

Derivation of textbook's equation 2.46

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January 18, 2007

Applying the voltage divider rule on the bridge's two legs yields

$$\frac{v_1 - v_2}{V_{ref}} = \frac{R + \delta R}{R + R_1 + \delta R} - \frac{R}{R + R_1}$$

Dividing by numerator and denominator R ,

$$\frac{v_1 - v_2}{V_{ref}} = \frac{1 + \delta}{1 + \frac{R_1}{R} + \delta} - \frac{1}{1 + \frac{R_1}{R}}$$

which can be expressed as

$$\frac{v_1 - v_2}{V_{ref}} = \frac{1 + \delta}{\left(\frac{R_1}{R} + 1\right) \left(1 + \frac{\delta}{\frac{R_1}{R} + 1}\right)} - \frac{1}{1 + \frac{R_1}{R}}$$

Multiplying the second term to get a common denominator,

$$\begin{aligned} \frac{v_1 - v_2}{V_{ref}} &= \frac{1 + \delta}{\left(\frac{R_1}{R} + 1\right) \left(1 + \frac{\delta}{\frac{R_1}{R} + 1}\right)} - \frac{1 + \frac{\delta}{\frac{R_1}{R} + 1}}{\left(\frac{R_1}{R} + 1\right) \left(1 + \frac{\delta}{\frac{R_1}{R} + 1}\right)} \\ &= \frac{1 + \delta - 1 - \frac{\delta}{\frac{R_1}{R} + 1}}{\left(\frac{R_1}{R} + 1\right) \left(1 + \frac{\delta}{\frac{R_1}{R} + 1}\right)} \\ &= \delta \frac{1 - \frac{1}{\frac{R_1}{R} + 1}}{\left(\frac{R_1}{R} + 1\right) \left(1 + \frac{\delta}{\frac{R_1}{R} + 1}\right)} \\ &= \delta \frac{\frac{R_1}{R} + 1 - 1}{\left(\frac{R_1}{R} + 1\right) \left(1 + \frac{\delta}{\frac{R_1}{R} + 1}\right) \left(\frac{R_1}{R} + 1\right)} \\ &= \delta \frac{\frac{R_1}{R}}{\left(\frac{R_1}{R} + 1\right) \left(\frac{R_1}{R} + 1 + \delta\right)} \end{aligned}$$

which is equivalent to

$$\frac{v_1 - v_2}{V_{ref}} = \delta \frac{1}{\left(\frac{R}{R_1}\right) \left(\frac{R_1}{R} + 1\right) \left(\frac{R_1}{R} + 1 + \delta\right)}$$

The two left bottom terms can be combined to get

$$\frac{v_1 - v_2}{V_{ref}} = \delta \frac{1}{\left(1 + \frac{R}{R_1}\right) \left(\frac{R_1}{R} + 1 + \delta\right)}$$

which can in turn be expressed as

$$\frac{v_1 - v_2}{V_{ref}} = \delta \frac{1}{\left(1 + \frac{R}{R_1}\right) \left(\frac{R_1}{R} + 1\right) + \delta \left(1 + \frac{R}{R_1}\right)}$$

To get the textbook's result, multiply the bottom left terms and simplify

$$\begin{aligned} \frac{v_1 - v_2}{V_{ref}} &= \delta \frac{1}{1 + \frac{R}{R_1} + \frac{R_1}{R} + 1 + \delta \left(1 + \frac{R}{R_1}\right)} \\ &= \delta \frac{1}{2 + \frac{R}{R_1} + \frac{R_1}{R} + \delta \left(1 + \frac{R}{R_1}\right)} \end{aligned}$$