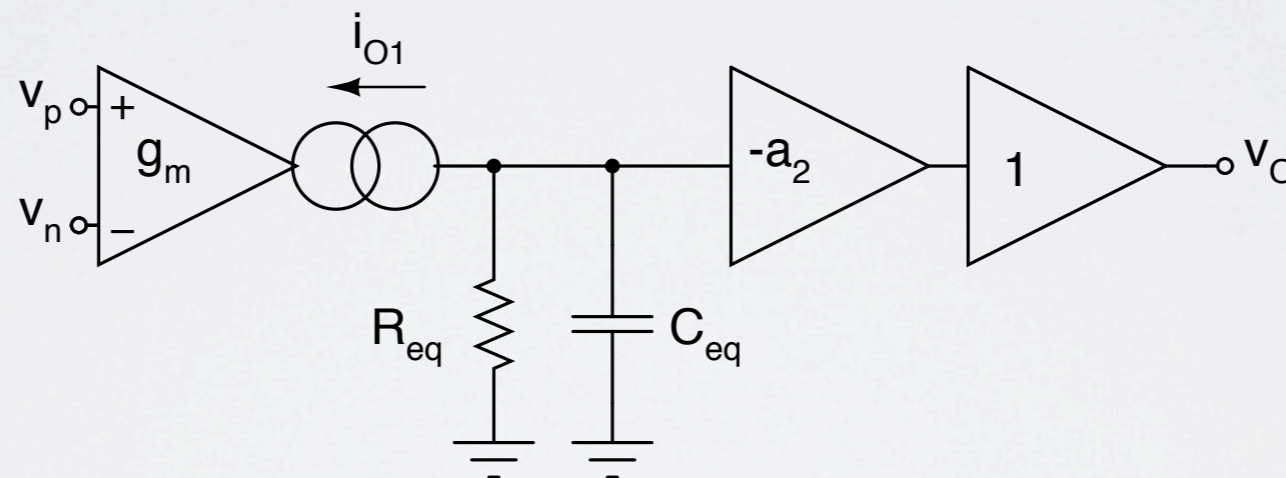


# BANDWIDTH

INEL 5207 - Spring 2013

## Frequency Limits

- Many opamps are internally compensated to have a single dominant pole at a relatively low frequency.



Noninverting Amplifier

- Open-loop gain can be written as:

$$a(s) = a_0 \frac{1}{1 + s/\omega_p}$$

$a_0 =$  d.c. open-loop gain;  $f_p = \frac{1}{2\pi R_{eq}C_{eq}} =$  pole freq.

- For the non-inverting amplifier

$$A = \frac{a}{1 + a\beta}$$

where  $\beta = \frac{R_1}{R_1 + R_2}$ .

- Using above  $a(s)$  and  $A_0 = \frac{a_0}{1 + \beta a_0}$ ,

$$A(s) = \frac{a(s)}{1 + a(s)\beta} = A_0 \frac{\omega_p(1 + \beta a_0)}{s + \omega_p(1 + \beta a_0)}$$

- Corner frequency is increased by  $1 + \beta a_0$ . Gain is decreased by the same factor.
- Gain-bandwidth product remains constant and equal to unity gain frequency,  $f_t$ .

$$GBP = f_t$$

$$\text{Since } A_f \simeq 1/\beta, \quad \boxed{f_B = \beta f_t}$$

- This is only true for  $\beta$  constant and compensated opamp (dominant pole at low freq.)

Gain of  $n$  identical noninverting stages

- If  $f_{cl} = \omega_p(1 + \beta a_0)/2\pi$ , then gain magnitude of one stage is

$$A = A_0 \frac{1}{\sqrt{1 + (f/f_{cl})^2}}$$

- Gain of  $n$  identical stages is

$$A^n = A_0^n \left(1 + (f/f_{cl})^2\right)^{-n/2}$$

- At corner frequency  $f_{3dB}$ ,  $A^n/A_0^n = 1/\sqrt{2}$  (i.e. -3dB). Thus,

$$f_{3dB} = f_{cl} \sqrt{2^{1/n} - 1} = \frac{f_t}{A_0} \sqrt{2^{1/n} - 1}$$

- To design an amplifier with bandwidth  $f_{bw}$  and gain  $K$ , we must select  $n$  such that  $K = A_0^n$  and  $f_{bw} \leq \frac{f_t}{A_0} \sqrt{\frac{1}{2^n} - 1}$ .

## Inverting Amplifier

- Bw:  $f_t \frac{R_1}{R_1 + R_2}$  ; same than non-inv with gain  $1 + R_2/R_1$
- $A = A_{ideal} \frac{1}{1 + 1/T}$ ;  $A_{ideal} = -\frac{R_2}{R_1}$ ;  $T = a\beta_{non-inv}$
- For the inverting amplifier, the gain-bandwidth product is equal to

$$GBP = f_t \frac{R_2}{R_1 + R_2}$$

so the bandwidth is always lower than that of a non-inverting amplifier with the same gain.

For inverting amp.  $f_B = \beta f_t$  using  $\beta$  from non-inverting....

- Equivalently, we can say that  $f_{bw} \times (1 + \frac{R_2}{R_1})$  is still constant and equal to  $f_t$ , but the the amplifier's gain magnitude of is only  $\frac{R_2}{R_1}$ .

## Input Impedance

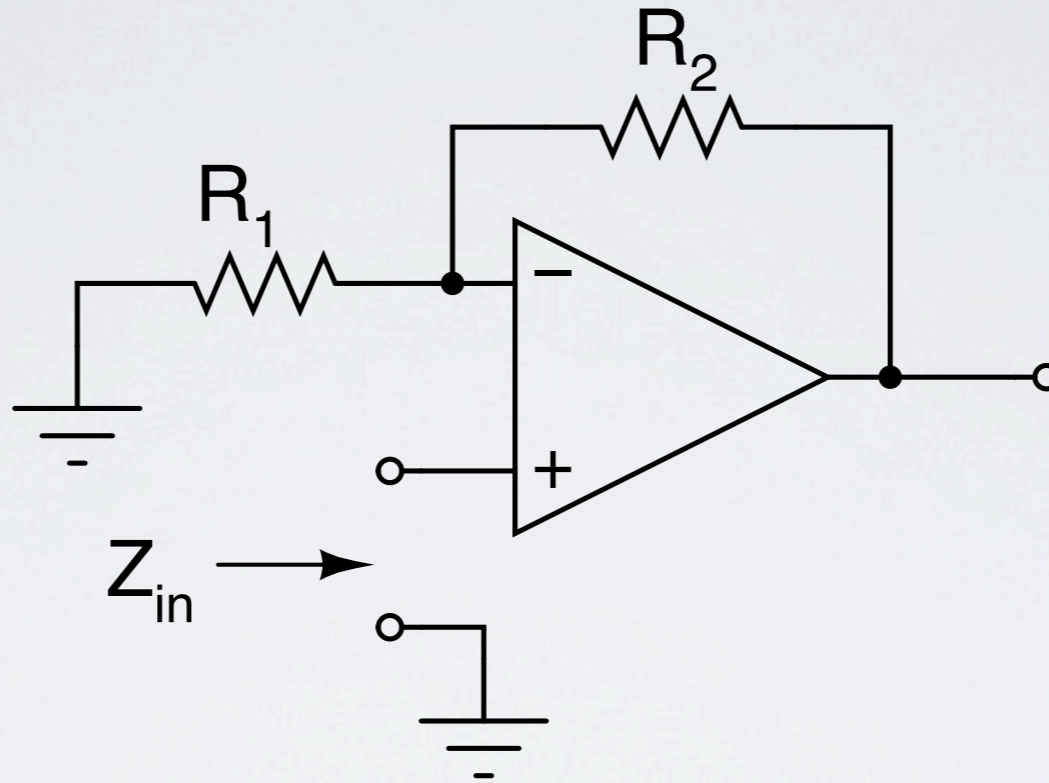
- Diff. input impedance  $r_d$  is typ. few  $M\Omega$ , common-mode  $r_c$  is in the  $G\Omega$  for BJT. FETs are in the 100's  $G\Omega$ .
- Input capacitance for uA741 is about 1 pF . Appears in parallel with  $r_d$  and/or  $r_c$ .
- For  $f = 100kHz$ ,  $X_c = 10M\Omega$  so it is comparable to  $r_d$ . At higher frequencies, input impedance drops due to the input

capacitance.

$$z_d = \frac{r_d}{1 + sC_d r_d}$$

- There is also a common-mode impedance.

**$Z_{in}$  for Input-series Feedback**



- $z_{df} = z_d(1 + a\beta)$ . Using  $a = \frac{a_0}{1 + j\frac{f}{f_a}}$  yields

$$z_{df} = z_d(1 + a_0\beta) \frac{1 + j \frac{f}{f_a(1 + \beta a_0)}}{1 + j \frac{f}{f_a}}$$

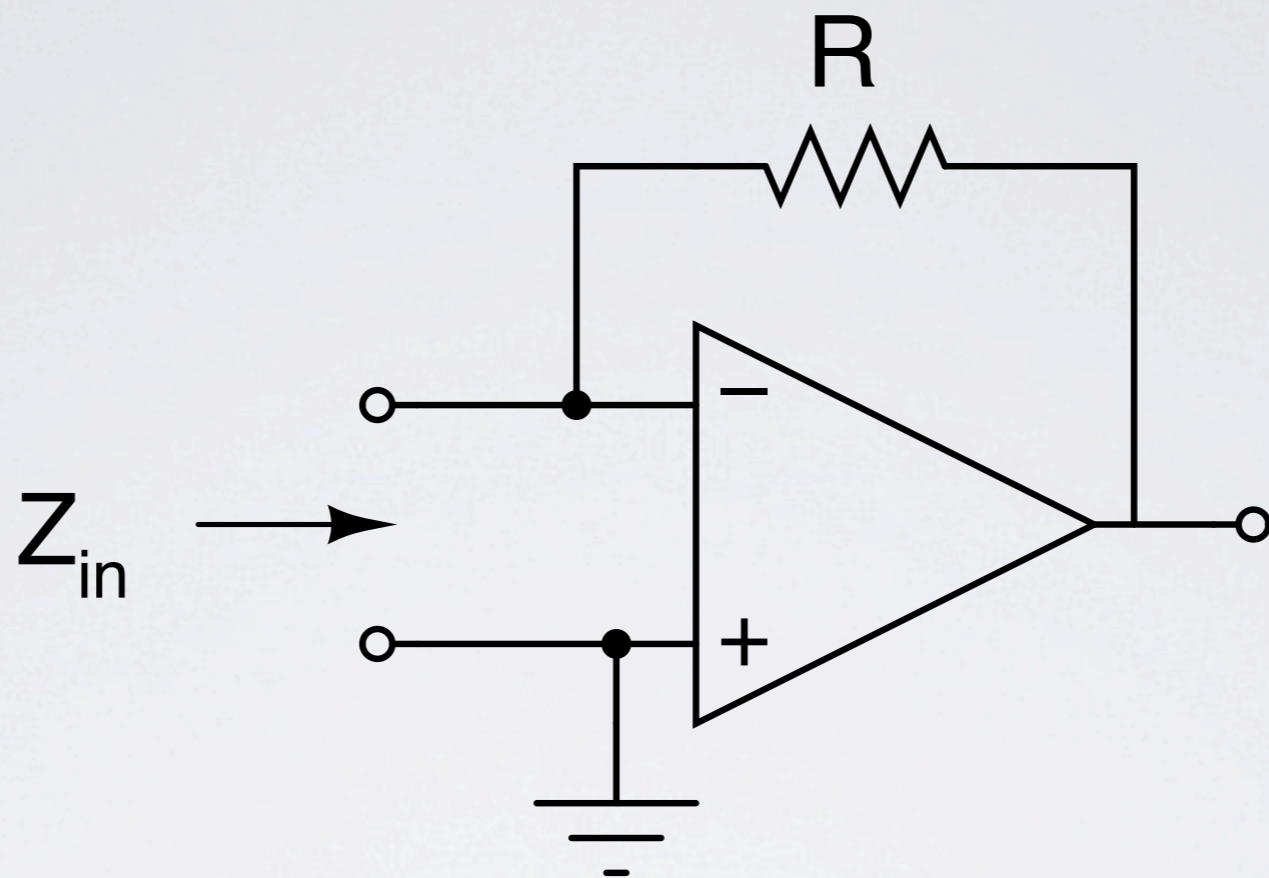
- Observe that  $f_a$  is the open-loop pole. Since  $a_0 f_a = GBP = f_t$ , we can write  $f_a = \frac{f_t}{a_0}$ . Generally  $1 + \beta a_0 \approx \beta a_0$ ; thus

$$f_a(1 + \beta a_0) \approx \beta a_0 f_a = \beta f_t$$

The above expression becomes

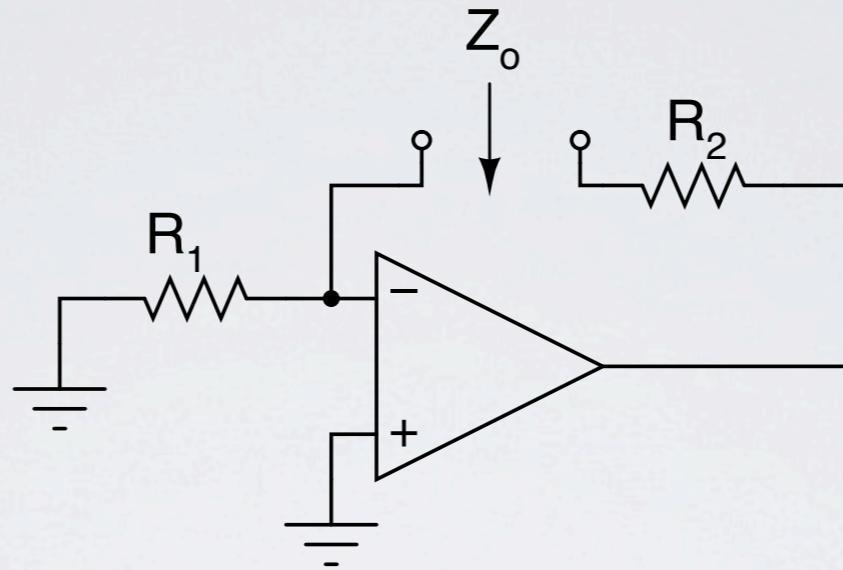
$$z_{df} = z_d(1 + a_0\beta) \frac{1 + j \frac{f}{\beta f_t}}{1 + j \frac{f}{f_a}}$$

**$Z_{in}$  for Input-shunt Feedback**



$$Z_{if} = R_i \frac{1}{1 + a\beta} = R_i \frac{1 + j\frac{f}{f_a}}{1 + j\frac{f}{f_t}}$$

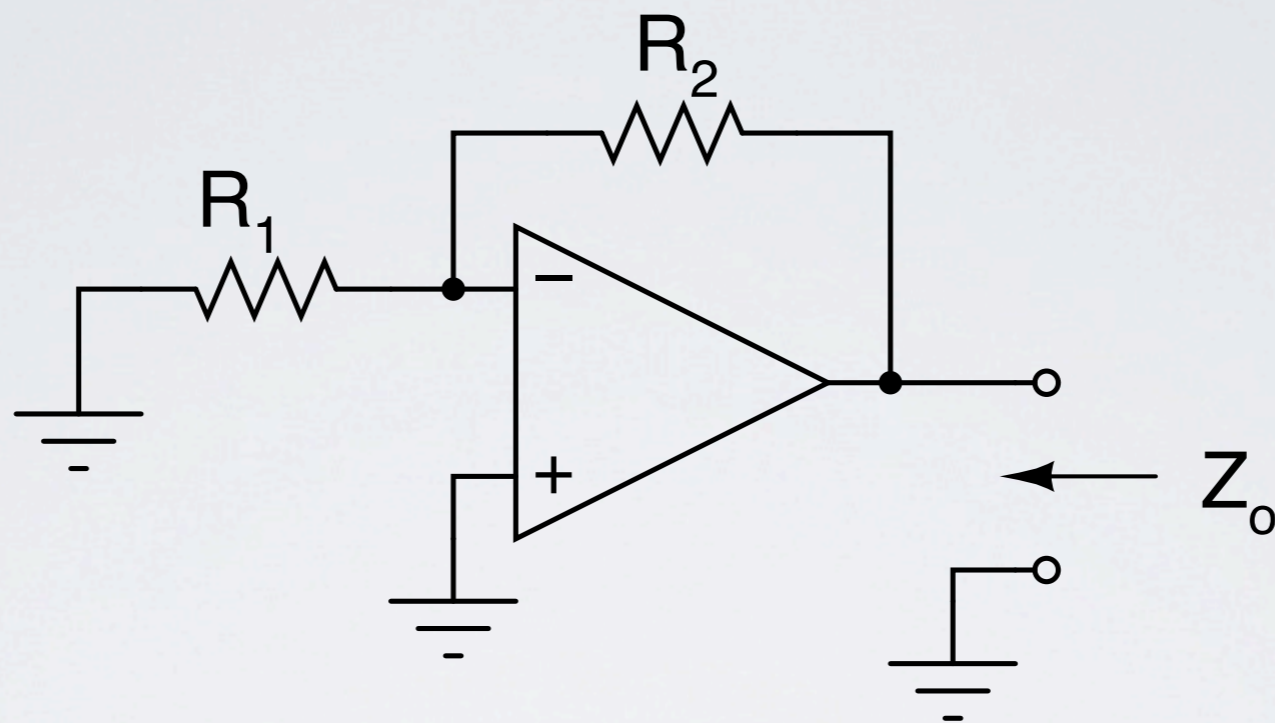
$Z_o$  for Output-series feedback



- $Z_{of} = R(1 + a\beta)$ . Using  $\beta = 1$  and our previous result

$$R_{of} = R(1 + a_0\beta) \frac{1 + j\frac{f}{f_t}}{1 + j\frac{f}{f_a}}$$

**$Z_o$  for Output-shunt feedback**

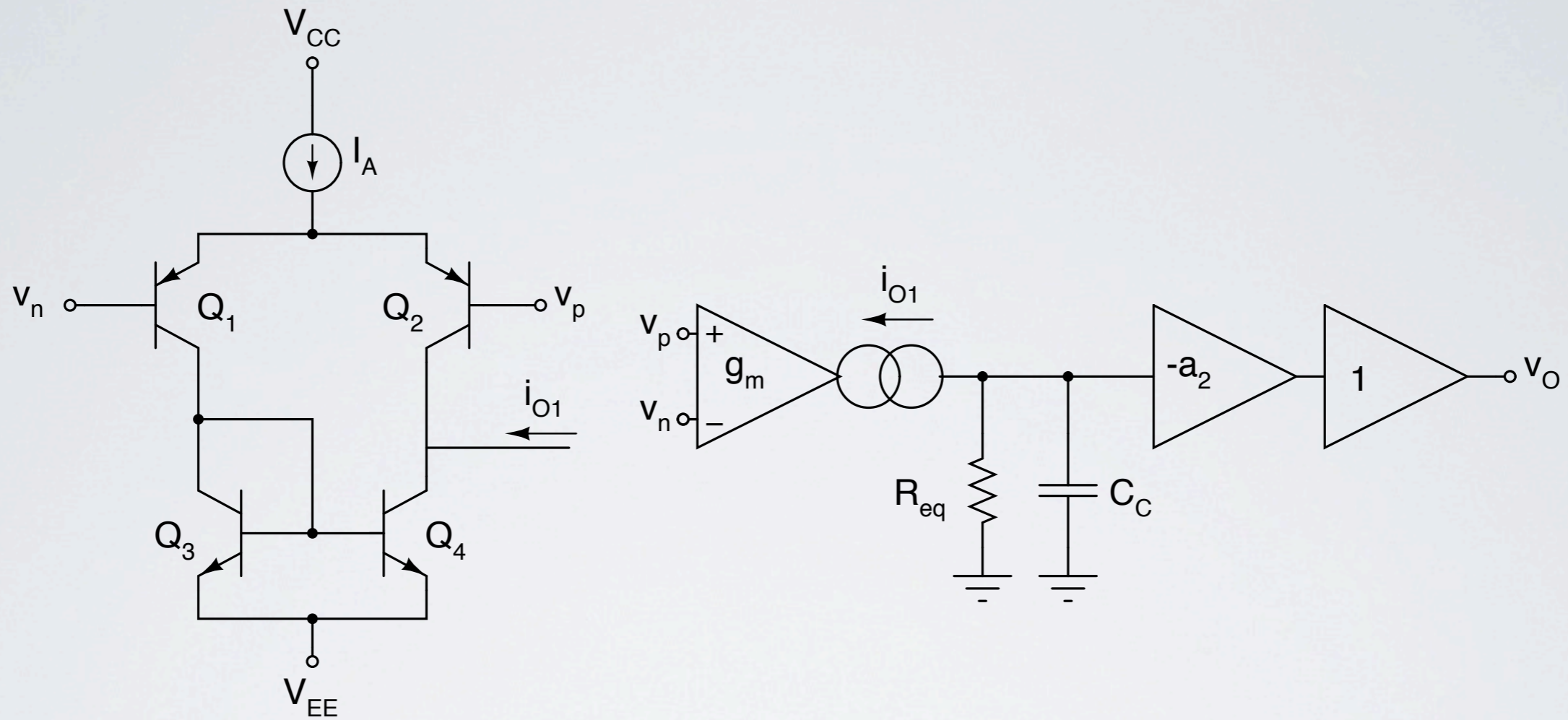


$$Z_o = \frac{r_o}{1 + \beta a} = r_o(1 + a_0\beta) \frac{1 + j\frac{f}{\beta f_t}}{1 + j\frac{f}{f_a}}$$

## Transient Response

- A follower have a gain given by  $A = \frac{1}{1+j\frac{f}{f_t}}$ .
- Step response is  $v_o = V_m(1 - e^{-t/\tau})$ , where  $\tau = \frac{1}{2\pi f_t}$ .
- Rise time  $t_r$  is time from 10% to 90% of  $V_m$ .
$$t_r = \frac{0.35}{f_t}$$
- Ringing due to higher frequency poles.

## Slew Rate



- $i_{O1}$  is limited to  $\pm I_A$ . Output can not change faster than

$$SR = \frac{I_A}{C_C}$$

- Fastest part of step response occurs at  $t = 0$ , and is

$$\frac{dv_O}{dt} \Big|_{t=0} = \frac{V_{om}}{\tau} = 2\pi f_t V_{om}$$

- If  $V_{om} > \frac{SR}{2\pi f_t}$  then the output is slew-rate limited and changes linearly, not exponentially. Also  $v_n \neq v_p$ .
- If gain is larger than 1, replace  $f_t$  with  $\beta f_t$  on the above expressions.
- Analysis shows that  $f_t \approx \frac{g_{m1}}{2\pi C_C}$  so  $C_C = \frac{g_{m1}}{2\pi f_t}$  and

$$SR = \frac{I_A}{C_C} = \frac{2\pi f_t I_A}{g_{m1}}$$

- SR can be increased by increasing  $I_A$
- SR can be increased by decreasing  $g_{m1}$  using a FET input stage or adding resistors to the differential stage's emitter (emitter degeneration).
- Using opamps with higher  $f_t$ .
- Full-power bandwidth (FPB): maximum frequency at which the opamp will yield an undistorted sinusoidal output with the largest possible amplitude. Assuming saturation at  $\pm V_{sat}$ ,

$$FPB = \frac{SR}{2\pi V_{sat}}$$

- Settling time: time it takes for the response to a large input step to remain within a specific error band. Example: AD843 will has  $t_s = 135ns$  to 0.01% of a 10V step.