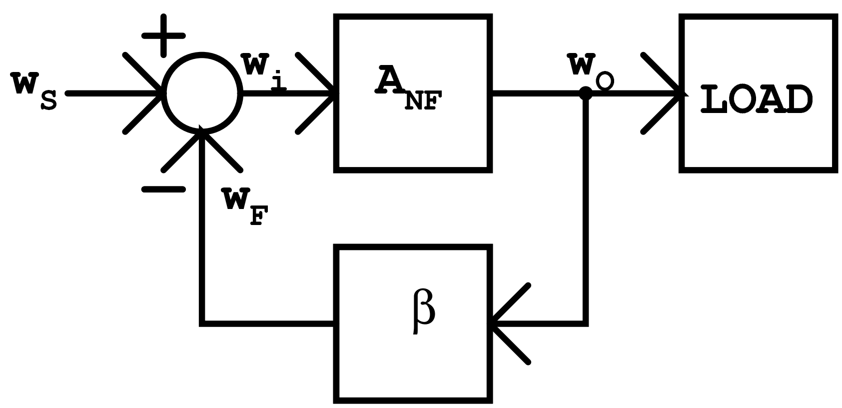
Capítulo 5

Amplificadores Retro-alimentados

5.1. Conceptos Básicos

El siguiente diagrama



muestra un sistema general que emplea retro-alimentación. En el mismo el símbolo w puede representar una señal de voltaje o de corriente.

Definiciones y observaciones:

- La señal de salida es w_O.
- La red de retro-alimentación, representada por β en el diagrama, normalmente se implementa con un circuito resistivo de dos compuertas - o sea un divisor de voltaje o de corriente en alguna de sus formas.
- La ecuación fundamental de cualquier sistema con retro-alimentación es

$$w_O = A_{NF} w_i = A_{NF}(w_S - \beta w_O)$$

$$w_O = \frac{A_{NF}}{1 + \beta A_{NF}} w_S = A_F w_S = \frac{A_{NF}}{D} \tag{5.1}$$

- A_{NF} es la ganancia del sistema cuando la retro-alimentación es removida, pero en su calculo se debe tomar en cuenta los efectos de carga de la red de retro-alimentación.
- La *ganancia del lazo*, también conocida como la *razón de retorno*, se define como el producto $L = \beta A_{NF}$.
- La cantidad D es llamada el *factor de mejoramiento* o la *diferencia de retorno*.
- Si $A_{NF} \gg 1$,

$$A_F \approx \frac{1}{\beta}$$

  Dado que la red β usualmente se implementa como un divisor de corriente o voltaje,

  - La ganancia del amplificador retro-alimentado es insensible a los parámetros del amplificador siempre y cuando $L \gg 1$.
  - Como los divisores envuelven la razón entre dos o más resistencias, errores en las resistencias tienden a cancelarse. También el efecto de cambios en temperatura es grandemente reducido.

Ventajas del uso de retro-alimentación:

- Reduce la sensitividad a cambios en los parámetros.
- Aumenta el ancho de banda.
- Reduce la distorsión.
- Mejora la resistencia de entrada y salida.

Costo del uso de retro-alimentación:
Reducción en la ganancia.

5.1.1. Efecto de la retro-alimentación en la sensitividad del amplificador

La sensitividad se define como

$$S_P^A = \frac{P}{A}\frac{dA}{dP} = \frac{dA/A}{dp/p}$$

La sensitividad es la razón entre cambios relativos en la cantidad A y el parámetro p.

Para el amplificador retro-alimentado,

$$\begin{aligned} S_P^{A_F} &= \frac{P}{A_F}\frac{dA_F}{dP} \\ &= \frac{P}{A_F}\frac{d\frac{A_{NF}}{1+\beta A_{NF}}}{dP} \end{aligned}$$

$$
\begin{aligned}
&= \frac{P}{A_F}\frac{d}{dA_{NF}}\frac{A_{NF}}{1+\beta A_{NF}}\frac{dA_{NF}}{dp} \\
&= \frac{P}{A_F}\frac{dA_{NF}}{dp}\left(\frac{1}{1+\beta A_{NF}} - \frac{\beta A_{NF}}{(1+\beta A_{NF})^2}\right) \\
&= \frac{P}{A_{NF}}\frac{dA_{NF}}{dp}\left(1 - \frac{\beta A_{NF}}{1+\beta A_{NF}}\right) \\
&= \frac{P}{A_{NF}}\frac{dA_{NF}}{dp}\frac{1}{1+\beta A_{NF}} \\
&= S_P^{A_{NF}}\frac{1}{1+\beta A_{NF}}
\end{aligned}
$$

Vemos que el uso de retro-alimentación reduce la sensitividad por el factor de mejoramiento.

5.1.2. Efecto de la retro-alimentación en la respuesta a señales de baja frecuencia

Se asume que podemos aproximar la función de transferencia del amplificador sin retro-alimentación por la ganancia de frecuencia intermedia, un polo y un cero en el origen. Entonces

$$A_{NF}(s) = A_{mid}\frac{s}{s+\omega_L}$$

La ganancia del amplificador retro-alimentado es

$$
\begin{aligned}
A_F(s) &= \frac{A_{mid}\frac{s}{s+\omega_L}}{1+\beta A_{mid}\frac{s}{s+\omega_L}} \\
&= \frac{A_{mid}s}{s+\omega_L+\beta A_{mid}s} \\
&= \frac{A_{mid}s}{s(1+\beta A_{mid})+\omega_L} \\
&= \frac{A_{mid}}{1+\beta A_{mid}} \times \frac{s}{s+\frac{\omega_L}{1+\beta A_{mid}}}
\end{aligned}
$$

Podemos ver que el polo de baja frecuencia se redujo por el factor de mejoramiento.

5.1.3. Efecto de la retro-alimentación en la respuesta a señales de alta frecuencia

Asumiendo que el amplificador sin retro-alimentación puede representarse por su ganancia de frecuencia intermedia y un solo polo,

$$A_{NF} = A_{mid}\frac{\omega_H}{s+\omega_H}$$

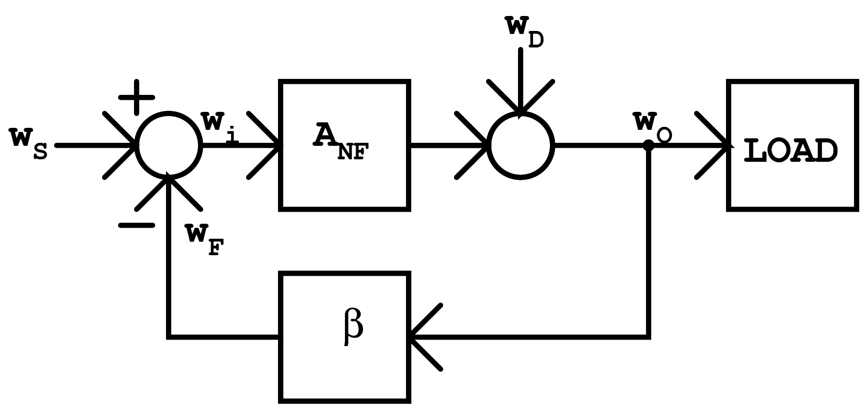
entonces

$$
\begin{aligned}
A_F(s) &= \frac{A_{mid}\frac{\omega_H}{s+\omega_H}}{1+\beta A_{mid}\frac{\omega_H}{s+\omega_H}} \\
&= \frac{A_{mid}\omega_H}{s+\omega_H+\beta A_{mid}\omega_H} \\
&= \frac{A_{mid}\omega_H}{s+\omega_H(1+\beta A_{mid})} \\
&= \frac{A_{mid}}{1+\beta A_{mid}} \times \frac{\omega_H(1+\beta A_{mid})}{s+\omega_H(1+\beta A_{mid})}
\end{aligned}
$$

Así que el polo se mueve a el producto de su frecuencia y el factor de mejoramiento.

5.1.4. Efecto de la retro-alimentación en la distorsión

Debido a los aspectos no-lineales de la respuesta del amplificador, las señales de entrada y salida tendrán formas diferentes. Si denotamos la distorsión por w_D, podemos representar al amplificador por



$$
\begin{aligned}
w_O &= w_D + A_{NF}w_i \\
&= w_D + A_{NF}(w_S - \beta w_O) \\
&= \frac{w_D}{1+\beta A_{NF}} + \frac{A_{NF}w_S}{1+\beta A_{NF}}
\end{aligned}
$$

Podemos ver que el efecto de la distorsión es reducido por el factor de mejoramiento.

5.2. Topologías Empleadas

Amplificadores usan retro-alimentación en una de cuatro configuraciones. Cada una se identifica por el tipo de señal que se muestrea (voltaje o corriente) y el que se sustrae en la entrada. La figura 5.1 muestra ejemplos de como se conecta la red de retro-alimentación para cada configuración o topología.

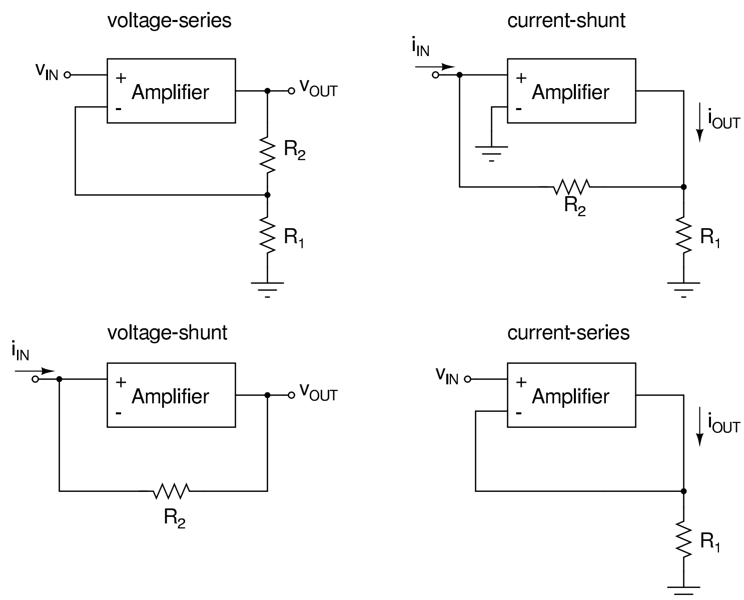


Figura 5.1: Diagrama simplificado que muestra los cuatro modos de conectar la red de retro-alimentación.

5.2.1. Resumen de las Formulas de Retro-alimentación

5.2.2. Discrete Feedback Amplifiers

The following four circuits are examples of feedback amplifiers that use discrete components. For each,

1. identify the type of feedback being used
2. draw a diagram of the feedback network
3. find the 11, 12 and 22 two-port network parameters that correspond to the type of feedback

Nombre en texto	series-shunt	series-series	shunt-series	shunt-shunt
Nombre abreviado	voltaje-voltaje	voltaje-corriente	corriente-corriente	corriente-voltaje
Señal de entrada	voltaje	voltaje	corriente	corriente
Señal de salida	voltaje	corriente	corriente	voltaje
β	$\frac{v_1}{v_2}\Big\|_{i_1=0}$	$\frac{v_1}{i_2}\Big\|_{i_1=0}$	$\frac{i_1}{i_2}\Big\|_{v_1=0}$	$\frac{i_1}{v_2}\Big\|_{v_1=0}$
A_f	$\frac{A_V}{1+\beta A_V}$	$\frac{G_M}{1+\beta G_M}$	$\frac{A_I}{1+\beta A_I}$	$\frac{R_M}{1+\beta R_M}$
R_{if}	$R_i(1+\beta A_V)$	$R_i(1+\beta G_M)$	$\frac{R_i}{1+\beta A_I}$	$\frac{R_i}{1+\beta R_M}$
R_{of}	$\frac{R_o}{1+\beta A_V}$	$R_o(1+\beta G_M)$	$R_o(1+\beta A_I)$	$\frac{R_o}{1+\beta R_M}$

Cuadro 5.1: Formulas para amplificadores retro-alimentados.

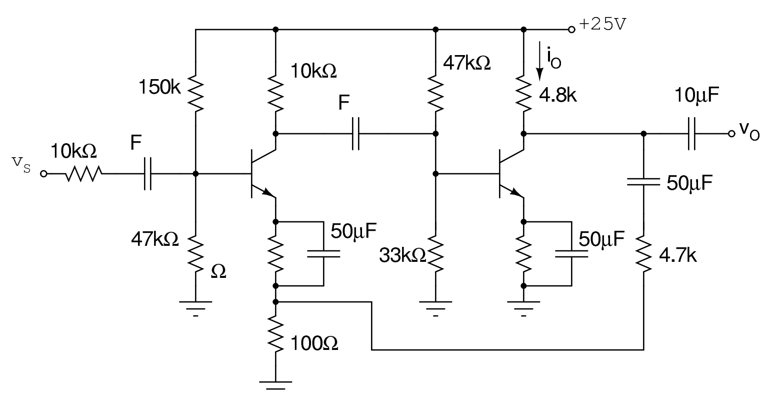
tipo	voltaje-voltaje	voltaje-corriente	corriente-corriente	corriente-voltaje
fuente de entrada	Thevenin	Thevenin	Norton	Norton
fuente dependiente	Norton	Thevenin	Thevenin	Norton
R_{11}	$\frac{v_1}{i_1}\|_{v_2=0}$	$\frac{v_1}{i_1}\|_{i_2=0}$	$\frac{v_1}{i_1}\|_{i_2=0}$	$\frac{v_1}{i_1}\|_{v_2=0}$
β	$\frac{v_1}{v_2}\|_{i_1=0}$	$\frac{v_1}{i_2}\|_{i_1=0}$	$\frac{i_1}{i_2}\|_{v_1=0}$	$\frac{i_1}{v_2}\|_{v_1=0}$
R_{22}	$\frac{v_2}{i_2}\|_{i_1=0}$	$\frac{v_2}{i_2}\|_{i_1=0}$	$\frac{v_2}{i_2}\|_{v_1=0}$	$\frac{v_2}{i_2}\|_{v_1=0}$

Cuadro 5.2: Formulas para simplificar la red de retro-alimentación.

4. draw a diagram of the non-feedback amplifier, including the loading effects of the feedback network. Assume caps are shorts.
5. find the non-feedback amplifier gain that correspond to the type of feedback being used (i.e. A_v, A_i, G_M or R_M)
6. find the non-feedback amplifier input and output resistance, R_i and R_o
7. find the feedback amplifier gain, input and output resistance.
8. find the feedback amplifier voltage and current gains, $A_v = \frac{v_o}{v_s}$ and $A_i = \frac{i_o}{i_s}$, respectively, where $v_s = R_{TH} i_s$.

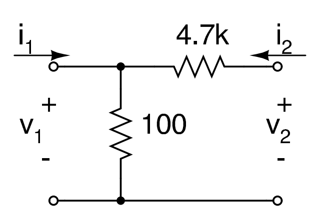
Use $r_\pi = 1100\Omega$ and $\beta = 50$.

Amplifier 1



ANSWER:

1. Feedback is voltage-sampling, voltage-mixing.
2. The feedback network is:



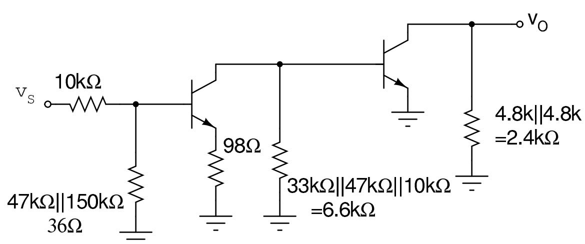
3. For voltage-sampling, voltage-mixing,

$$R_{11} = \frac{v_1}{i_1} |_{v_2=0} = 4{,}7k\Omega \mid\mid 100\Omega = 98\Omega$$

$$R_{22} = \frac{v_2}{i_2} |_{i_1=0} = 4{,}7k\Omega + 100\Omega = 4{,}8k\Omega$$

$$\beta = \frac{v_1}{v_2} |_{i_1=0} = \frac{100\Omega}{4{,}7k\Omega + 100\Omega} = \frac{1}{48} V/V$$

4. The non-feedback amplifier is:



5. The voltage gain is

$$\begin{aligned}
A_v &= \frac{10k\Omega}{10k\Omega + R_{in}} \times A_{v1} \times A_{v2} \\
A_{v2} &= -g_{m2} R_{c2} = -\frac{50}{1100\Omega} 2{,}4k\Omega = -109{,}1 V/V \\
R_{c1} &= 6{,}6k\Omega \mid\mid r_{\pi 2} = 6{,}6k\Omega \mid\mid 1{,}1k\Omega = 943\Omega \\
A_{v1} &= -\frac{g_{m1} R_{c1}}{1 + g_{m1} R_{e1}} \\
&= -\frac{\frac{50}{1100\Omega} 943\Omega}{1 + \frac{50}{1100\Omega} 98\Omega} \\
&= -7{,}86 V/V \\
R_{in} &= 36k\Omega \mid\mid (r_{\pi 1} + (h_{fe1} + 1) R_{e1}) \\
&= 36k\Omega \mid\mid (1{,}1k\Omega + 51 \times 98\Omega) \\
&= 5214\Omega \\
A_v &= \frac{5214}{15214} \times -7{,}86 \times -109{,}1 V/V = 294 V/V
\end{aligned}$$

6.

$$\begin{aligned}
R_i &= 10k\Omega + R_{in} = 15214\Omega \\
R_o &\approx 2{,}4k\Omega
\end{aligned}$$

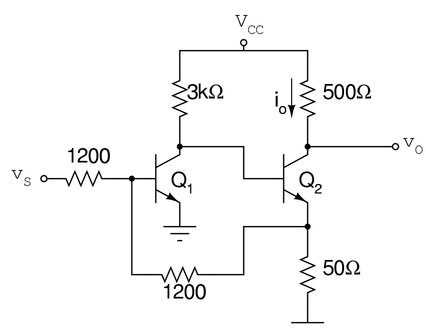
7.

$$
\begin{aligned}
A_{vf} &= \frac{A_v}{1+\beta A_v} \\
&= \frac{294}{1+\frac{1}{48}294} = \frac{294}{7{,}12} = 41{,}3V/V \\
R_{if} &= R_i(1+\beta A_v) = 7{,}12 \times 15214\Omega = 108{,}3k\Omega \\
R_{of} &= \frac{R_o}{1+\beta A_v} = \frac{2{,}4k\Omega}{7{,}12} = 337\Omega
\end{aligned}
$$

8. The voltage gain has been found. The current gain can be found from the voltage gain by observing that $i_o = -v_o/4{,}7k\Omega$, so that

$$
\begin{aligned}
A_{if} &= \frac{i_o}{i_s} \\
&= -\frac{v_o/4{,}7k\Omega}{v_s/10k\Omega} \\
&= -\frac{10}{4{,}7}A_{vf} \\
&= -87{,}9A/A
\end{aligned}
$$

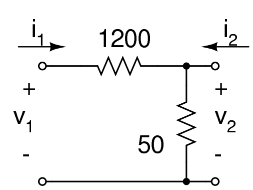
Amplifier 2



ANSWER:

1. The feedback type is current-sampling, current mixing.

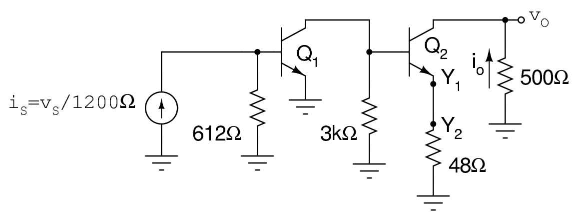
2. The feedback network is:



3.

$$\begin{aligned} R_{11} &= \frac{v_1}{i_1}\,|_{i_2=0} = 1250\Omega \\ R_{22} &= \frac{v_2}{i_2}\,|_{v_1=0} = 1200\Omega\ ||\ 50\Omega = 48\Omega \\ \beta &= \frac{i_1}{i_2}\,|_{v_1=0} = -\frac{50}{1250} = -\frac{1}{25}A/A \end{aligned}$$

4. The small-signal equivalent non-feedback amplifier is:



5. For this amplifier the non-feedback gain is $A_i = \frac{i_o}{i_s}$. Observe that $i_o = i_{c2} = h_{fe}i_{b2}$, Using $R_{in2} = r_{\pi 2} + (h_{fe}+1)R_{e2} = 1{,}1k\Omega + 51 \times 48\Omega = 3548\Omega$, a current divider on the base of Q_2 yields

$$\frac{i_{b2}}{i_{c1}} = -\frac{3000}{6548} = -0{,}46$$

where the negative sign accounts for the fact that current flows into the collector. Similarly, for Q_1,

$$i_{c1} = 50 i_{b1}$$

Another current divider at the input gives

$$\frac{i_{b1}}{i_s} = \frac{612}{1712} = 0{,}36$$

Thus

$$\begin{aligned} A_i &= \frac{i_o}{i_s} \\ &= 0{,}36 \times 50 \times (-0{,}46) \times 50 \\ &= -414A/A \end{aligned}$$

6. The input resistance is

$$R_i = 1{,}1k\Omega\ ||\ 612\Omega = 393{,}2\Omega$$

The output resistance that should be used in the formulae is the resistance seen by an ideal load (a short) at the point where the output current is being measured, i.e. by the piece of wire between Y_1 and Y_2.Thus

$$R_o \approx 48\Omega + \frac{1{,}1k\Omega + 3k\Omega}{51} = 128{,}4\Omega$$

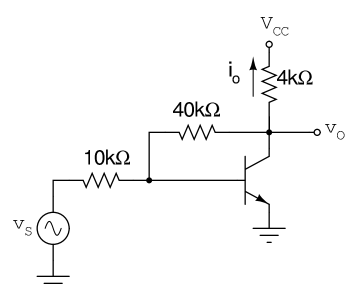
7. Now we can apply the feedback formulae:

$$
\begin{aligned}
A_{if} &= \frac{A_i}{1+\beta A_i} \\
&= \frac{-414A/A}{1+\frac{1}{25}414} \\
&= \frac{-414A/A}{17{,}56} = -23{,}6A/A \\
R_{if} &= \frac{R_i}{1+\beta A_i} \\
&= \frac{393{,}2\Omega}{17{,}56} = 22{,}4\Omega \\
R_{of} &= R_o(1+\beta A_i) = 17{,}56 \times 128{,}4\Omega \\
&= 2255\Omega
\end{aligned}
$$

8. The current gain has been found. To find the voltage gain, use $v_o = -500\Omega \times i_o$ and $v_s = 1200\Omega \times i_s$, so that

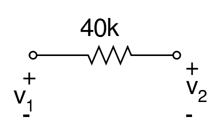
$$A_{vf} = -\frac{500}{1200}A_{if} = 9{,}8V/V$$

Amplifier 3



ANSWER:

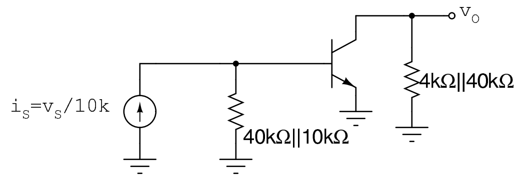
1. The feedback type is voltage-sampling, current-mixing.
2. The feedback network is just the $40k\Omega$ resistor,



3. The feedback parameters are:

$$
\begin{aligned}
R_{11} &= \frac{v_1}{i_1}\,|_{v_2=0} = 40k\Omega \\
R_{22} &= \frac{v_2}{i_2}\,|_{v_1=0} = 40k\Omega \\
\beta &= \frac{i_1}{v_2}\,|_{v_1=0} = -\frac{1}{40k\Omega}
\end{aligned}
$$

4. The small-signal equivalent circuit for the non-feedback amplifier is



5. For this amplifier the non-feedback gain is $R_M = \frac{v_o}{i_s}$. Observing that

$$v_o = -3636 \times i_c = -3636 \times 50 \times i_b$$

and that

$$i_b = \frac{8}{9{,}1} i_s$$

yields

$$R_M = -\frac{8}{9{,}1} \times 3636 \times 50 \approx -160k\Omega$$

6. The input resistance is

$$R_i = 1{,}1k\Omega \mid\mid 8k\Omega = 967\Omega$$

The output resistance is just about 3636Ω.

7. Now we can apply the feedback formulae:

$$\begin{aligned}
R_{Mf} &= \frac{R_M}{1+\beta R_M} \\
&= \frac{-160k\Omega}{1+\frac{1}{40k\Omega}160k\Omega} \\
&= \frac{-160k\Omega}{5} = -32k\Omega \\
R_{if} &= \frac{R_i}{1+\beta R_M} \\
&= \frac{967\Omega}{5} = 193{,}4\Omega \\
R_{of} &= \frac{R_o}{1+\beta A_i} \\
&= 727\Omega
\end{aligned}$$

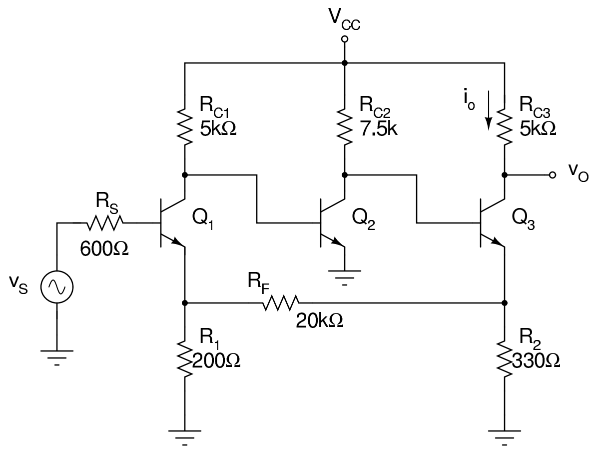
8. To find the voltage and current gains, use $v_o = 4k\Omega \times i_o$ and $v_s = 10k\Omega \times i_s$, so that

$$A_{vf} = \frac{R_{Mf}}{10k\Omega} = -3{,}2V/V$$

and

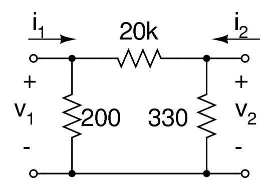
$$A_{if} = \frac{R_{Mf}}{4k\Omega} = -8A/A$$

Amplifier 4



ANSWER:

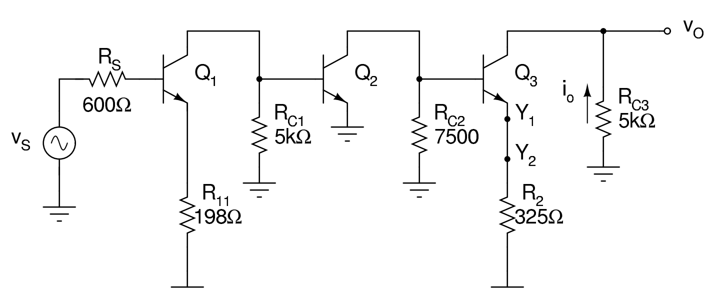
1. The feedback type is current-sampling, voltage-mixing.
2. The feedback network is



3. The feedback parameters are:

$$
\begin{aligned}
R_{11} &= \frac{v_1}{i_1} \mid_{i_2=0} = 200\Omega \mid\mid 20k\Omega \approx 198\Omega \\
R_{22} &= \frac{v_2}{i_2} \mid_{i_1=0} = 330\Omega \mid\mid 20k\Omega = \approx 325\Omega \\
\beta &= \frac{v_1}{i_2} \mid_{i_1=0} = -\frac{200\Omega \times 330\Omega}{20{,}53k\Omega} = 3{,}21V/A
\end{aligned}
$$

4. The small-signal equivalent circuit for the non-feedback amplifier is



5. For this amplifier the non-feedback gain is $G_M = \frac{i_o}{v_s}$. Observing that

$$i_o = i_{c3} = h_{fe} i_{b3} = 50 i_{b3}$$

To find the current gain i_o/i_{b1}, we can apply the current divider rule at the bases of Q_2 and Q_3.

$$\begin{aligned}
R_{in3} &= 1{,}1k\Omega + 51 \times 325\Omega = 17675\Omega \\
\frac{i_{b3}}{i_{c2}} &= -\frac{7500}{25175} = -0{,}3 \\
R_{in2} &= 1{,}1k\Omega \\
\frac{i_{b2}}{i_{c1}} &= -\frac{5000}{6100} = -0{,}82 \\
\frac{i_o}{i_{b1}} &= 50 \times (-0{,}82) \times 50 \times -0{,}3 \times 50 = 30750A/A
\end{aligned}$$

Now use

$$i_{b1} = \frac{v_s}{600\Omega + 1100\Omega + 51 \times 198} = \frac{v_s}{11{,}8k\Omega}$$

gives

$$G_M = \frac{i_o}{v_s} = 2{,}6A/V$$

6. The input resistance is

$$R_i = 600\Omega + 1100\Omega + 51 \times 198\Omega = 11{,}8k\Omega$$

The output resistance is that seen by the short between Y_1 and Y_2,

$$R_o = 325\Omega + \frac{1100\Omega + 7500\Omega}{51} = 494\Omega$$

7. Now we can apply the feedback formulae:

$$\begin{aligned}
G_{Mf} &= \frac{G_M}{1 + \beta G_M} \\
&= \frac{2{,}6A/V}{1 + 3{,}2V/A \times 2{,}6A/V} \\
&= \frac{2{,}6A/V}{9{,}3} = 0{,}28A/V \\
R_{if} &= \frac{R_i}{1 + \beta G_M} \\
&= 9{,}3 \times 11{,}8k\Omega = 109{,}7k\Omega \\
R_{of} &= 9{,}3 \times 494\Omega = 4{,}6k\Omega
\end{aligned}$$

8. To find the voltage and current gains, use $v_o = -5000\Omega \times i_o$ and $v_s = 600\Omega \times i_s$, so that

$$A_{vf} = -5000\Omega \times G_{Mf} = -1400V/V$$

and

$$A_{if} = 600\Omega \times G_{Mf} = 168A/A$$

5.3. Amplifier Stability

Basic feedback equation:

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)}$$

Thus, feedback moves the poles of the amplifier's transfer function.

Poles of A_f are roots of

$$1 + \beta A(s)$$

5.3.1. Nyquist Theorem

Stated simply, it says that with $\omega_{180°}$ defined by $\angle \beta A(\omega_{180°}) = 180°$, if

$$| \beta A(\omega_{180°}) | > 1$$

the amplifier is unstable. Otherwise, it is stable.

Nyquist theorem allows us to answer questions about the stability of A_f by analysing βA.

Gain Margin

Defined as the number of decibels below zero of the loop gain's magnitude at $\omega_{180°}$.

Loop gain: $L(\omega) \equiv \beta A(\omega)$

Phase Margin ϕ_m

Number of degrees above $-180°$ at ω_1, where ω_1 is defined as the frequency at which the amplifier's gain magnitude is 1, or 0 db.

Stability

The amplifier is unstable if the gain/phase margins are negative. If the margins are zero, the amplifier is *marginally stable*.

We normally want $\phi_m \geq 45°$.

Easier Way

Observation: for us $\angle \beta = 0$.

Express $L(\omega)$ in decibels,

$$L_{db} = 20 \log L(\omega) = 20 \log \beta A = A_{db} - 20 \log \frac{1}{\beta}$$

Thus examine stability by examining the difference between the two plots.

5.4. Practice Problems

From chapter 8: problems 1, 3, 7, 9, 11, 15, 19, 21, 23, 31, 35, 37, 39, 41, 43, 48, 50, 51, 53, 55, 61, 63, 65, 67 and 69.