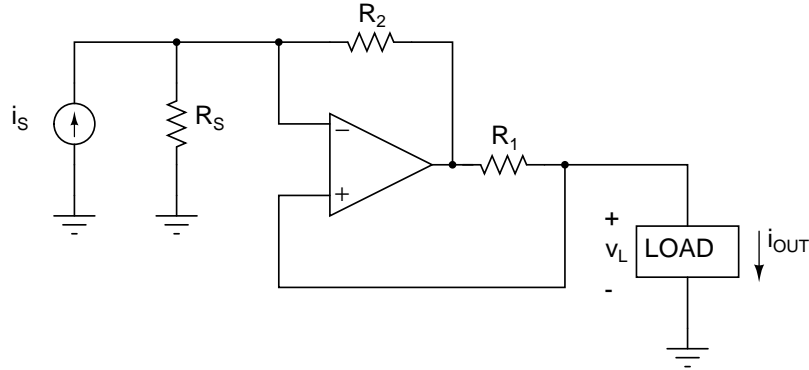


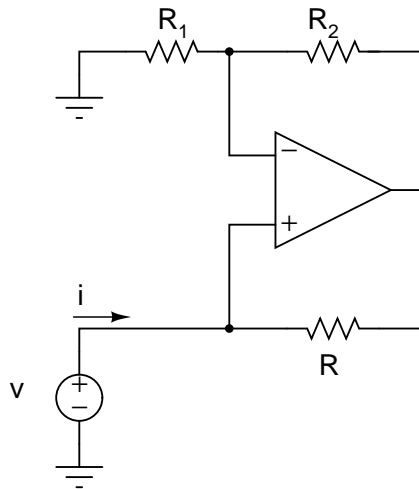
INEL5207 - Spring 2013 - Problem set 1 - Solutions
 Each problem counts for 3 points of the first partial exam

1. Show that $i_{OUT} = Ai_S - \frac{1}{R_O}v_L$, where $A = -\frac{R_2}{R_1}$ and $R_O = -\frac{R_1}{R_2}R_S$, for the following circuit.



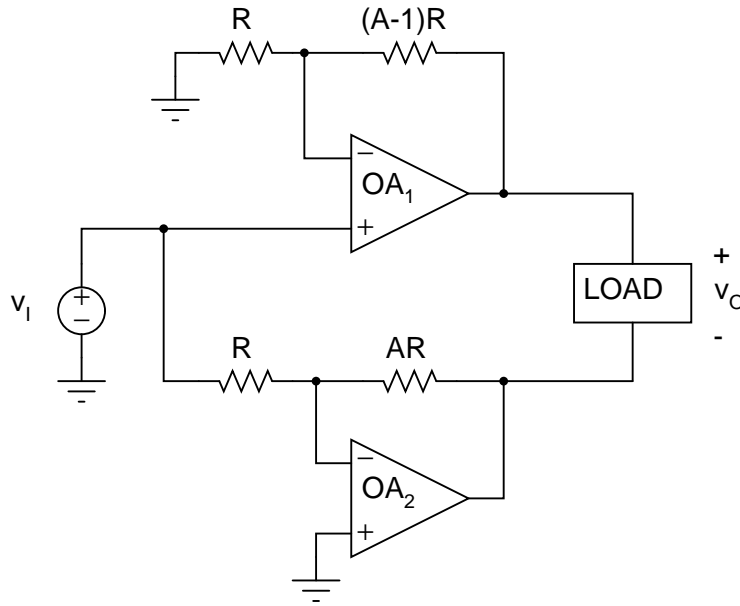
ANSWER: Observe that $v_L = v_+ = v_-$. Thus $i_{R_S} = \frac{v_L}{R_S}$ flowing towards ground, and $i_{R_2} = i_S - \frac{v_L}{R_S}$. The op amp output voltage is $v_O = v_L - R_2 i_{R_2} = v_L + \frac{R_2}{R_S} v_L - R_2 i_S$. Since no current flows into the op amp input, $i_{R_1} = i_L = \frac{v_O - v_L}{R_1} = -\frac{R_2}{R_1} i_S + \frac{R_2}{R_1} \frac{1}{R_S} v_L$.

2. Show that $R_{EQ} = \frac{v}{i} = -\frac{R_1}{R_2}R$ for the following circuit.



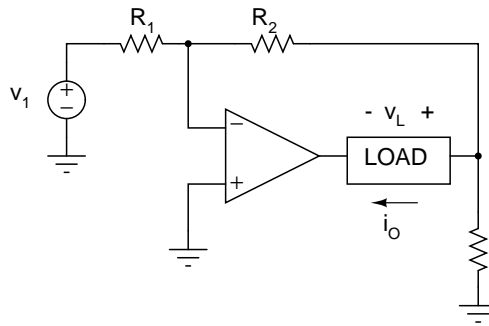
ANSWER: Since $v_+ = v$, the op amp's output voltage is $v_O = \left(1 + \frac{R_2}{R_1}\right)v = v - iR$. Solving for $\frac{v}{i}$ yield the desired expression.

3. The circuit in the figure below, a *bridge amplifier*, allows one to double the linear output range as compared with a single op amp. (a) Show that if the resistances are in the ratios shown, then $v_O/v_I = 2A$. (b) If the individual op amps saturate at $\pm 13V$, what is the maximum peak-to-peak output voltage that the circuit can provide without distortion?



ANSWER: Let v_{O1} and v_{O2} denote OA_1 and OA_2 's output voltages, respectively. Observe that $v_{O1} = \left(1 + \frac{(A-1)R}{R}\right) v_i = AR$ and $v_{O2} = -\frac{AR}{R} v_i = -AR$. Thus, $v_O = 2Av_i$. Since each op amp output can swing $\pm 13V$, the v_O can be up to $26V_{peak}$, or $52V_{peak-peak}$.

4. (a) Show that the floating-load V-I converter shown below yields $i_O = v_I/(R_1/k)$, $k = 1 + R_2/R_3$, (b) Specify standard 5% resistances for a sensitivity (gain) of $1mA/V$ and $R_i = 1M\Omega$, where R_i is the resistance seen by the input source.

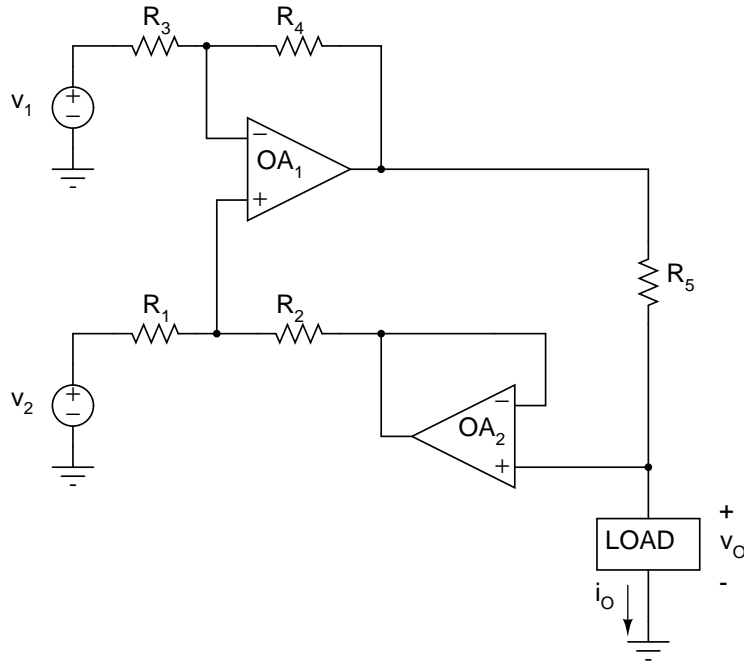


ANSWER:

(a) $i_o = i_{R_2} - i_{R_3} = i_{R_1} - i_{R_3} = \frac{v_i}{R_1} + \left(\frac{R_2}{R_1} v_i\right) \frac{1}{R_3} = \frac{v_i}{R}$; $R = \frac{R_1 R_2}{1 + \frac{R_2}{R_3}}$.

(b) $R_i = R_1$; $R_1 = 1M\Omega$. The sensitivity is $1 \frac{mA}{V} = \frac{1}{R}$ so $R = 1k\Omega = \frac{1M\Omega}{1 + \frac{R_2}{R_3}}$ so $\frac{R_2}{R_3} = 999 \approx 1k\Omega$. You can select $R_2 = 1M\Omega$ and $R_3 = 1k\Omega$.

5. (a) For the circuit shown below, given that $i_O = av_I - (1/R_O)v_L$, $v_I = v_2 - v_1$, find expressions for A and R_O , as well as the condition among resistances that yields $R_O = \infty$.



ANSWER: Let v_{O1} and v_{O2} be OA_1 and OA_2 's output voltages. Observe that $v_{O2} = v_O$, and that $v_{+OA_1} = v_{p1} = v_2 - \frac{v_2 - v_O}{R_1 + R_2} R_1 = \frac{1}{R_2 + 1} v_2 + \frac{1}{1 + \frac{R_2}{R_1}} v_O$.

For OA_1 , $v_{O1} = \left(1 + \frac{R_4}{R_3}\right) v_{p1} - \frac{R_4}{R_3} v_1 = v_{p1} + \frac{R_4}{R_3} (v_{p1} - v_1)$. Since no current flows into OA_2 ,

$$\begin{aligned} i_O &= \frac{v_{O1} - v_O}{R_5} \\ &= \frac{\left(1 + \frac{R_4}{R_3}\right) v_{p1} - \frac{R_4}{R_3} v_1 - v_O}{R_5} \end{aligned}$$

Using $\left(1 + \frac{R_4}{R_3}\right) v_{p1} = \frac{1 + \frac{R_4}{R_3}}{1 + \frac{R_1}{R_2}} v_2 + \frac{1 + \frac{R_4}{R_3}}{1 + \frac{R_1}{R_2}} v_O$, we get

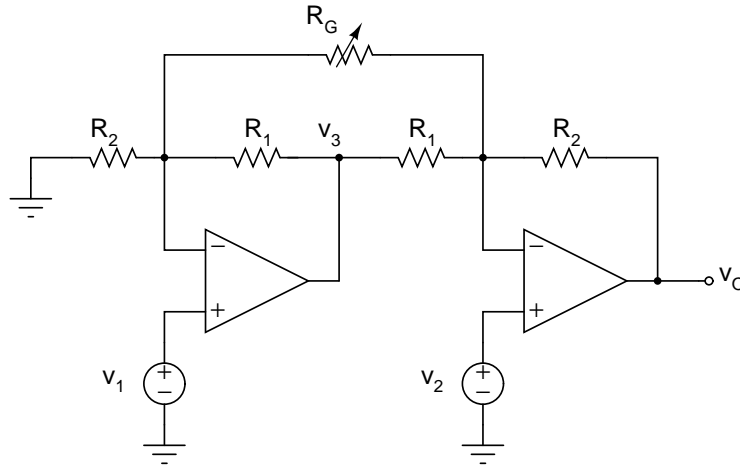
$$\begin{aligned} i_O &= \frac{1}{R_5} \left(\frac{1 + \frac{R_4}{R_3}}{1 + \frac{R_1}{R_2}} v_2 - \frac{R_4}{R_3} v_1 + \left(\frac{1 + \frac{R_4}{R_3}}{1 + \frac{R_2}{R_1}} - 1 \right) v_O \right) \\ &= \frac{1}{R_5} \left(\frac{v_2 - \frac{R_1 R_4}{R_2 R_3} v_1 + \frac{R_4}{R_3} (v_2 - v_1)}{1 + \frac{R_1}{R_2}} + \frac{\frac{R_4}{R_3} - \frac{R_2}{R_1}}{1 + \frac{R_2}{R_1}} v_O \right) \end{aligned}$$

Thus if $\frac{R_1 R_4}{R_2 R_3} = 1$, $\frac{1}{R_O} = \frac{\frac{R_4}{R_3} - \frac{R_2}{R_1}}{1 + \frac{R_2}{R_1}} = 0 = \frac{1}{\infty}$, and

$$\begin{aligned} i_O &= \frac{1}{R_5} \frac{1 + \frac{R_2}{R_1}}{1 + \frac{R_1}{R_2}} (v_2 - v_1) \\ &= \frac{1}{R_5} \frac{R_2}{R_1} v_I \end{aligned}$$

In summary, selecting $\frac{R_4}{R_3} = \frac{R_2}{R_1}$ makes $R_O = \infty$ and the gain $a = \frac{i_O}{v_2 - v_1} = \frac{R_2}{R_1 R_5}$.

6. The following sketch shows a variable-gain, dual-op amp instrumentation amplifier. Show that $A = \frac{v_O}{v_2 - v_1} = 1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G}$.



ANSWER: By superposition

$$v = \left(1 + \frac{R_1}{R_2 \parallel R_G}\right) v_1 - \frac{R_1}{R_G} v_2$$

$$v_O = -\frac{R_2}{R_1} v - \frac{R_2}{R_G} v_1 + \left(1 + \frac{R_2}{R_G \parallel R_1}\right) v_2$$

This can be re-written as

$$v_O = -\frac{R_2}{R_1} v_1 - \frac{R_2}{R_2 \parallel R_G} v_1 + \frac{R_2}{R_G} v_2 - \frac{R_2}{R_G} v_1 + v_2 + \frac{R_2}{R_G \parallel R_1} v_2$$

$$= \left(1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G}\right) (v_2 - v_1)$$

which is the desired relationship.

7. For the circuit shown below, a power supply of $\pm 15V$ is used. The opamp utilizes a bias current $I_Q = 50mA$ and exhibits a short-circuit current $I_{ss} = 20mA$ but can otherwise be considered ideal. If $i_S = 1mA$, $R_2 = 10k\Omega$ and $R_1 = R_L = 1k\Omega$, determine the following quantities:

- a) output voltage v_O ,

ANSWER: $v_O = -i_S R_2 = -1mA \times 10k\Omega = -10V$

- b) opamp output current and direction,

ANSWER: A current of $10V/1k\Omega = 10mA$ flows upward through R_L . The opamp output current is $i_S + i_L = 11mA$ flowing into the opamp.

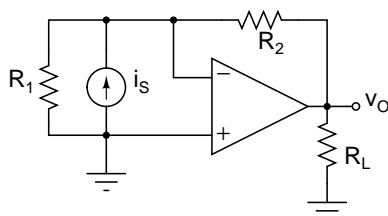
c) power dissipated by the opamp,

ANSWER: Since $i_S = I_{SS}$, the opamp won't limit the current. Thus

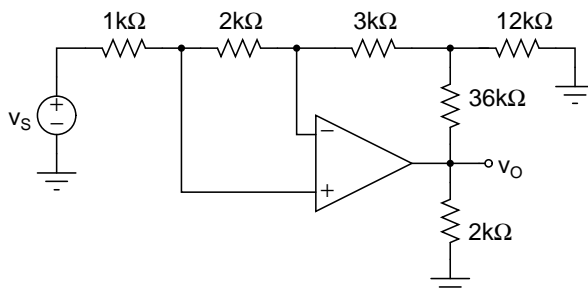
$$P_{opamp} = I_Q \times 30V + (-10V - (-15V))i_O = 50mA \times 30V + 11mA \times 5V = 1555mW$$

d) power dissipated by the whole circuit.

$$P_{total} = P_{opamp} + i_L^2 R_L + i_S^2 R_2 = 1555mW + 110mW = 1660mW$$



8. Assuming the opamp of the following circuit has an open-loop gain $a = 3000V/V$, $r_d = \infty$ and $r_o = 0$, find the voltage gain $A_v = \frac{v_O}{v_S}$ for the following circuit.



ANSWER: Let v_A be the voltage at the node common to the $3k$, $12k$ and $36k$ resistors.

$$\begin{aligned} i_{2k} &= \frac{v_d}{2k} = i_{1k} = i_{3k} \\ v_A &= v_S - i_{2k} \times 6k = v_S - 3v_d \\ i_{12k} &= v_A/12k \rightarrow \\ v_O &= v_A - (i_{2k} - i_{12k})36k \\ &= v_S - 3v_d - 36 \left(\frac{v_d}{2} - \frac{v_S}{12} + \frac{3v_d}{12} \right) \\ &= v_S - 3v_d - 18v_d + 3v_S - 9v_d \\ &= 4v_S - 30v_d = 4v_S - \frac{30v_O}{3000} = 4v_S - 0.01 \times v_O \\ \frac{v_O}{v_S} &= \frac{4}{1.01} \end{aligned}$$

9. For the circuit shown in problem 8, determine the loop gain T using the same parameters for the opamp. Show that the result is consistent with the feedback formula $A_V = A_{ideal} \frac{1}{1+T}$.

ANSWER:

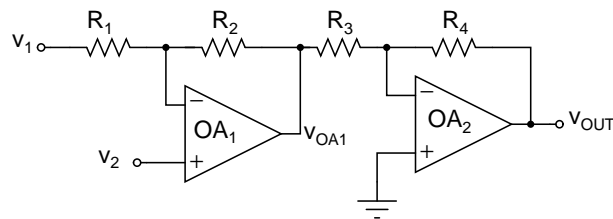
$$\beta = \frac{2}{6} \times \frac{12 \parallel 6}{12 \parallel 6 + 36} = \frac{1}{3} \times \frac{4}{40} = \frac{1}{30}$$

$$T = a\beta = \frac{3000}{30} = 100V/V$$

$$A_{ideal} = \frac{v_S + 36 \times v_S/12}{v_S} = 4V/V$$

$$A_V = 4V/V \times \frac{1}{1 + \frac{1}{100}} = \frac{4}{1.01}$$

10. For the following circuit,

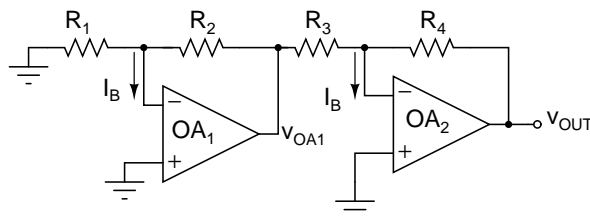


let both v_1 and v_2 be connected to ground and consider only the effect of each transistor's bias current I_B . Assume that current I_B flows into the + and - inputs of each opamp and that the opamps are matched (i.e. identical).

- a) Find the relationship between the resistance values that causes the cancelation of the error due to the bias current. Consider only the resistors shown in the above figure.

ANSWER:

The circuit looks as follows:



So $v_{OA1} = i_B R_2$ and $i_{R3} = \frac{R_2}{R_3} I_B$. If $i_{R3} = I_B$, then no current flows through R_4 and $v_{OUT} = 0$. So the relationship that we are seeking is

$$\boxed{R_2 = R_3}$$

- b) Determine the worse-case error due to the offset voltage $|V_{OS}| \leq 1mV$ if R_1 , R_2 , R_3 and R_4 are chosen so that: (i) the overall voltage gain is $25V/V$; (ii) the input is v_1 and (iii) no resistor is less than $1k\Omega$; and (iv) the relationship found in part (a) is satisfied.

ANSWER:

To have an overall gain of $25V/V$, select $R_1 = 1k\Omega$, $R_2 = R_3 = 5k\Omega$ and $R_4 = 25k\Omega$.

The first stage V_{OS} will be amplified by a factor $6 \times 5 = 30$. The gain for the second stage V_{OS} is 6. To obtain the worse-case error, neglect signs and add the two contributions. Thus

$$E_O = 36mV$$