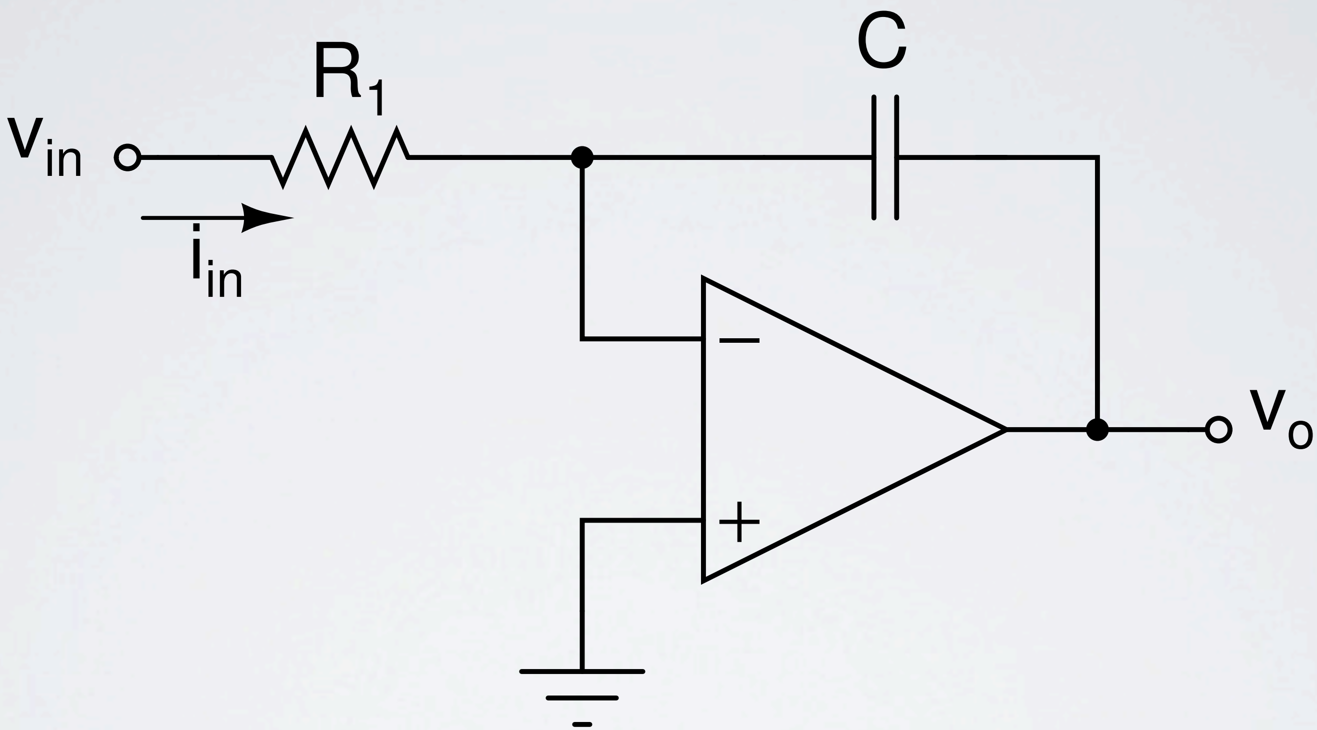
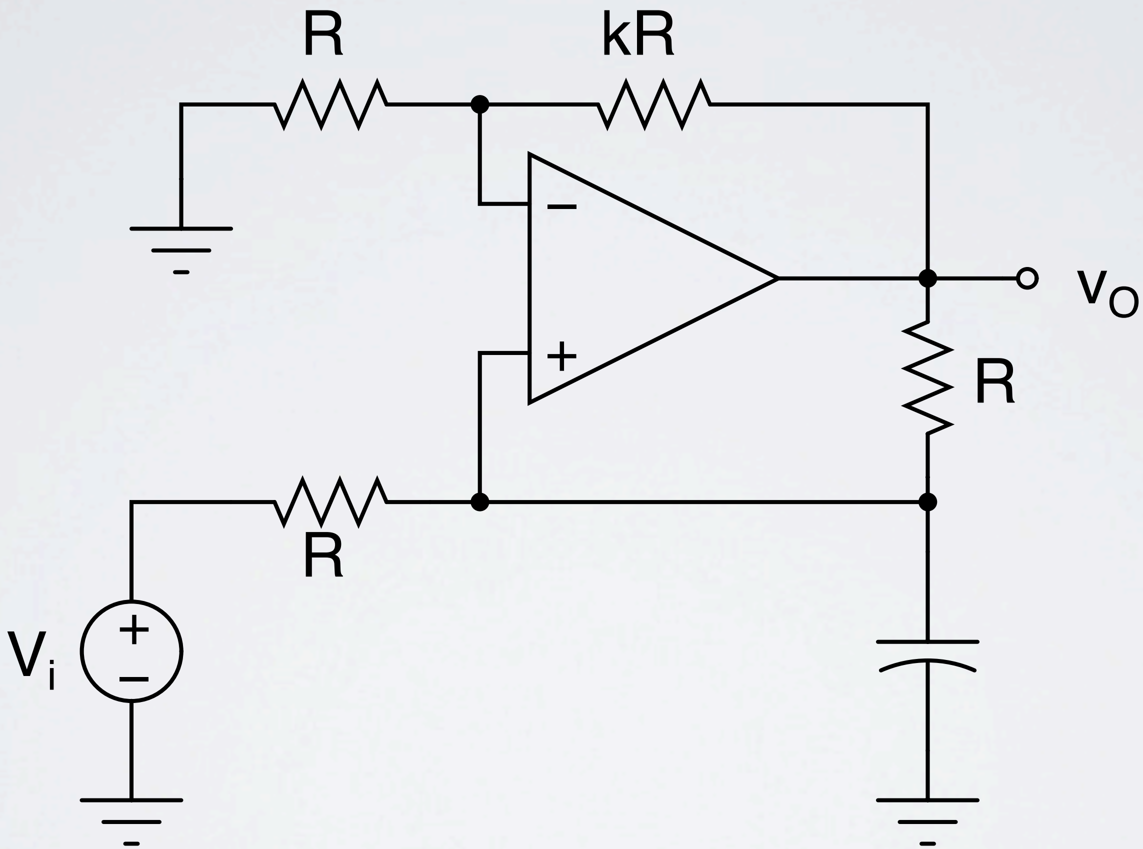
FILTERS

INEL 5207 - Spring 2013

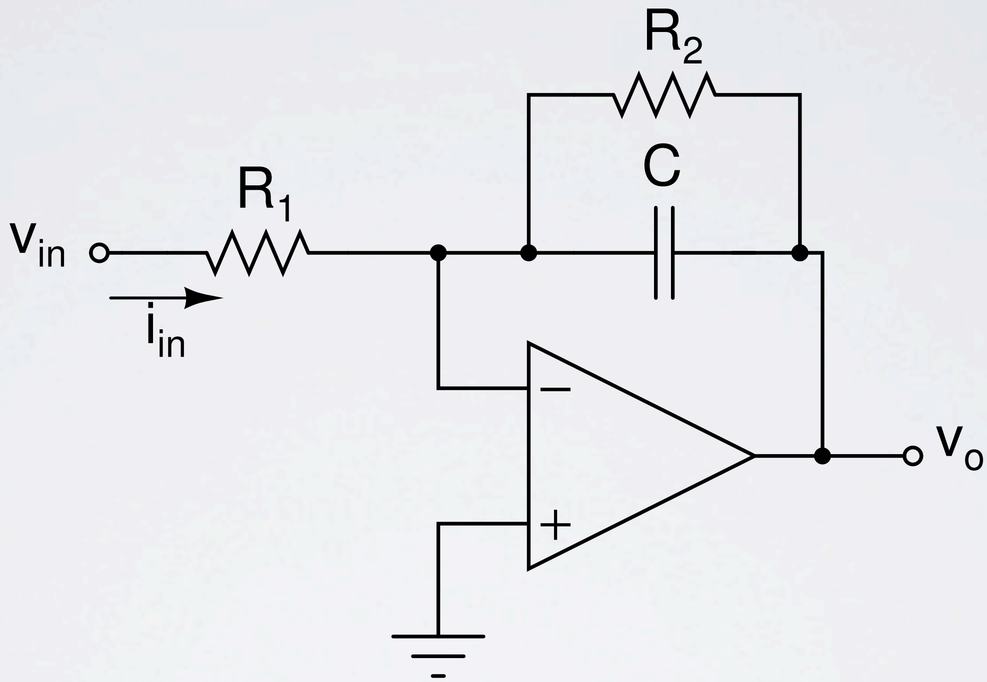
INTEGRATOR



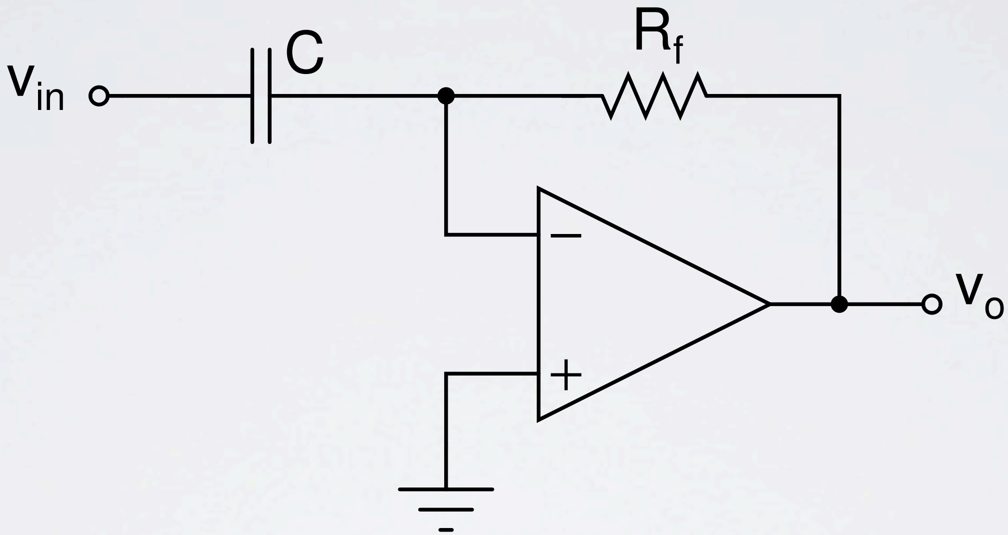
DEBOO INTEGRATOR



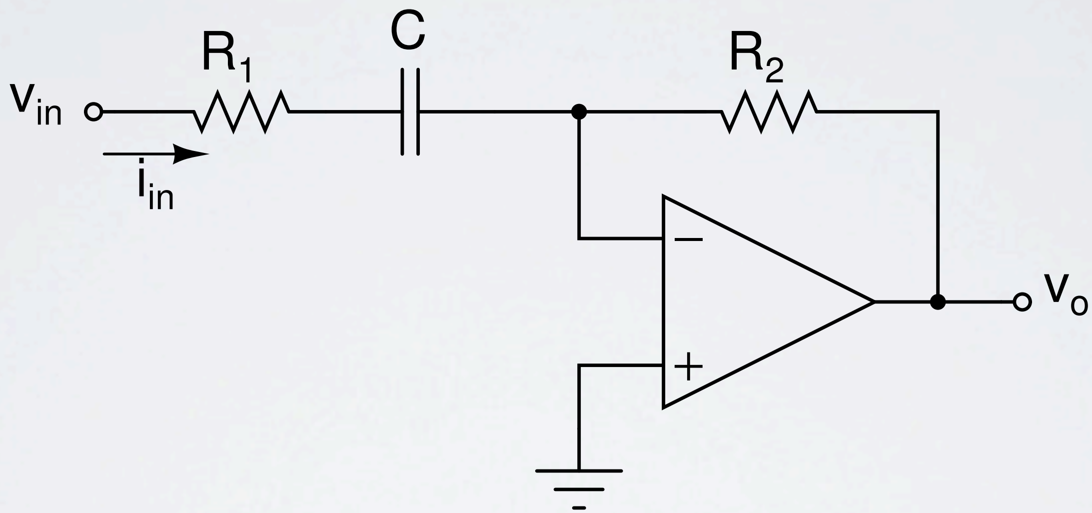
LOW-PASS FILTER



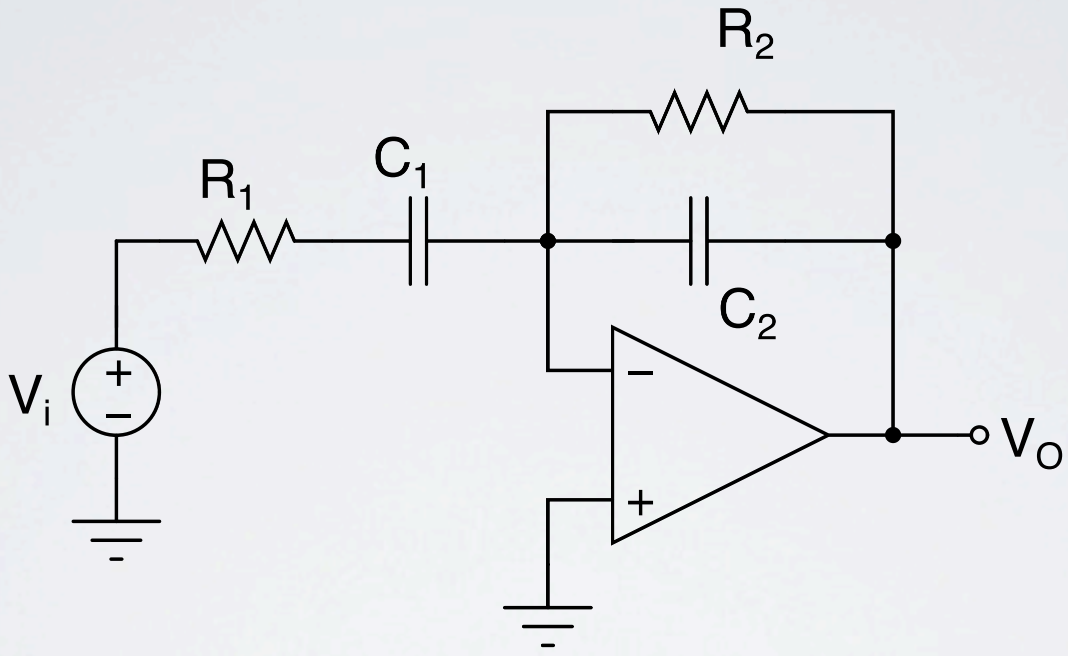
DIFFERENTIATOR



HIGH-PASS FILTER

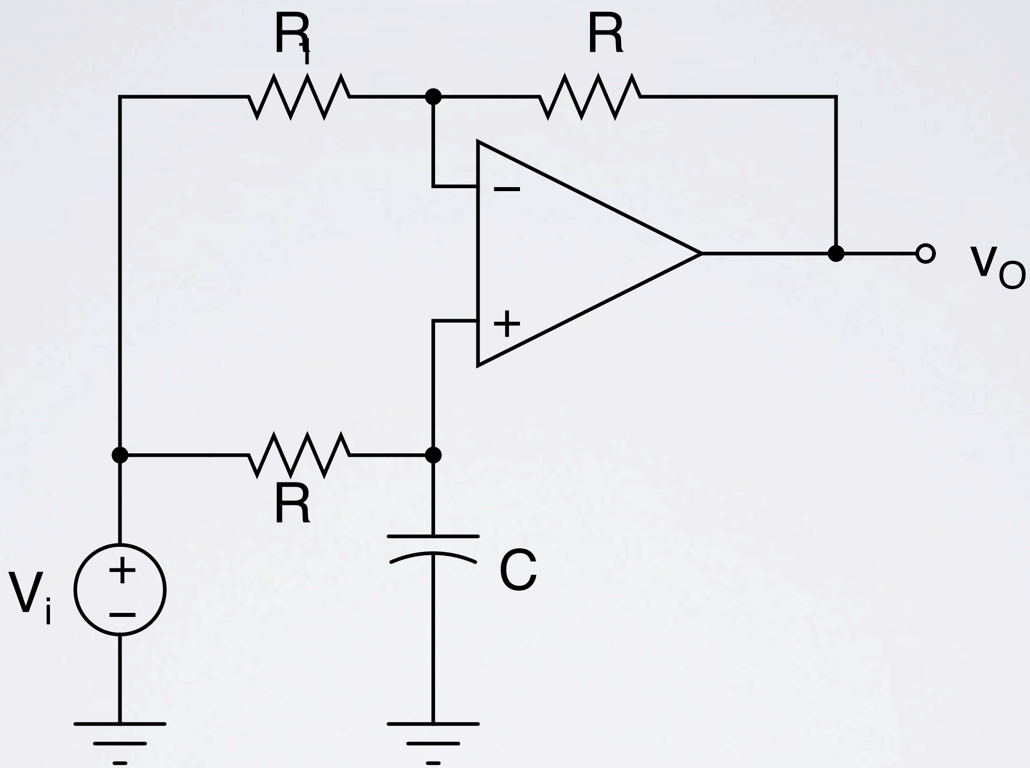


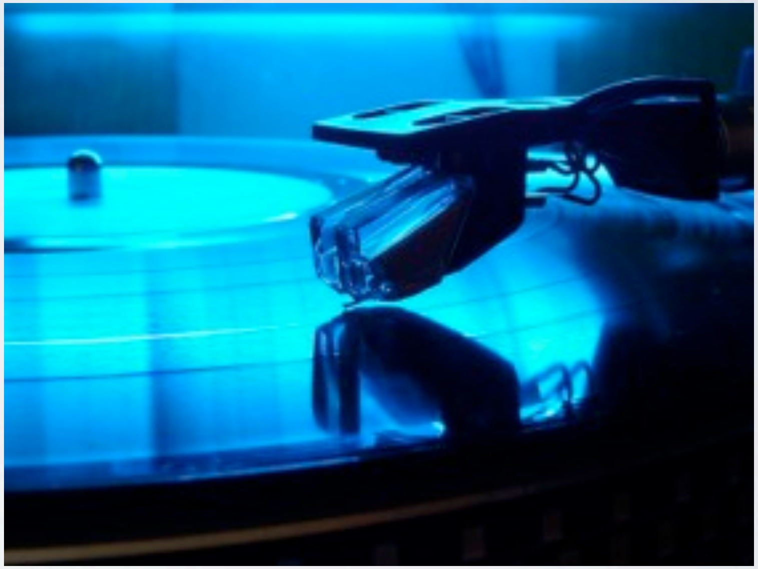
BANDPASS



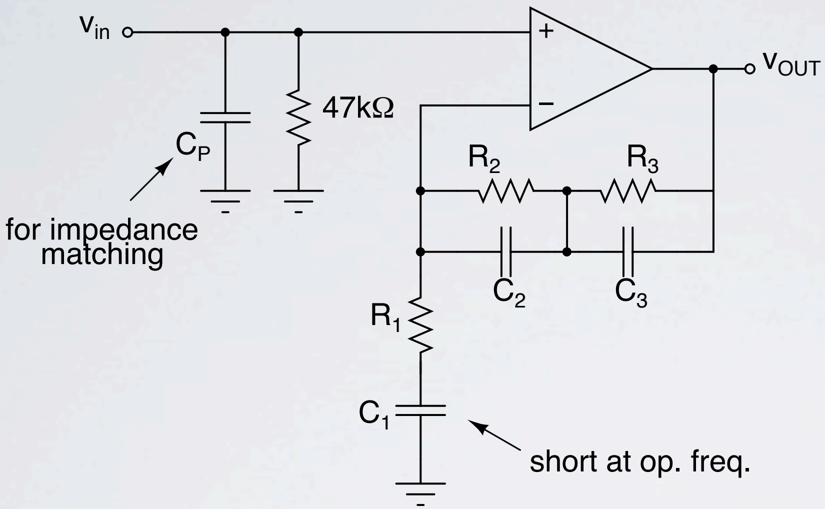
Choose components for a gain of 20dB over the audio range.

PHASE SHIFTER

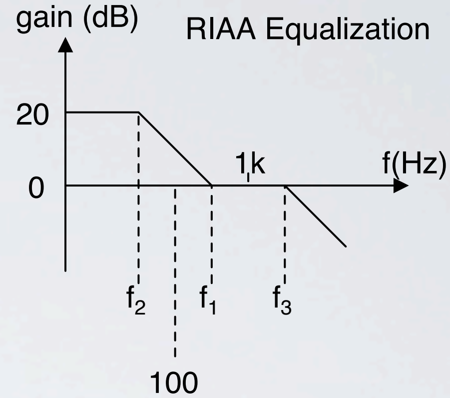


Phono pre-amp



$f_1 = 500\text{Hz},\ f_2 = 50\text{Hz},\ f_3 = 2122\text{Hz}$



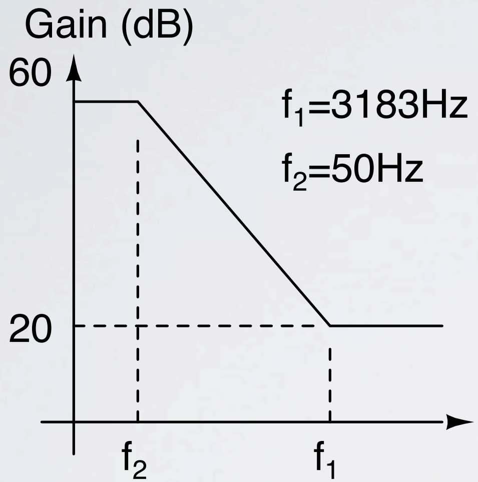
shift by 30-40dB for moving magnet cartridge, 50-60dB for moving-coil cartridge

$$H(jf) \simeq 1 + \frac{R_2 + R_3}{R_1} \frac{1 + jf/f_1}{(1 + jf/f_2)(1 + jf/f_3)}$$

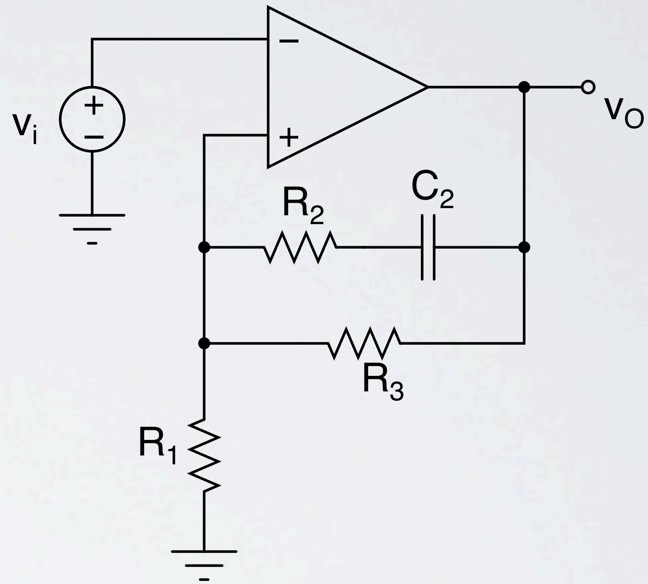
$$f_1 = \frac{1}{2\pi(R_2 \| R_3)(C_2 + C_3)}$$

$$f_2 = \frac{1}{2\pi R_2 C_2} \qquad f_3 = \frac{1}{2\pi R_3 C_3}$$

TAPE AMP



NAB Equalization Curve
and Tape Amplifier



$$H(jf) \simeq 1 + \frac{R_3}{R_1}\frac{1 + jf/f_1}{1 + jf/f_2}$$

$$f_1 = \frac{1}{2\pi R_2 C_2} \qquad f_2 = \frac{1}{2\pi(R_2 + R_3)C_2}$$

2^{nd} order filters

$$H(s) = \frac{N(s)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

ω_0 = undamped natural frecuency
ζ = damping ratio

$\zeta > 1$	Overdamp: Real, neg. poles. Response shows decaying exponentials
$0 < \zeta < 1$	Underdamped: poles are comples conj. Response is a damped sinusoid
$\zeta = 0$	Undamped: Poles on imag. axis. Response shows sustained oscillations.
$\zeta < 0$	Unstable: rhp poles. Filters must have $\zeta < 0$.

Let $s \rightarrow j\omega$,

$$H(j\omega) = \frac{N(j\omega)}{1 - j(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

$$Q \equiv \frac{1}{2\zeta}$$

Low-pass

$$H_{LP}(j\omega) = \frac{1}{1 - j(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

- For $\omega << \omega_0$: $|H_{LP}| = 1 = 0dB$.
- For $\omega >> \omega_0$: $|H_{LP}| = \frac{1}{(\omega/\omega_0)^2)} = -40dB/dec.$
- For $\omega = \omega_0$: $|H_{LP}| = Q$

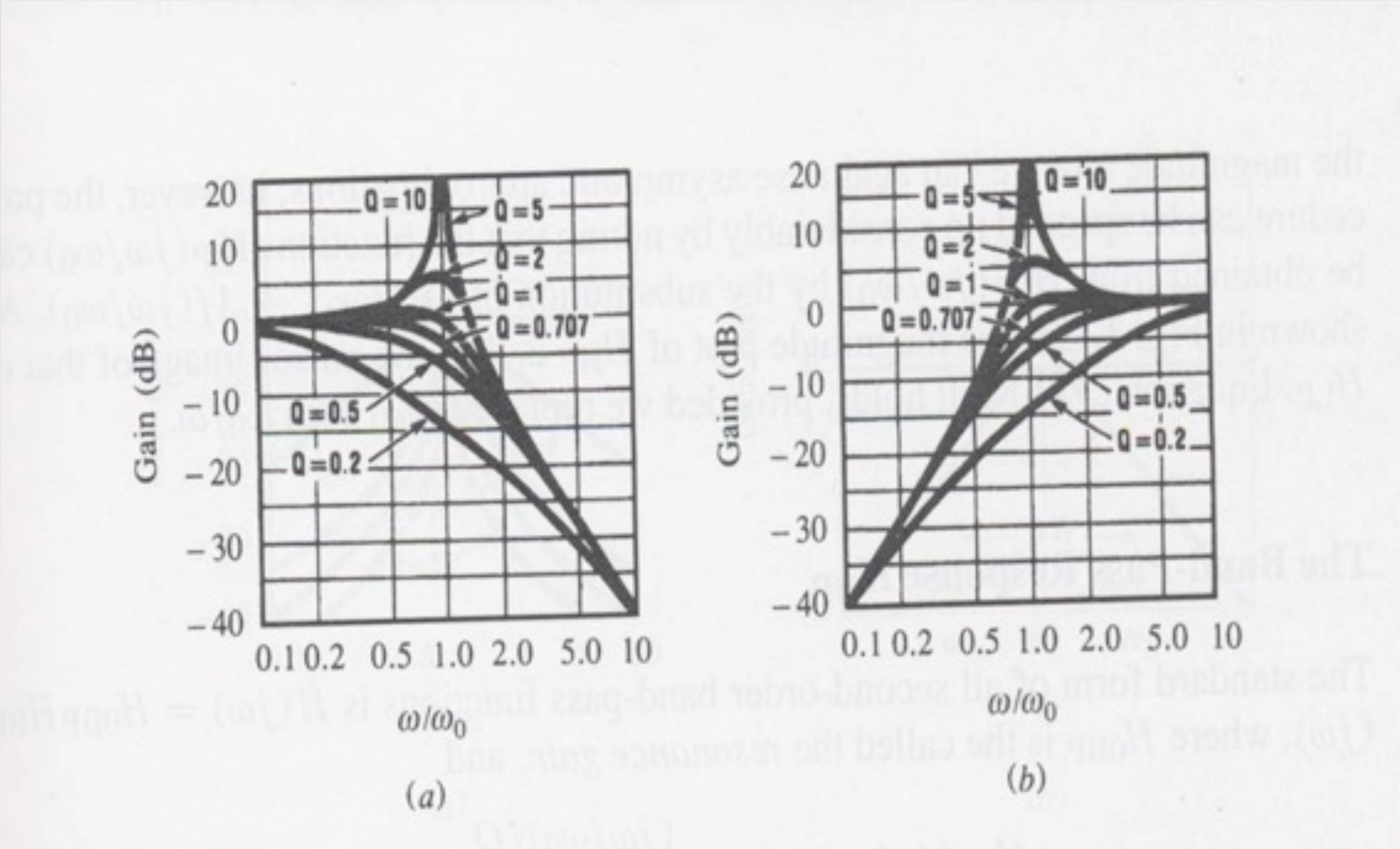
Largest Q before the unset of peaking: $\frac{1}{\sqrt{2}}$

For $Q > \frac{1}{\sqrt{2}}$

$$\frac{\omega_{peak}}{\omega_0} = \sqrt{1 - \frac{1}{2Q^2}}$$

$$|H_{LP}|_{max} = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}$$

2nd order Response



Lowpass and highpass

High-pass

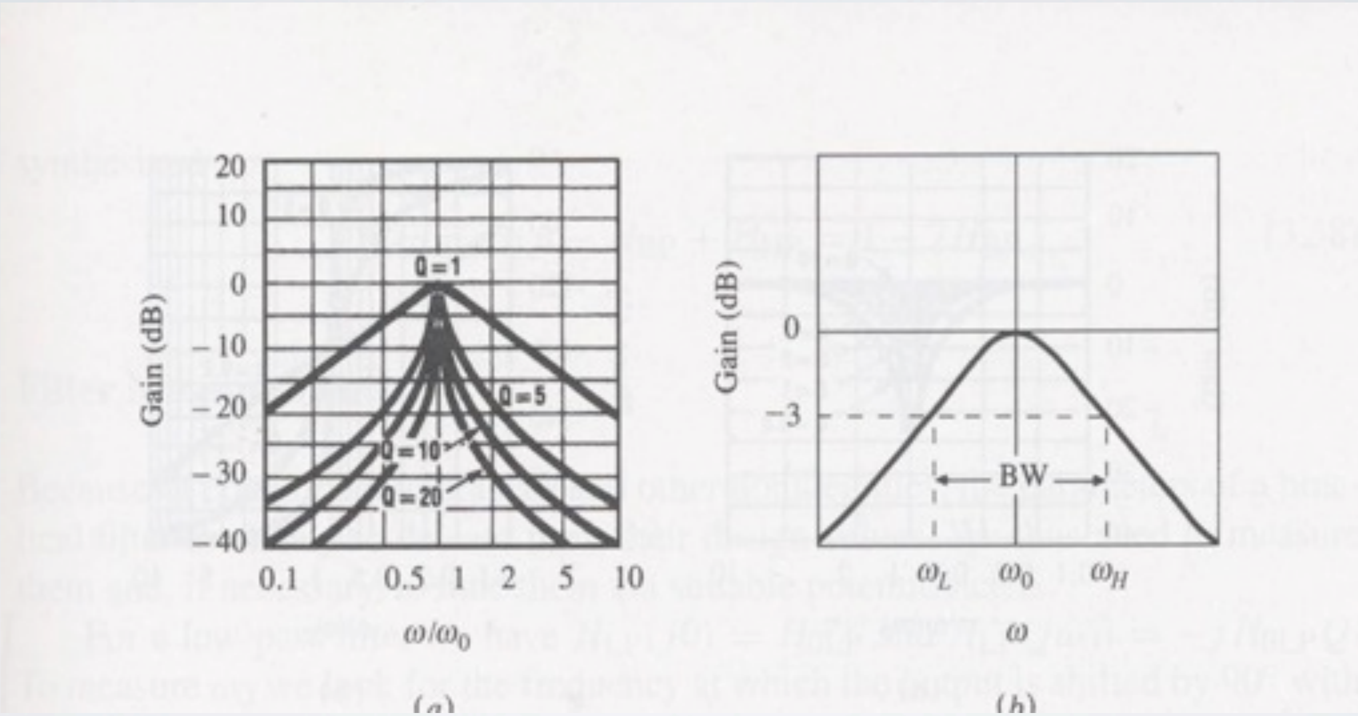
$$H_{HP}(j\omega) = \frac{-(\omega/\omega_0)^2}{1 - j(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

Band-pass

$$H_{BP}(j\omega) = \frac{j(\omega/\omega_0)/Q}{1 - j(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

- $j(\omega/\omega_0)/Q \rightarrow$ zero at origin.
- For $\omega << \omega_0 : |H_{BP}| \simeq (\omega/\omega_0)/Q = (\frac{\omega}{\omega_0})_{dB} - Q_{dB}$.
- For $\omega >> \omega_0 : |H_{BP}| \simeq -(\frac{\omega}{\omega_0})_{dB} - Q_{dB}$.
- For $\omega = \omega_0 : |H_{LP}| = 0$

$$\begin{aligned} BW &= \omega_H - \omega_L \\ &= \omega_0\left(\sqrt{1-\frac{1}{4Q^2}} - \frac{1}{2Q}\right) - \omega_0\left(\sqrt{1-\frac{1}{4Q^2}} + \frac{1}{2Q}\right) \\ \omega_0 &= \sqrt{\omega_L\omega_H} \\ Q &= \omega_0/BW \end{aligned}$$


20
10
0
−10
−20
−30
−40
Gain (dB)
Q = 1
Q = 5
Q = 10
Q = 20
0.1
0.5
1
2
5
10
ω/ω0
(a)
Gain (dB)
0
−3
BW
ωL
ω0
ωH
ω
(b)

Bandpass

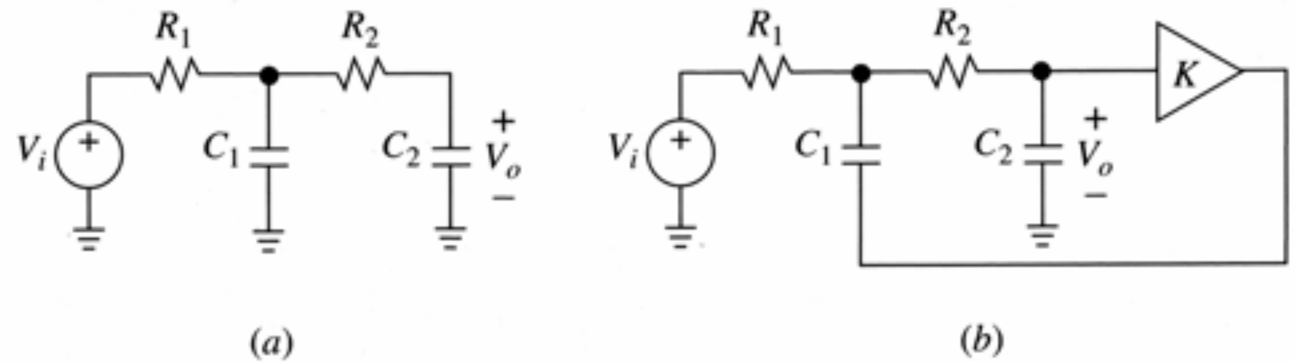
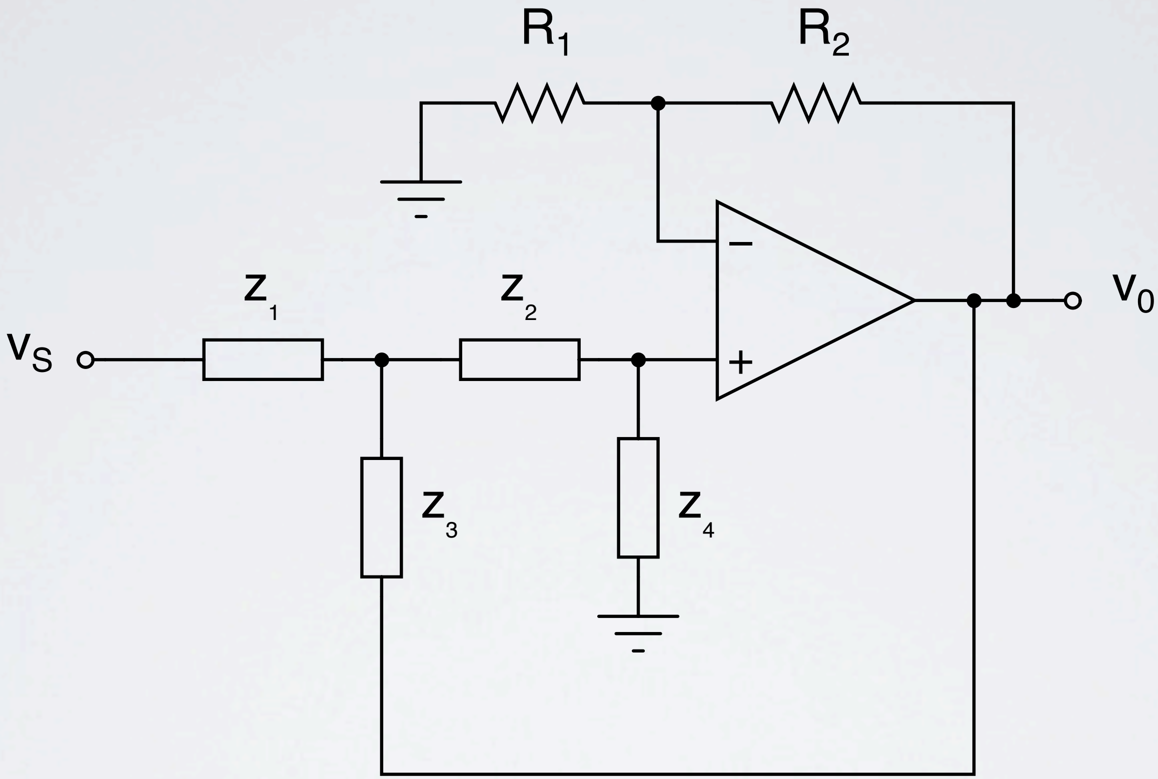


FIGURE 3.22
(a) Passive and (b) active realization of a second-order low-pass filter.

For (a) $Q \leq 0.5$

Textbook
$Z_1 = R_1$
$Z_2 = R_2$
$Z_3 = 1/sC_1$
$Z_4 = 1/sC_2$
$K = A_{v0}$

Sallen-Key (KRC) Filter



$$v_+ = \frac{z_4}{z_2 + z_4}\left(\frac{z_3(z_2 + z_4)}{z_1(z_2 + z_3 + z_4) + z_3(z_2 + z_4)} v_S + \frac{z_1(z_2 + z_4)}{z_3(z_1 + z_2 + z_4) + z_1(z_2 + z_4)} v_O\right)$$

Setting $v_+ = V_O/A_{v0}$ and rearranging gives

$$\frac{v_O}{A_{V0}} = \frac{z_3 z_4}{z_1(z_2 + z_3 + z_4) + z_3(z_2 + z_4)} v_S + \frac{z_1 z_4}{z_3(z_1 + z_2 + z_4) + z_1(z_2 + z_4)} v_O$$

or

$$\left(A_{V0} - \frac{z_1 z_4}{z_3(z_1 + z_2 + z_4) + z_1(z_2 + z_4)} \right) v_O = \frac{z_3 z_4}{z_1(z_2 + z_3 + z_4) + z_3(z_2 + z_4)} v_S$$

which, after rearranging yields

$$\frac{v_O}{v_S} = \frac{z_3 z_4}{z_3(z_1 + z_2 + z_4) + z_1 z_2 + z_1 z_4(1 - A_{v0})} A_{v0}$$

where $A_{v0} = 1 + R_2/R_1$.

Component choices are for the *equal component* design

To obtain a Sallen-Key filter, let $z_1 = z_2 = R$ and $z_3 = z_4 = \frac{1}{sC}$. Then

$$\frac{v_O}{v_S} = \frac{\frac{1}{s^2C^2}}{\frac{1}{sC}(2R + \frac{1}{sC}) + R^2 + \frac{R}{sC}(1 - A_{v0})} A_{v0}$$

which can be rearranged to obtain

$$\frac{v_O}{v_S} = \frac{A_{v0}}{s^2(RC)^2 + (3 - A_{v0})(RC)s + 1}$$

Letting RCs represent a scaled frequency $s' = j\omega'$,

$$\frac{v_O}{v_S} = \frac{A_{v0}}{s'^2 + (3 - A_{v0})s' + 1}$$

whose denominator is of the form

$$s^2 + 2\zeta s + 1$$

$Q = 1/(3 - A_{v0})$

Example: Design an equal-component low-pass filter with $f_0 = 1kHz$ and $Q = 5$.
Example: Modify the design to get a dc gain of 0dB.

Unity-gain option

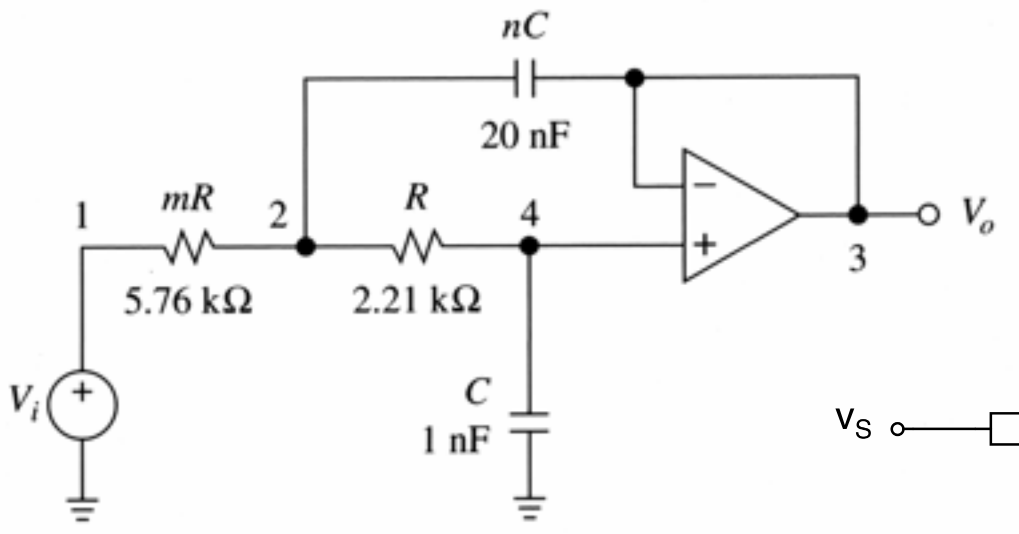
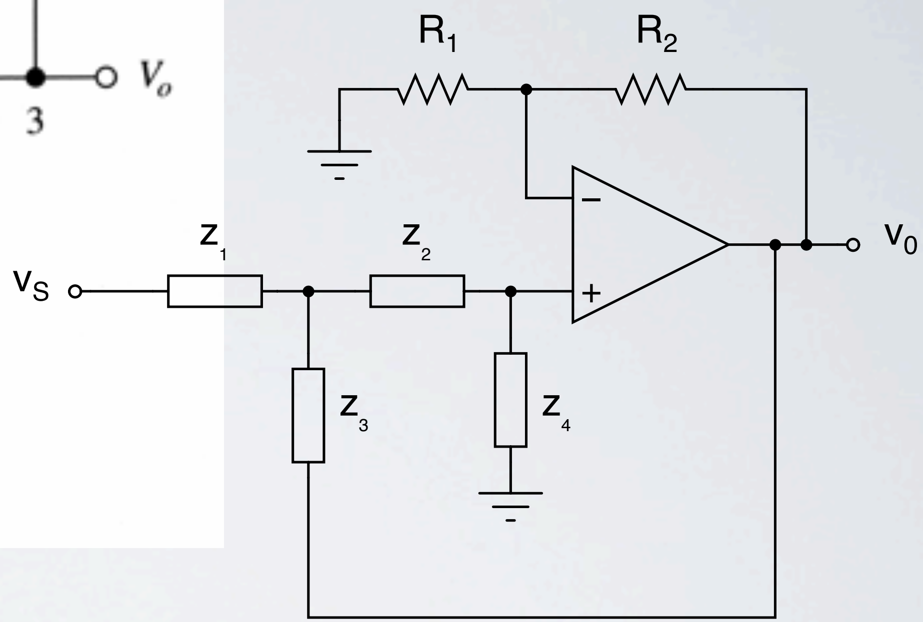


FIGURE 3.25
Filter of Example 3.10.



$$\frac{v_O}{v_S} = \frac{z_3 z_4}{z_3(z_1 + z_2 + z_4) + z_1 z_2 + z_1 z_4 (1 - A_{v0})} A_{v0}$$

$$z_1 = mR \qquad z_2 = R \qquad z_3 = 1/nsC \qquad z_4 = 1/sC$$

For $m = 1$, $n = 4Q^2 \Rightarrow$ select n based on available cap values, then $m = k + \sqrt{k^2 - 1}$ where $k = n/2Q^2 - 1$.

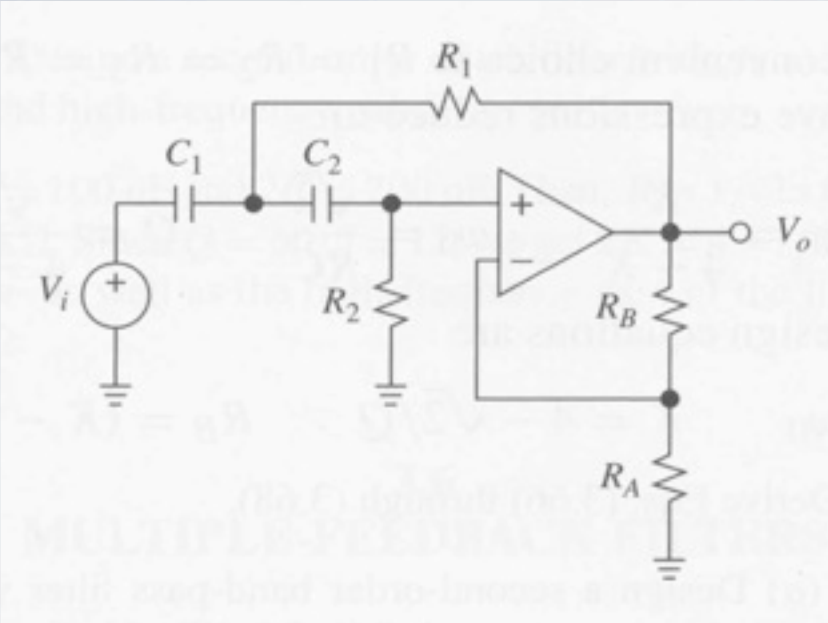
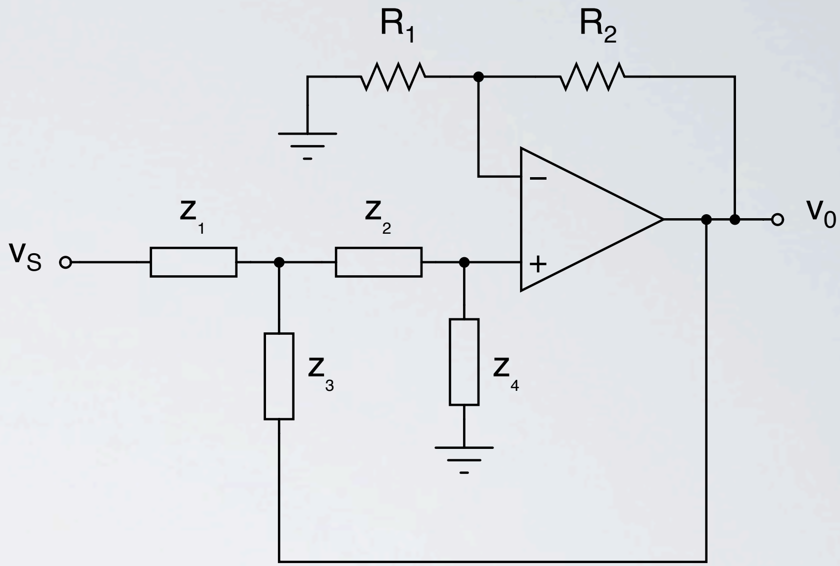


FIGURE 3.27
High-pass *KRC* filter.



$$\frac{v_O}{v_S} = \frac{z_3 z_4}{z_3(z_1 + z_2 + z_4) + z_1 z_2 + z_1 z_4 (1 - A_{v0})} A_{v0}$$

$$z_1 = 1/sC_1 \qquad z_2 = 1/sC_2 \qquad z_3 = R_1 \qquad z_4 = R_2$$

$$H_{0HP} = K \qquad \omega_0 = 1/\sqrt{R_1 C_1 R_2 C_2}$$

$$Q = \frac{1}{(1-K)\sqrt{R_2C_2/R_1C_1} + \sqrt{R_1C_2/R_2C_1} + \sqrt{R_1C_1/R_2C_2}}$$

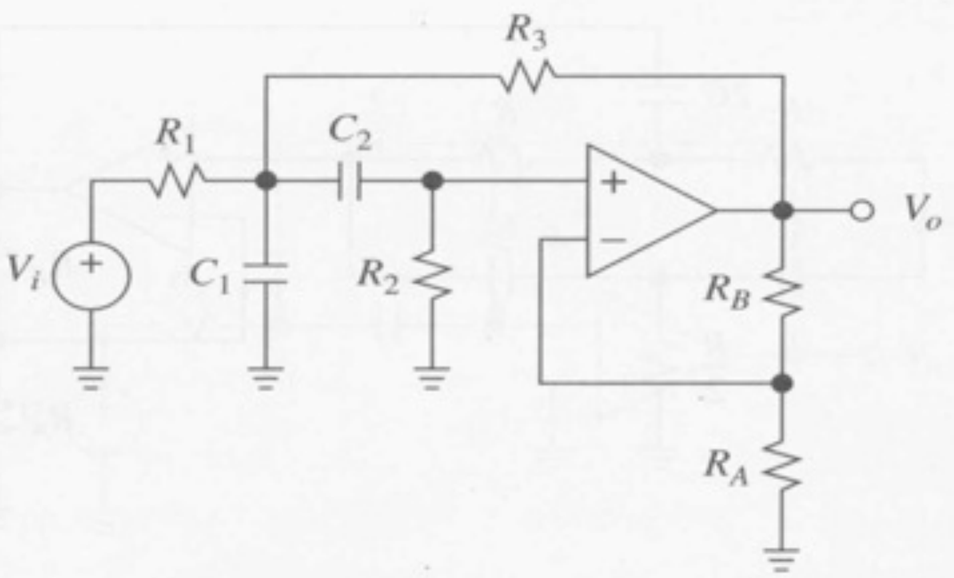


FIGURE 3.28
Band-pass *KRC* filter.

Band-Pass KRC: $V_O/V_i = H_{0BP}H_{BP}$ where H_{BP} is given in slide 12 and, for $Q > \sqrt{2}/3$, $R_1 = R_2 = R_3 = R$ and $C_1 = C_2 = C$,

$$H_{0BP} = \frac{K}{4-K} \quad \omega_0 = \frac{\sqrt{2}}{RC} \quad Q = \frac{\sqrt{2}}{4-K}$$

Design equations:

$$RC = \sqrt{2}/\omega_0 \quad K = 4 - \sqrt{2}/Q \quad R_B = (K-1)R_A$$

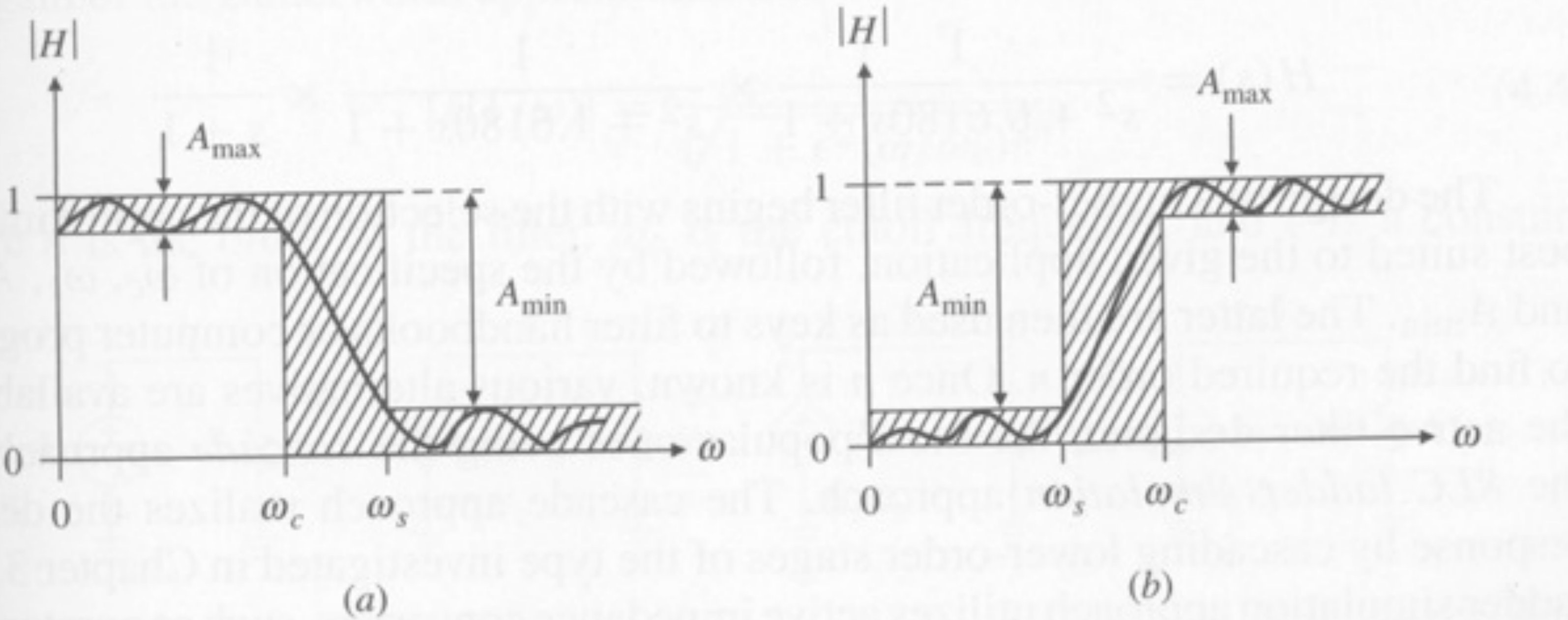


FIGURE 4.1
Magnitude limits for (*a*) the low-pass and (*b*) the high-pass responses.

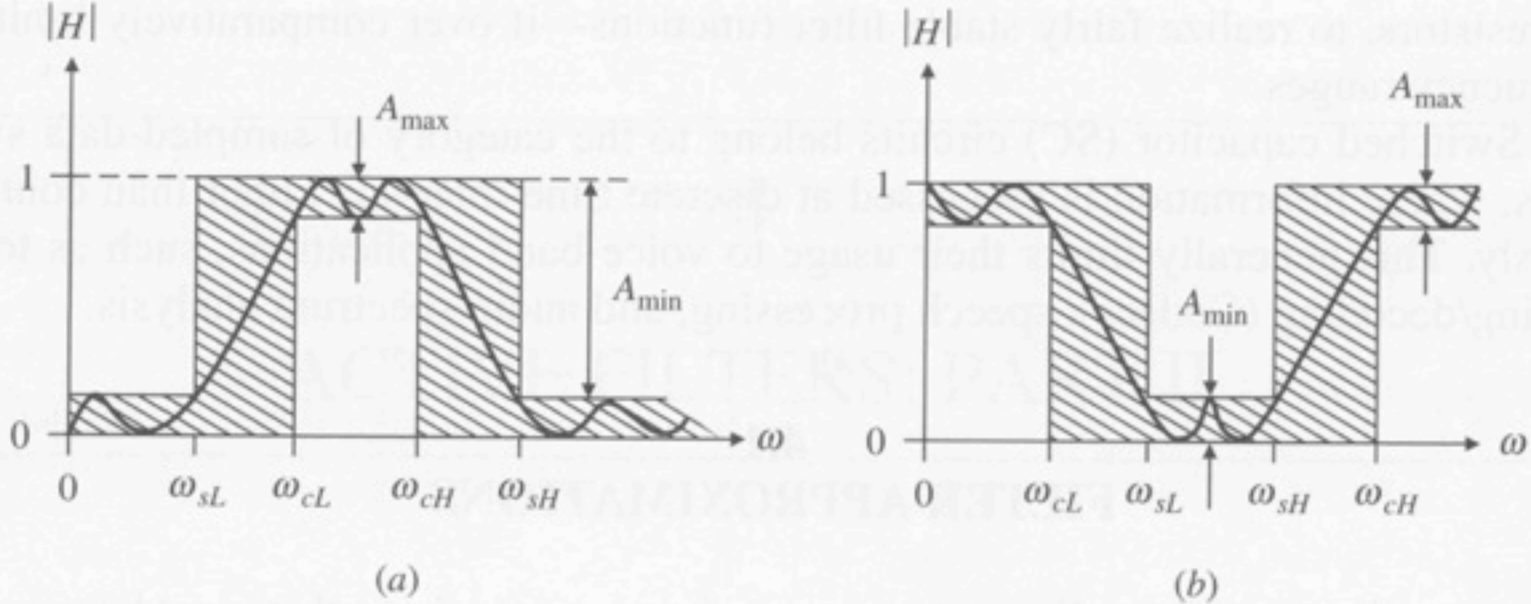


FIGURE 4.2
Magnitude limits for (*a*) the band-pass and (*b*) the band-reject responses.

- Butterworth filter: $|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2(\omega/\omega_C)^{2n}}}$
  - $n \geq \frac{\log\left(10^{A_{min}/10}-1)/(10^{A_{max}/10}-1)\right)}{\log(\omega_s/\omega_c)}$: order of the filter
  - ω_C: cutoff frequency
  - ϵ: constant that determines maximum passband variation
- Chebyshev: $|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 C_n^2(\omega/\omega_C)}}$
  - $n \geq \frac{\cosh^{-1}\sqrt{(10^{A_{min}/10}-1)/(10^{A_{max}/10}-1)}}{\cosh^{-1}(\omega_s/\omega_c)}$: order of the filter
  - $C_n(\omega/\omega_C)$: Chebyshev polynomial of order n
  - $C_n(\omega/\omega_C \leq 1) = \cos(n \arccos(\omega/\omega_C))$
  - $C_n(\omega/\omega_C > 1) = \cosh(n \cosh^{-1}(\omega/\omega_C))$
- Cauer (or elliptic) filters and Bessel (or Thomson) filters

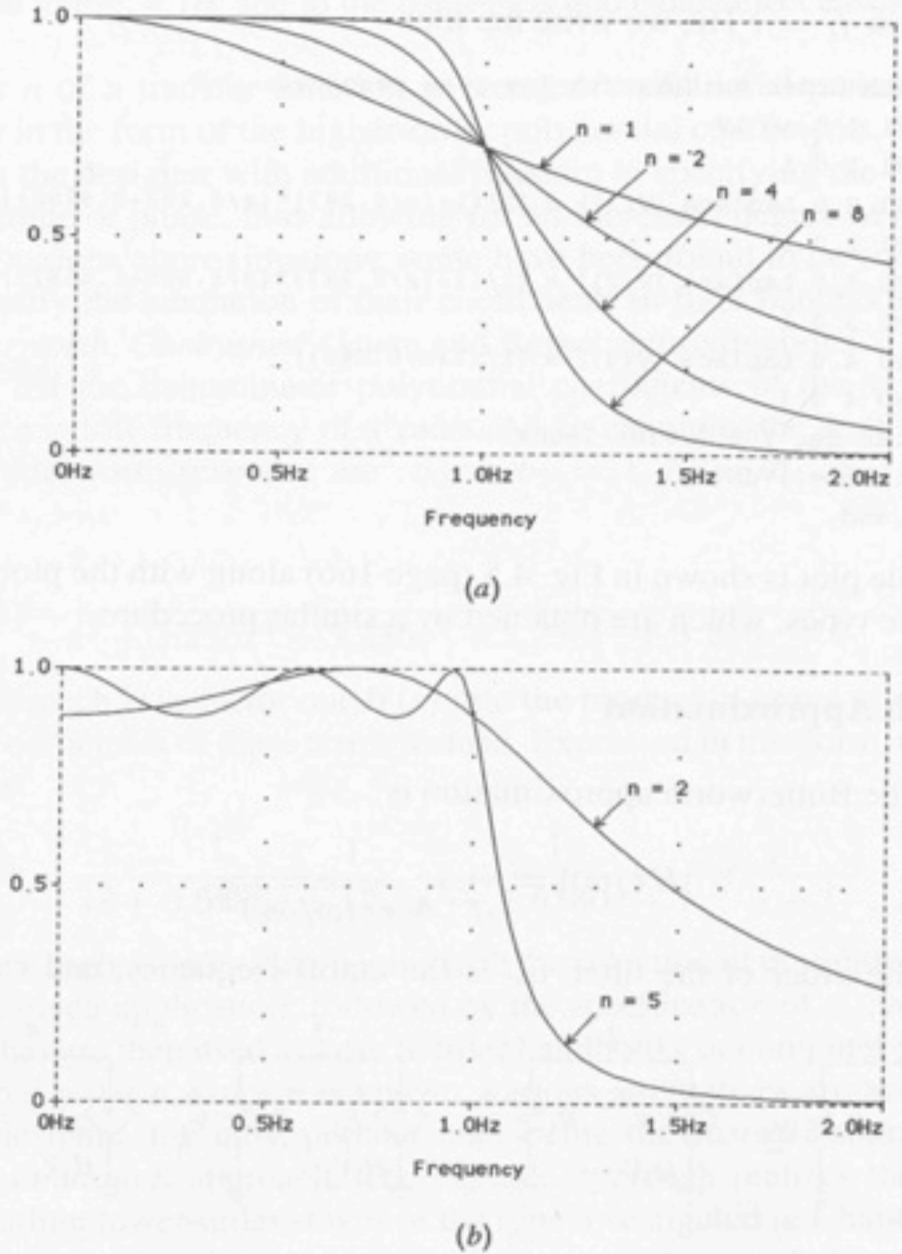


FIGURE 4.4
(*a*) Butterworth and (*b*) 1-dB Chebyshev responses.

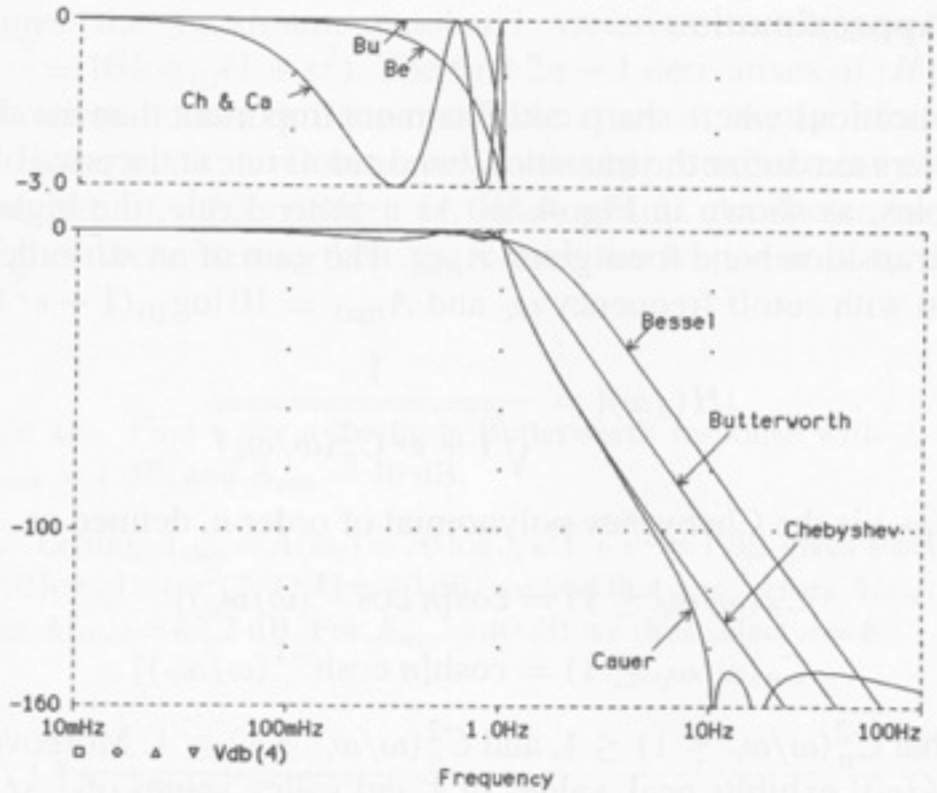


FIGURE 4.5
Comparison of fifth-order 3-dB responses.

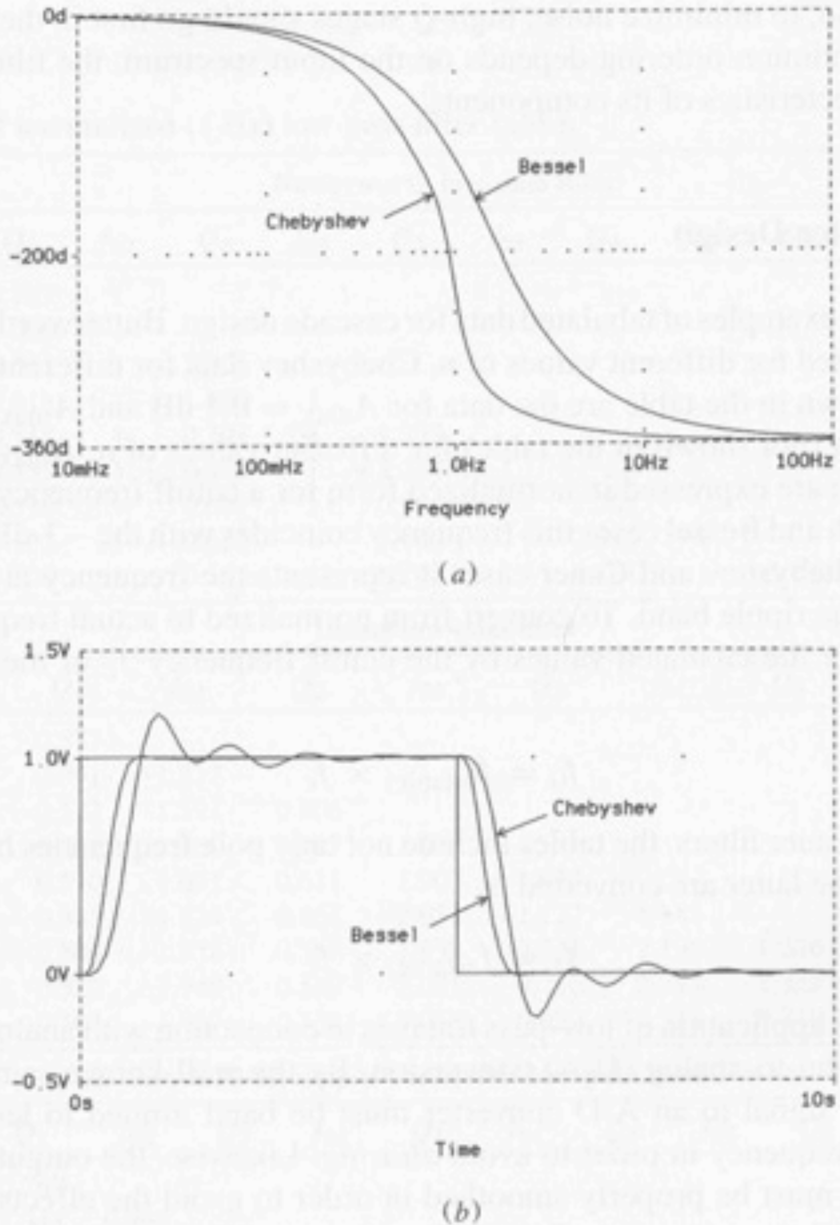


FIGURE 4.6
Phase and pulse responses of the fourth-order Bessel and 1-dB Chebyshev filters.

Thursday, March 21, 13

TABLE 4.1

Examples of normalized (1 Hz) low-pass filter tables

Butterworth low-pass filter

n	f_{01}	Q_1	f_{02}	Q_2	f_{03}	Q_3	f_{04}	Q_4	f_{05}	Q_5	Att (dB) at $2f_c$
2	1	0.707	1								12.30
3	1	1.000	1								18.13
4	1	0.541	1	1.306							24.10
5	1	0.618	1	1.620	1						30.11
6	1	0.518	1	0.707	1	1.932					36.12
7	1	0.555	1	0.802	1	2.247	1				42.14
8	1	0.510	1	0.601	1	0.900	1	2.563			48.16
9	1	0.532	1	0.653	1	1.000	1	2.879	1		54.19
10	1	0.506	1	0.561	1	0.707	1	1.101	1	3.196	60.21

Bessel low-pass filter

n	f_{01}	Q_1	f_{02}	Q_2	f_{03}	Q_3	f_{04}	Q_4	f_{05}	Q_5
2	1.274	0.577								
3	1.453	0.691	1.327							
4	1.419	0.522	1.591	0.806						
5	1.561	0.564	1.760	0.917	1.507					
6	1.606	0.510	1.691	0.611	1.907	1.023				
7	1.719	0.533	1.824	0.661	2.051	1.127	1.685			
8	1.784	0.506	1.838	0.560	1.958	0.711	2.196	1.226		
9	1.880	0.520	1.949	0.589	2.081	0.760	2.324	1.322	1.858	
10	1.949	0.504	1.987	0.538	2.068	0.620	2.211	0.810	2.485	1.415

0.10-dB ripple Chebyshev low-pass filter

n	f_{01}	Q_1	f_{02}	Q_2	f_{03}	Q_3	f_{04}	Q_4	f_{05}	Q_5	Att (dB) at $2f_c$
2	1.820	0.767									3.31
3	1.300	1.341	0.969								12.24
4	1.153	2.183	0.789	0.619							23.43
5	1.093	3.282	0.797	0.915	0.539						34.85
6	1.063	4.633	0.834	1.332	0.513	0.599					46.29
7	1.045	6.233	0.868	1.847	0.575	0.846	0.377				57.72
8	1.034	8.082	0.894	2.453	0.645	1.183	0.382	0.593			69.16
9	1.027	10.178	0.913	3.145	0.705	1.585	0.449	0.822	0.290		80.60
10	1.022	12.522	0.928	3.921	0.754	2.044	0.524	1.127	0.304	0.590	92.04

1.00-dB ripple Chebyshev low-pass filter

n	f_{01}	Q_1	f_{02}	Q_2	f_{03}	Q_3	f_{04}	Q_4	f_{05}	Q_5	Att (dB) at $2f_c$
2	1.050	0.957									11.36
3	0.997	2.018	0.494								22.46
4	0.993	3.559	0.529	0.785							33.87
5	0.994	5.556	0.655	1.399	0.289						45.31
6	0.995	8.004	0.747	2.198	0.353	0.761					56.74
7	0.996	10.899	0.808	3.156	0.480	1.297	0.205				68.18
8	0.997	14.240	0.851	4.266	0.584	1.956	0.265	0.753			79.62
9	0.998	18.029	0.881	5.527	0.662	2.713	0.377	1.260	0.159		91.06
10	0.998	22.263	0.902	6.937	0.721	3.561	0.476	1.864	0.212	0.749	102.50

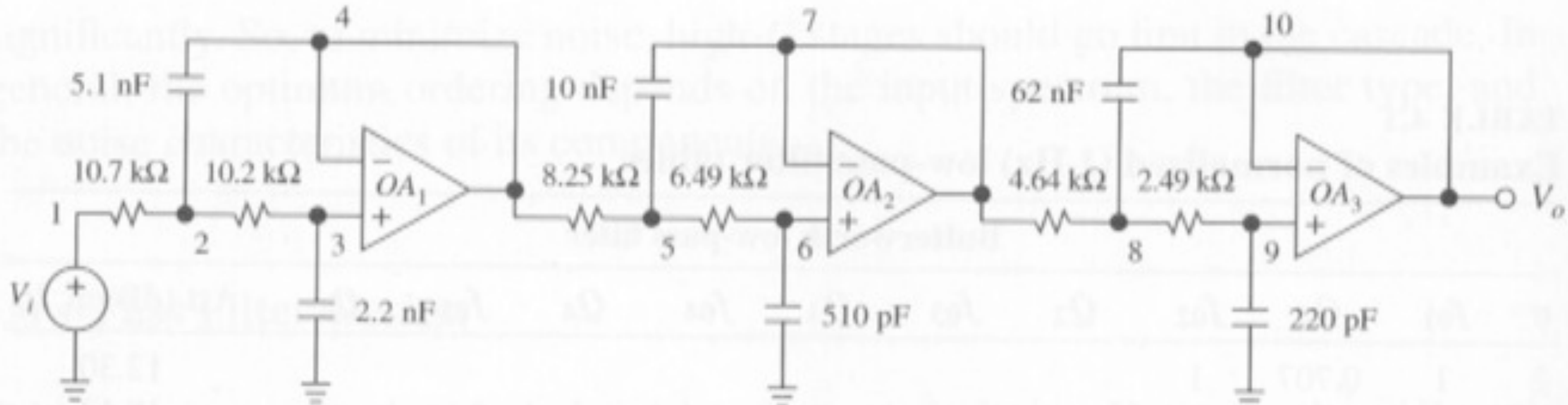


FIGURE 4.7
Sixth-order 1-dB Chebyshev low-pass filter.

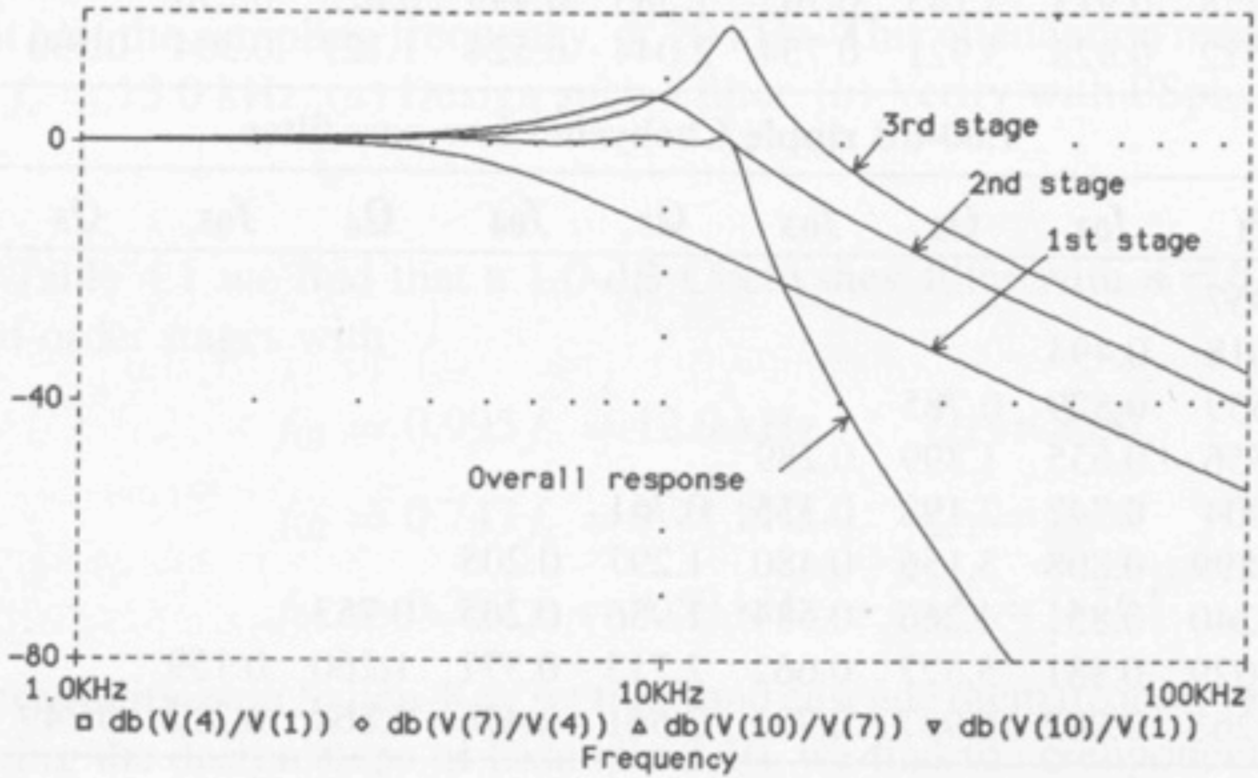


FIGURE 4.8
Overall as well as individual-stage responses of the filter of Fig. 4.7.

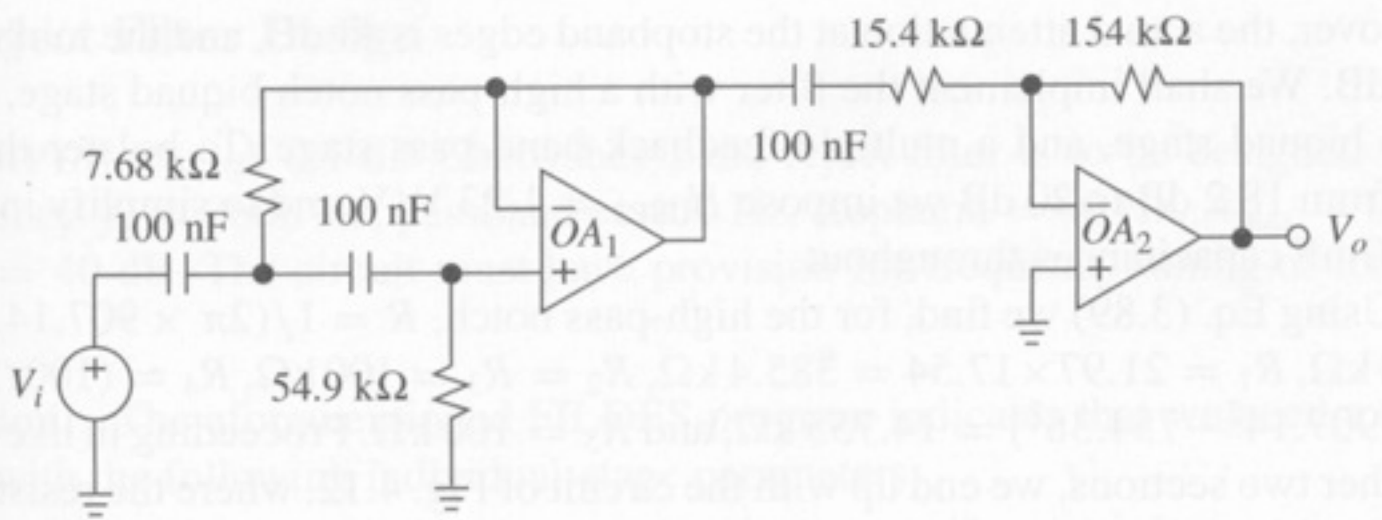


FIGURE 4.10
Third-order 0.1-dB Chebyshev high-pass filter of Example 4.4.

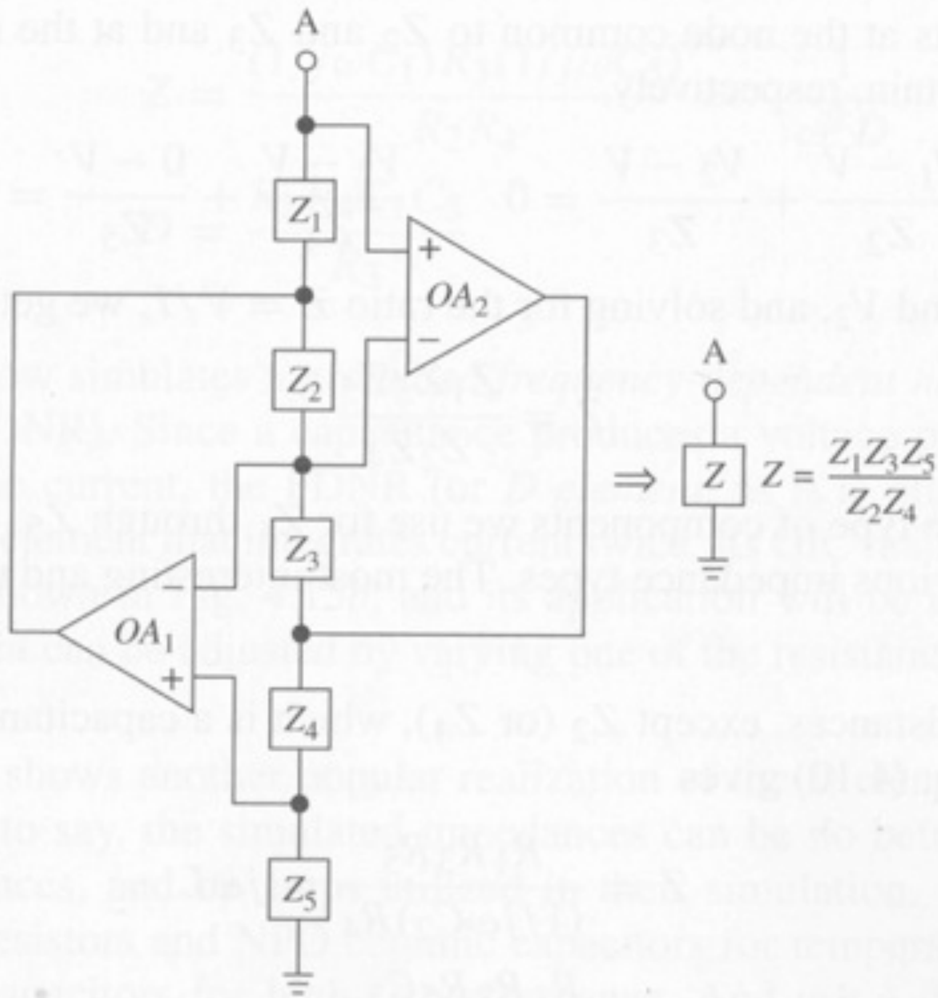


FIGURE 4.13
Generalized impedance converter (GIC).

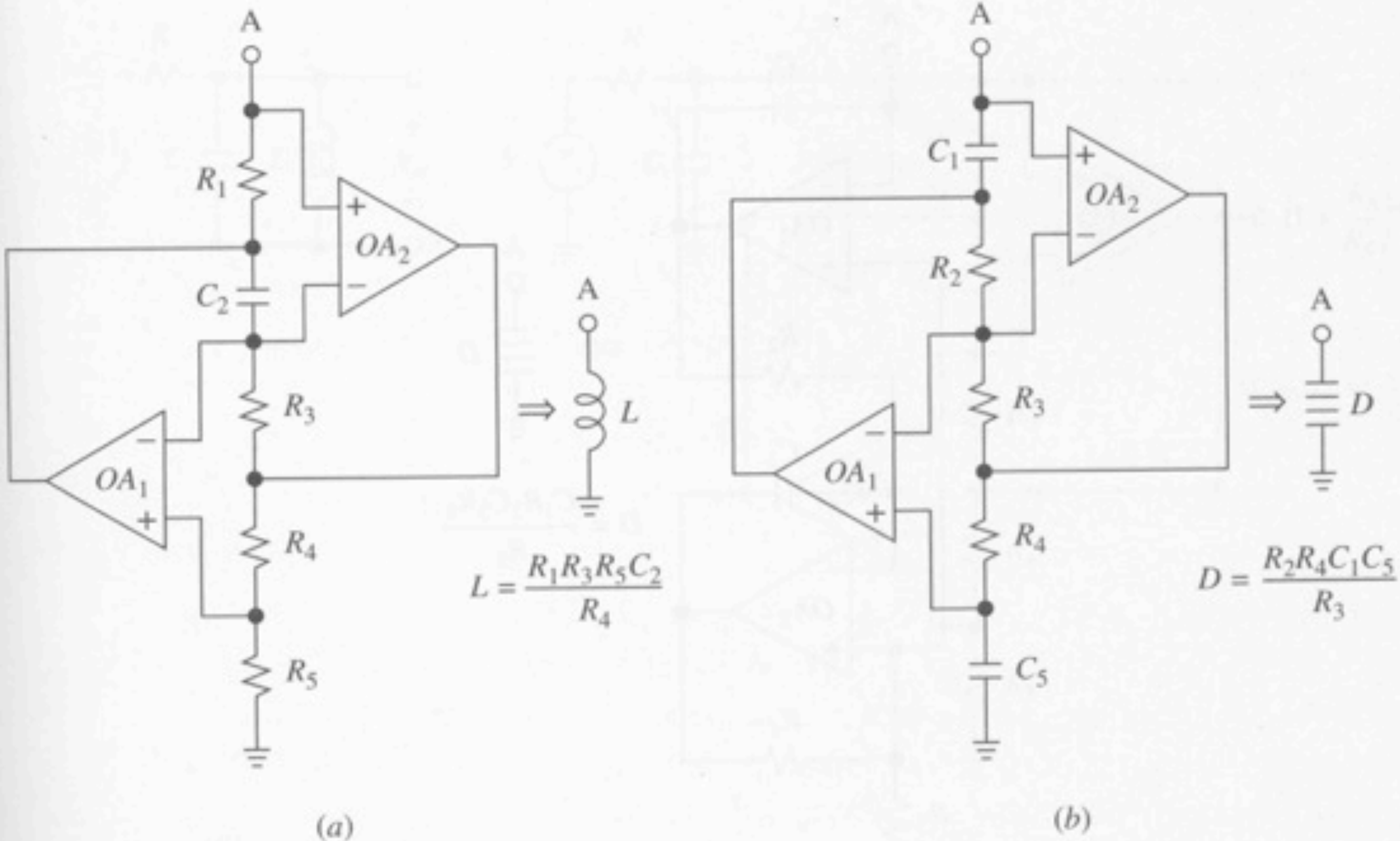


FIGURE 4.15
(*a*) Inductance simulator and (*b*) *D*-element realization.

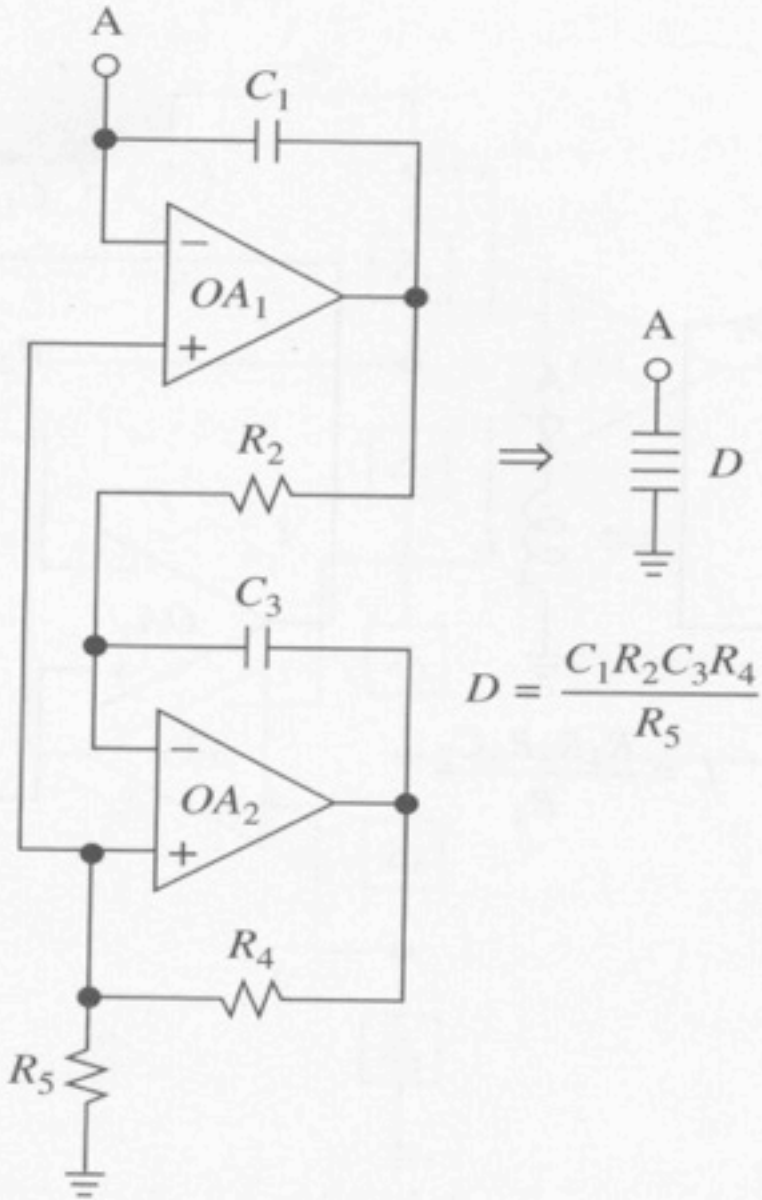


FIGURE 4.16
Alternative *D*-element realization.

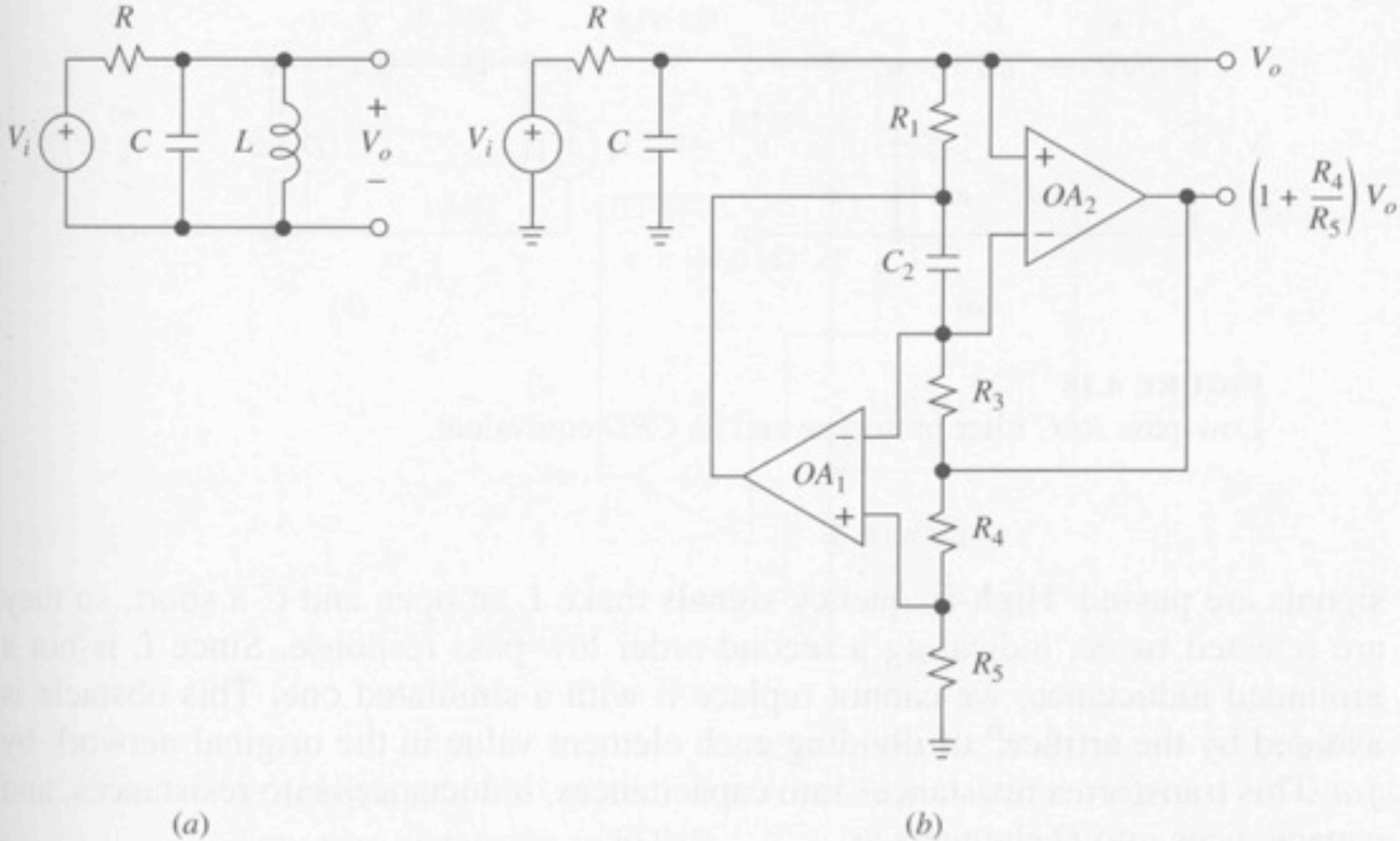


FIGURE 4.17
(*a*) Passive band-pass filter prototype and (*b*) active realization using an inductance simulator.

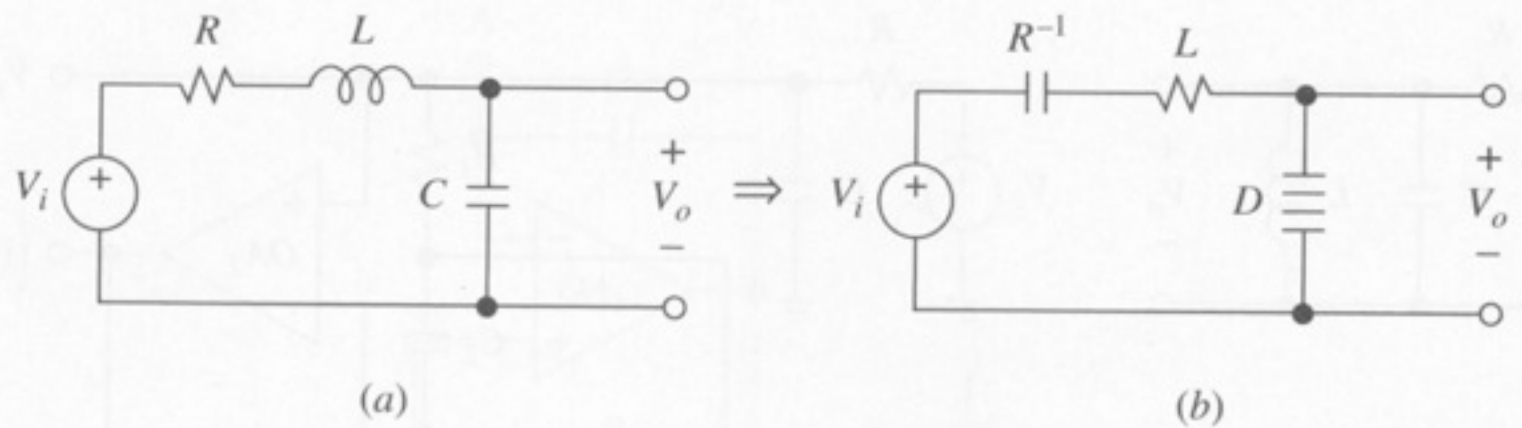


FIGURE 4.18
Low-pass *RLC* filter prototype and its *CRD* equivalent.