrix}}{\begin{vmatrix} 11 + j6 & -6 - j4 \\ -6 - j4 & 8 + j3 \end{vmatrix}} = \frac{60 + j40}{50 + j33} = 1.2\,\angle 0.28^\circ$$

Now $V = -j5\,I_2 = 6.0\,\angle -89.72^\circ$

$\therefore$ $\underline{v(t) = 6\sin(2t - 89.7^\circ)\ \text{V}}$

Section 11-10: The Ideal Transformer

P11.10-1



$$Z = Z_s + \frac{Z_2}{n^2} = (2 + j3) + \frac{(100 - j75)}{25} = 6\Omega$$

$$\therefore I_1 = \frac{12\angle 0^\circ}{Z} = \frac{12\angle 0^\circ}{6} = 2\ A$$

$$\Rightarrow V_1 = I_1\left(\frac{Z_2}{n^2}\right) = (2)(4 - \text{j}3) = 10\angle -36.9^\circ \text{V}$$

$$\textit{Now}\ V_2 = nV_1 = 5\ (10\angle -36.9^\circ) = \underline{50\ \angle -36.9^\circ\ V}$$

$$\textit{and}\ \ \underline{I_2 = \frac{I_1}{n} = \frac{2}{5} A}$$

P11.10-2

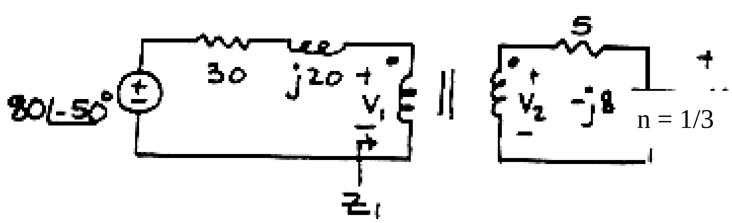
(a) $V_0 = (5\times 10^{-3}\text{A})(10{,}000\Omega) = 50\text{ V}$

$$\therefore \frac{N_1}{N_2} = \frac{V_1}{V_0} = \frac{10}{50} = \frac{1}{5} \Rightarrow \underline{n = 5}$$

(b) $R_{ab} = \dfrac{1}{n^2} R_2 = \dfrac{1}{25}(10\text{k}\Omega) = \underline{400\Omega}$

(c) $I_s = \dfrac{10}{R_{ab}} = \dfrac{10}{400} = \underline{0.025\text{A} = 25\text{mA}}$

P11.10-3



$$Z_1 = \frac{1}{n^2} Z_2 = 9Z_2 = 9(5 - \text{j}8) = 45 - \text{j}72$$

From voltage division, voltage across Z_1 is:

$$V_1 = 80\angle -50^\circ \left(\frac{45 - j72}{45 - j72 + 30 + j20}\right) = 74.4\angle -73.3^\circ$$

$\therefore$ the voltage across the secondary coil is:

$$V_2 = nV_1 = \frac{74.4}{3}\angle -73.3^\circ = 24.8\ \angle -73.3^\circ$$

Then from voltage division again

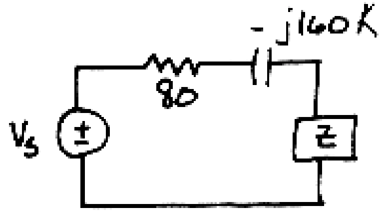
$$V_c = V_2\left(\frac{-j8}{5 - j8}\right) = \left(24.8\angle -73.3^\circ\right)\frac{\left(8\angle -90^\circ\right)}{\sqrt{89}\angle -58^\circ} = \underline{21.0\angle -105.3^\circ V}$$

P11.10-4 $n = 5,\ Z_1 = \frac{200}{(5)^2} = 8 \ \Rightarrow V_1 = \frac{8}{10}(50) = 40\text{V} = 40\angle 0^\circ$

now $\underline{V_2 = nV_1 = 200\angle 0^\circ \text{ V}}$

P11.10-5

$\omega = 10^5$



$$Z = \frac{320}{n^2} + \frac{j\omega L}{n^2}$$

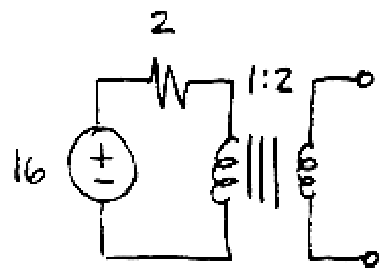
For maximum power transfer, need $\frac{j\omega L}{n^2} = j160\text{k}\Omega$

and $\frac{320}{n^2} = 80 \ \therefore \underline{n = 2}$

If $n = 2$, then $\omega L = 640\text{k}\Omega$

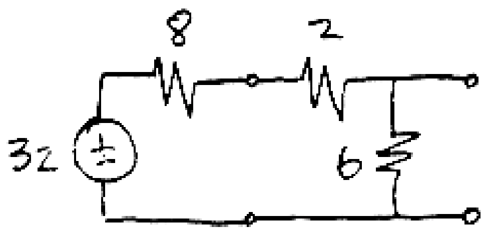
$$L = \frac{640\text{k}\Omega}{10^5} = \underline{6.4 \text{ H}}$$

P11.10-6



$V_{oc} = 32$ V

$Z_{Th} = (2)^2 2 = 8\Omega$



$V_{oc} = \frac{6}{16}(32) = 12$ V

$Z_{Th} = 6\|10 = 3.75\Omega$

P11.10-7

$$V_2 = \frac{1}{2}V_1$$

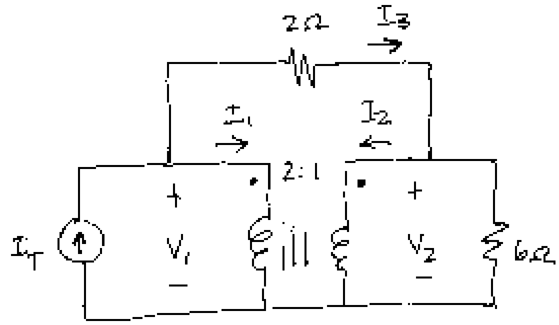
$$I_3 = \frac{V_1 - V_2}{2} = \frac{V_1}{4}$$

$$I_2 = I_3 - \frac{V_2}{6} = \frac{V_1}{6}$$

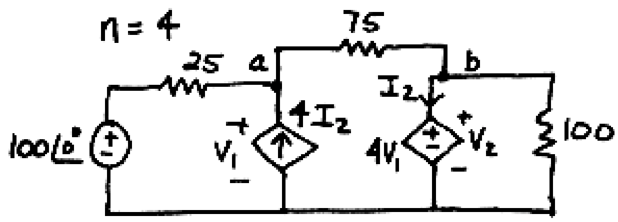
$$I_1 = -\frac{1}{2}I_2 = -\frac{V_1}{12}$$

$$I_T = I_3 - I_1 = \frac{V_1}{6}$$

$$Z = \frac{V_1}{I_T} = 6$$



P 11.10-8



$$I_1 = -4I_2$$

$$V_2 = 4V_1$$

KCL at a : $\dfrac{V_a - 200}{25} + \dfrac{V_a - V_o}{75} - 4I_2 = 0$ (1)

at b: $\dfrac{V_b}{100} + \dfrac{V_b - V_a}{75} + I_2 = 0$ (2)

also $V_b = 4V_a$ (3)

Solving (1) – (3) yields $V_a = \dfrac{25}{2}$ V, $V_b = 4V_a = 50$ V

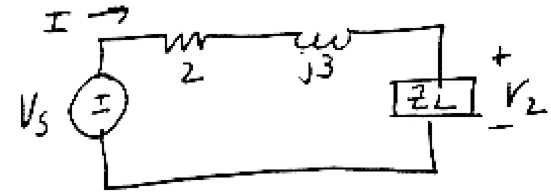
P11.10-9

Maximum Power requires $X_{Th} = X_L^*$

$$X_{C1}\left(\frac{1}{n_2^2}\right) = X_{L1},\ \frac{1}{n_2^2} = \frac{2}{10} = \frac{1}{5} \Rightarrow \underline{n_2 = \sqrt{5}}$$

$$\text{Now}\left[R_L\left(\frac{1}{n_2^2}\right) + 1\Omega\right]\left(\frac{1}{n_1^2}\right) = 100\Omega \quad \Rightarrow \frac{1}{n_1^2} = \frac{100}{3} \text{ or } \underline{n_1 = \frac{\sqrt{3}}{10}}$$

P11.10-10



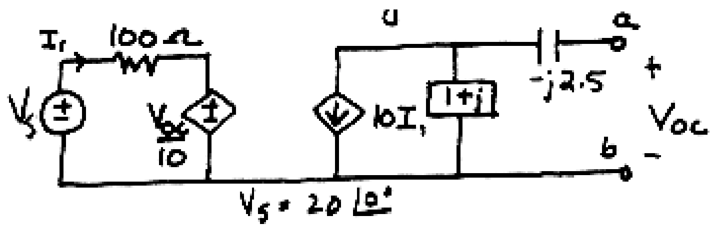
$$Z_L = \frac{20(1+j\,7.54)}{20+10+j7.54} = 8.1\angle 23^\circ\,\Omega$$

$$Z_L{}^1 = \frac{Z_L}{n^2} = \frac{8.1\angle 23^\circ}{25} = 0.3+j0.13\Omega$$

$$P_L = \frac{|V_L|^2}{2R_2} = \frac{|V_2|^2}{2R_L} = \frac{(230)^2}{2(0.3)} = 88\text{kW for 1 home} \quad \therefore \underline{529\text{kW for six homes}}$$

P11.10-11

Phasor circuit finding V_{oc} first

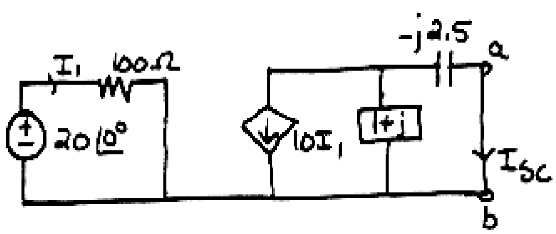


KVL: $100I_1 + 0.1V_{oc} = 20\angle 0^\circ$ (1)

KCL: $10I_1 + \frac{V_{oc}}{1+j} = 0$ (2)

above yields $\underline{V_{oc} = -2.86\angle 45.6^\circ\text{ V}}$

Now find I_{sc}



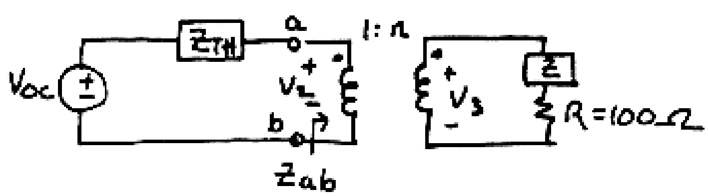
$$I_1 = \frac{20\angle 0^\circ}{100} = 0.2\angle 0^\circ\text{ A}$$

$$I_{sc} = -\frac{(1+j)}{(1+j)-j2.5}(10I_1)$$

$$= -1.57\angle 101.3^\circ\text{ A}$$

So $Z_{th} = \frac{V_{oc}}{I_{sc}} = \frac{-2.86\angle 45.6^\circ}{-1.57\angle 101.3^\circ} = \underline{1.03-j1.5\Omega}$

So have Thévenin Equivalent circuit



Assume $Z = jX$

$$Z_{ab} = \frac{1}{n^2}(R+Z)$$

$$= \frac{1}{n^2}(R+jX)$$

when $Z_{ab} = Z_{th}^* \Rightarrow$ max. power transfer

So $\frac{1}{n^2}(R+jX) = 1.03+j1.15$ So $\frac{R}{n_2} = 1.03$ or $\underline{n = 9.85 \text{ turns}}$

$$\frac{X}{n^2} = 1.5 \quad \text{or} \quad X = 145.6\Omega$$

$\underline{\text{So } Z = jX = j145.6\Omega} \leftarrow$ an inductor

So have $j\omega L = j145.6\,\Omega$ yields $\underline{L = 145.6\text{H}}$

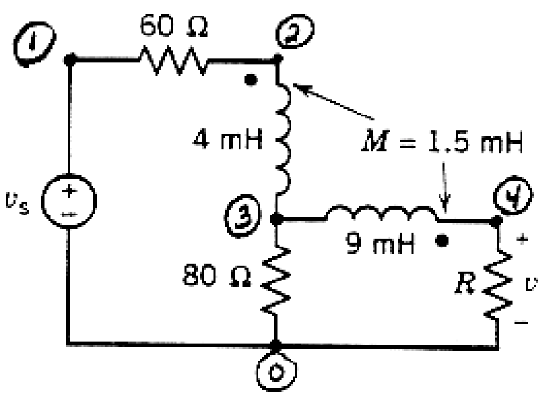
$$\text{Now } V_2 = \frac{Z_{ab}}{Z_{ab}+Z_{th}} V_{oc} = \frac{1.03+j1.5}{1.03+1.03}\left(-2.86\angle 45.6^\circ\right) = -2.53\angle 101.1^\circ$$

$$V_3 = nV_2 = -24.9\angle 101.1^\circ \text{ V}$$

$$\text{So } P_{max} = \frac{1}{2}\frac{V_m^{\ 2}}{R} = \frac{1}{2}(24.9)^2\left(\frac{1}{1.03}\right) = \underline{300 \text{ W}}$$

PSpice Problems

SP 11-1



$$k = \frac{m}{\sqrt{L_1 L_2}} = 0.25$$

$$\omega = 10^4 \text{ rad/sec}$$

$$v_s = 200\sin\left(\omega t + 45^\circ\right) \text{ V}$$

<u>Input File:</u>

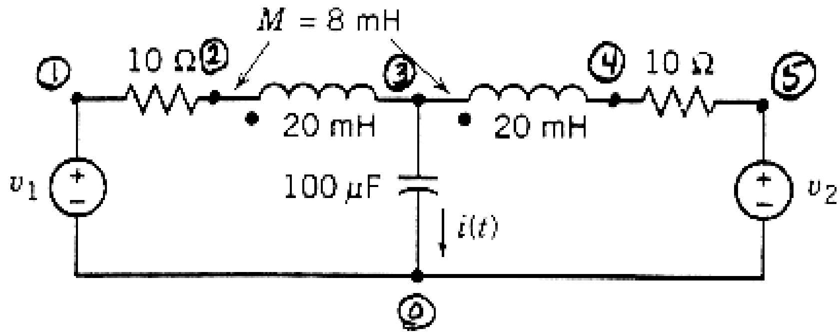
```
Vs   1   0   ac        200   45
R1   1   2   60
R2   3   0   80
R3   4   0   100
L1   2   3   4m
L2   2   3   9m
K1   L1  L2  0.25

.ac lin 1 1591.5 1591.5
.print ac Vm(4) Vp(4)
.end
```

<u>Output:</u>

```
 FREQ        VM(4)       VP(4)
I.592E+03   7.001E+01   7.177E+00
```

$\underline{\text{So } v(t) = 70\sin(\omega t + 7.2^\circ) \text{ V}}$

SP 11-2



$\omega = 500\ \text{rad/sec}$
$V_1 = 100\angle 0^\circ$
$V_2 = 100\angle -90^\circ$
$k = 0.4$

Input File:

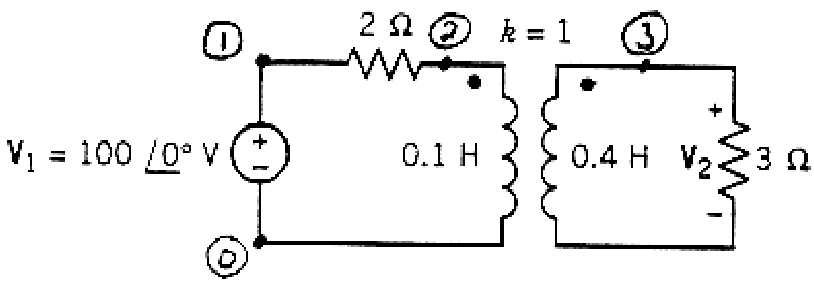
```
V1   1   0   ac   100  0
V2   5   0   ac   100  -90
R1   1   2   10
R2   4   5   10
L1   2   3   20m
L2   3   4   20m
K1   L1  L2  0.4
C1   3   0   100u

.ac lin 1 79.6 79 .6
.print ac Im(C1)  Ip(C1)
.end
```

Output:

```
 FREQ         IM(C1)       IP(C1)
7.960E+01   3.992E+00   2.860E+01
```

So $\underline{i(t) = 4.0\cos\left(500t + 28.6^\circ\right)\ \text{A}}$

SP 11-3



$V_1 = 100\angle 0^\circ$

$\omega = 1000\ \text{rad/sec}$

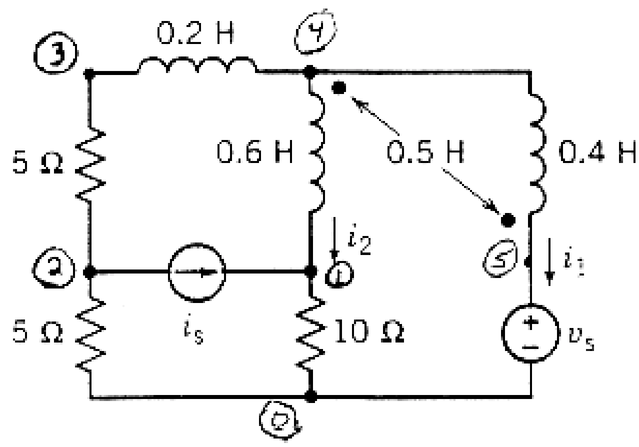
$n^2 = \frac{L}{L} = \frac{0.4}{0.1} = 2$

Input File:
```
Vs    1   0    ac   100   0
R1    1   2    2
R2    3   0    3
L1    2   0    0.1
L2    3   0    0.4
K1    L1  L2   1

.ac lin 1 159.15 159.15
.print ac Vm(3) Vp(3)
.end
```

Output:
```
 FREQ          VM(3)        VP(3)
1.592E+02   5.454E+01   3.125E-01
```

So $\frac{V_2}{V_1} = \frac{54.5}{100} = \underline{0.545} \angle 0^\circ$

SP 11-4



$K = \frac{m}{\sqrt{L_1 L}} = \frac{-0.5}{\sqrt{(0.6)(0.4)}} = -1$

$f = 1.59$

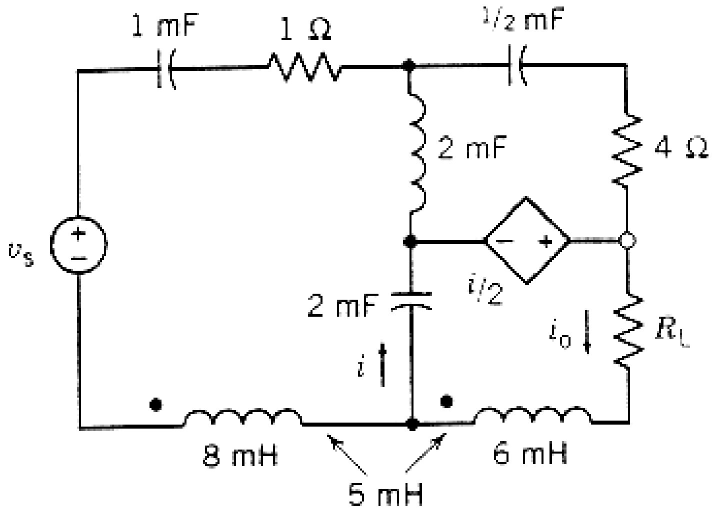
$i_s(t) = 6\cos\left(10t + 45^\circ\right)$

$v_s(t) = 12\sin(10t)$

$= 12\cos\left(10t - 90^\circ\right)$

Input:
```
R1    1  0     10
R2    2  0      5
R3    3  2      5
L1    3  4    0.2
L2    4  1    0.6
L3    4  5    0.4
K1   L2 L3     -1
Is    2  1     ac  6  45
Vs    5  0     ac  12 -90

.ac lin 1  1.59 1.59
.print ac  Im(L3)  Ip(L3)  Im(L2)  Ip(L2)
.end

Output:
FREQ         IM(L3)      IP(3)        IM(L2)      IP(L2)
1.590E+00   1.814E+00   -6.498E+01  3.729E+00   -1.677E+02
```

SP 11-5



$$K = \frac{5}{\sqrt{(8)(6)}} = 0.72$$

$v_s = 2\cos 500\,t$

$R_L = 3\Omega$

Input:

```
Vs       1    0   ac     2   0
R1       2    3   1
R2       4    5   4
RL       5    9   3
C1       1    2   1e-3
C2       3    4   0.5e-3
C3       7    6   2e-3
L1       3    6   2e-3
L2       0    8   8e-3
L3       8    9   6e-3
K1      L2   L3   0.72
Vdummy   8    7   0
H1       5    6   Vdummy   0.5

.ac lin  1  80 80
.print  ac    Im(RL)   Ip(RL)
.end
```

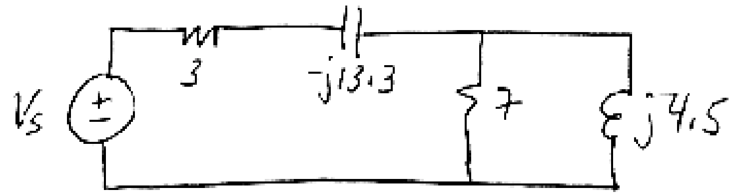
Output:

```
FREQ         IM(RL)       IP(RL)
8.000E+01   5.463E-01   -1.262e+02
```

So $i_0 = 0.546\cos\left(500t + 126^\circ\right)$A

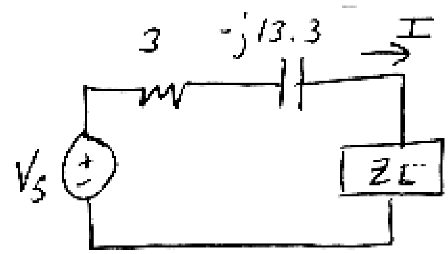
Verification Problems

VP 11-1 Phasor circuit



$$V_S = 163\angle 0^\circ \text{ V}$$

Circuit reduces to $\rightarrow$



$$Z_L = \frac{(7)(j4.5)}{7-j4.5} = \frac{31.5\angle 90^\circ}{8.32\angle 33^\circ} = 3.8\angle 57^\circ = 2.1 + j3.2$$

$$I = \frac{V}{Z_T} = \frac{163\angle 0^\circ}{3-j13.3+2.1+j3.2} = \frac{163\angle 0^\circ}{5-j10}$$

$$= \frac{163\angle 0^\circ}{11.2\angle -63^\circ} = 14.5\angle 63^\circ \text{ A}$$

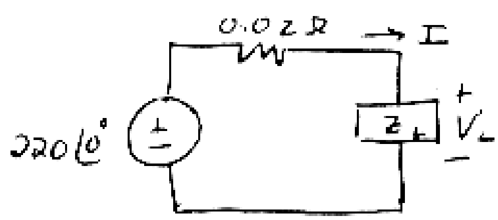
$$S = P + j\theta = VI^* = (163\angle 0^\circ)(14.5\angle -63^\circ) = 2372\angle -63^\circ \text{ VA}$$

$$S = 2372\angle -63^\circ = 1077 - j2114 = P + jQ$$

$\therefore$ <u>P = 1077W and S = 2114 VAR</u> <u>Not as reported</u>

VP 11-2

Simplify circuit to



$$P_L = \frac{|V_L|^2}{2}$$

$$Z_L = \frac{(15+j6)(20+j4)}{15+j6+20+j4} = \frac{(16.2\angle 22^\circ)(20.4\angle 11^\circ)}{36.4\angle 16^\circ} = 9.1\angle 17^\circ = 8.65 + j2.68$$

$$V_L = \left(\frac{Z_P}{Z_P + 0.02}\right) V_S = \frac{(9.06\angle 17^\circ)(220\angle 0^\circ)}{9.08\angle 17^\circ} = 220\angle 0^\circ$$

$$P_L = \frac{|V_L|^2}{R_L} = \frac{(220)^2}{8.65} = \underline{5595\text{W}}$$ <u>Not as reported</u>

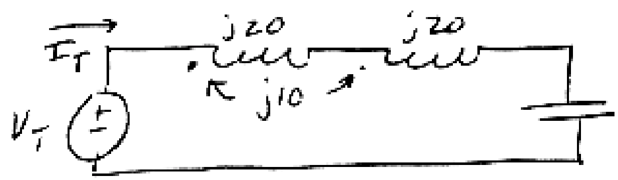
VP 11-3

$$v(t)=\begin{cases}5 & 0<t<5\\ -10 & 5<t<20\\ 5 & 20<t<25\end{cases}$$

$$V_{eff}=\sqrt{\frac{1}{25}\left[\int_0^5 25dt+\int_5^{20}100dt+\int_{20}^{25}25dt\right]}=\sqrt{\frac{1750}{25}}=\underline{8.37\text{ V}}\text{ as stated}$$

VP 11-4

Apply test voltage



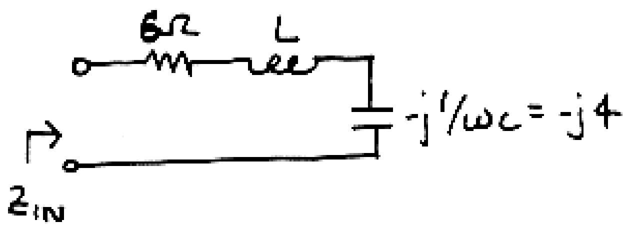
A -j40

loop equation : $j20I_T+j10I_T+j30I_T+j10I_T-j40I_T=V_T$

so $\frac{V_T}{I_T}=Z=\underline{j\,30}\ \Omega$ as reported

Design Problems

DP 11-1



Desire $Z_{IN}=\ 7.21\angle 33.7^\circ$

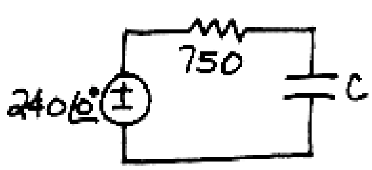
Now $Z_{IN}=6+jX=7.21\angle 33.7^\circ$

$|Z_{IN}|^2=(7.21)^2=6^2+X^2\ \Rightarrow X^2=16.0$ or $X=4$

$\therefore\ Z_{IN}=6+j4=6+j\omega L-j4$

Thus $4=\omega L-4$ or $\underline{L=8/4=2.0\text{ H}}$

DP 11-2



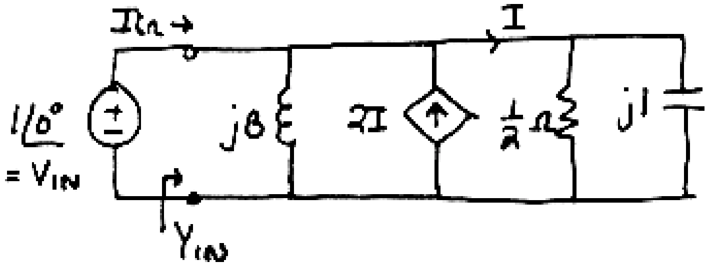
$|I|=\ .2$ desire $|Z|=\frac{240}{2}=1200\Omega$ or less

Now $Z=750-jX$ where $X=1/\omega C$

$|Z|^2=(1200)^2=(750)^2+X^2\ \Rightarrow X=937\Omega$

$\therefore C=\frac{1}{\omega X}=\frac{1}{(2\pi)(400)(937)}=\underline{0.425\mu F}$

DP 11-3



Apply test source $V_{in} = 1\angle 0^\circ$

$$I = V_{in}(2+j) = 1\angle 0^\circ(2+j) = 2+j$$

$$\text{KCL at top node}: \ -2I + I + V_{in}(-jB) = I_{in} \qquad \text{where } B = 1/\omega L$$

$$\Rightarrow \qquad I_{in} = -I - jB = -(2+j) - jB = -2 - j(1+B)$$

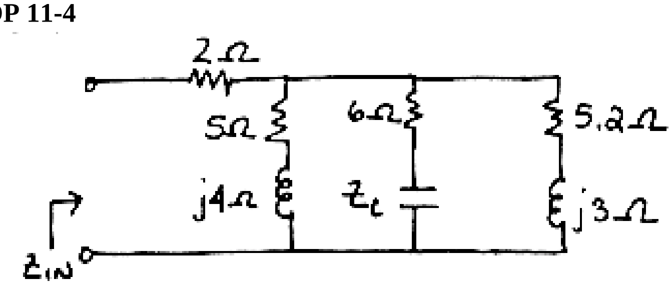
$$\text{Now } Y_{in} = I_{in} = Y\angle 0^\circ \text{ where } Y = \sqrt{2^2 + (1+B)^2}$$

$$\text{Note if } B = 1, \text{ then } Y = \sqrt{8} = 2.83$$

$$\text{and } Y_{in} = 2.83\angle -135^\circ$$

$$\therefore L = \frac{1}{\omega B} = \frac{1}{5\times 10^4} = \underline{20\mu H}$$

DP 11-4



$$\omega = 100 \text{ rad/sec}$$

$$Z_L = -jX \text{ where } X = 1/\omega C$$

$$Z_{in} = 2 + \frac{1}{Y}$$

$$Y = \frac{1}{5+j4} + \frac{1}{6-jX} + \frac{1}{5.2+j3}$$

$$\text{Try } X = 3\Omega \Rightarrow Y = .156\angle -38.7^\circ + .149\angle 26.6^\circ + .167\angle -30^\circ = 0.416\angle -16^\circ$$

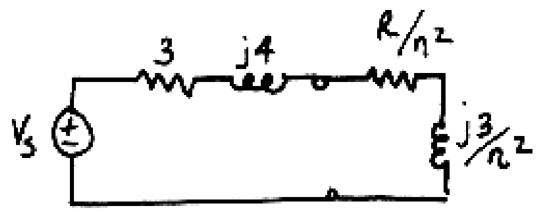
$$\therefore Z_{in} = 2 + 2.4\angle 16^\circ = 4.36\angle 8.72^\circ$$

$$\therefore |Z_{in}| = 4.36 \text{ is inside designed range}$$

$$\text{Thus } C = \frac{1}{\omega X} = \frac{1}{(100)(3)} = \underline{3.33\text{mF}}$$

DP 11-5

Equivalent circuit



$$I = \frac{V}{\left(3+\frac{R}{n^2}\right)+j\left(\frac{3}{n^2}+4\right)}$$

$$P = \frac{\left(\frac{R}{n^2}\right)V^2}{\left(3+\frac{R}{n^2}\right)^2+\left(\frac{3}{n^2}+4\right)^2}$$

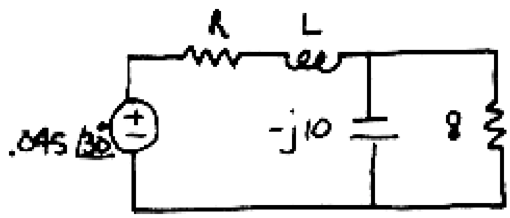
If $R = 4\Omega$

$$P = \frac{n^2RV^2}{25n^4+48n^2+25} \quad \text{thus} \quad \frac{dP}{dn} = 0 = RV^2\left[\frac{2n(25n^4+48n^2+25)-n^2(100n^3+96n)}{(25n^4+48n^2+25)^2}\right]$$

$$\Rightarrow -50n^5+50n = 0 \ \Rightarrow n^4 = 1 \text{ or } \underline{n = 1}$$

If $R = 8\Omega$ similar analysis yields $\underline{n = 1.31}$

DP 11-6



$\omega = 10^6$ rad/sec

$$Z_L = \frac{8(-j10)}{8-j10} = 4.88 - j\,3.9$$

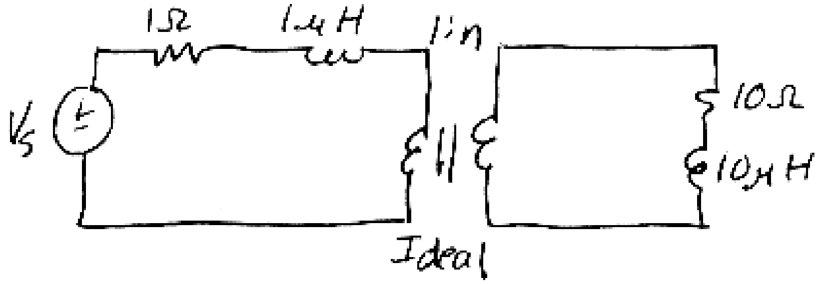
So need $j\omega L = j\,3.9 \Rightarrow$ then $\underline{L = 3.9\mu H}$

For maximum power to 4.88Ω of Z , we need $\underline{R = 0}$

Then all power goes to 4.88Ω.

$$\text{So } P_{max} = \frac{(.045/\sqrt{2})^2}{4.88} = \underline{208\mu W}$$

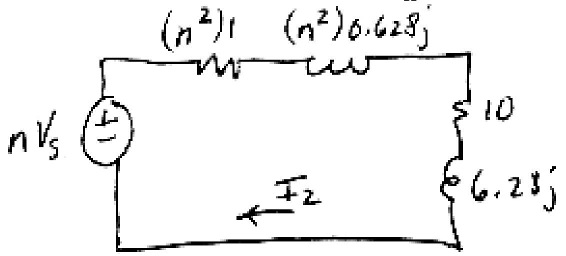
DP 11-7



f = 100KHz

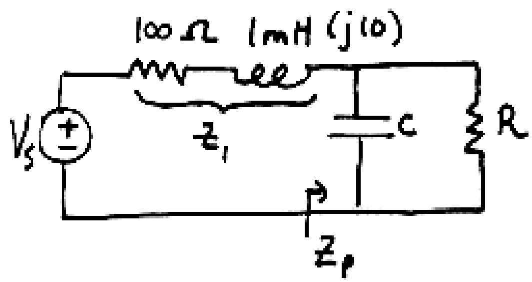
$Z_T = Z_L$ for maximum power transfer

$R_T = 1\Omega$, $R_L = 10\Omega$, so $1 = \frac{10}{n^2} \Rightarrow n^2 = 10 \Rightarrow \underline{n=3.16}$



$$I_2 = \frac{nV_s}{(10+6.28j)2} = \frac{nV_s}{17\angle 32^\circ} = \underline{(0.06\angle -32^\circ)nV_s}$$

DP 11-8



$\omega_0 = 10^7$ rad/sec
$v_s = V_0 \sin\omega_0 t$
$Z_1 = 100 + j10$

$\therefore$need $Z_p = 100 - j10$ for maximum power to R

$$Y_p = j\omega C + \frac{1}{R} = G + j\omega C$$

$$Z_p = \frac{1}{G + j\omega C} = \frac{G - j\omega C}{G^2 + (\omega C)^2}$$

need $\frac{G}{G^2 + (\omega C)^2} = 100$ (1) and $\frac{\omega C}{G^2 + (\omega C)^2} = 10$ (2)

Taking the ratio of (1) & (2) leads to $\frac{G}{\omega C} = 10$ or $\underbrace{G^2 = 100(\omega C)}_{(3)}$

Now plugging (3) into (2) yields $\underline{C = 99\text{pf}}$

and thus $G = 9.9 \times 10^{-3}$ or $\underline{R = \frac{1}{G} = 101\Omega}$