NOISE

- Interference noise - caused by unwanted interaction between the circuit and the outside
- Inherent noise - Internal to the devices. Due to random phenomena in devices.
- Signal-to-noise ratio (SNR)

$$SNR = 20log\left(\frac{X_s}{X_n}\right)$$

  where X_s is the rms value of the signal, X_n is the rms value of noise.

1. Noise Statistics

- rms value of noise

$$X_n = \frac{1}{T}\sqrt{\int_0^T x_n^2(t)dt}$$

  where T is the averaging interval and x_n is the instantaneous noise signal - i.e. a voltage or a current.
- mean square value - average power dissipated ina 1Ω resistor. Equal to X_n^2.
- crest factor: ratio of peak value to rms value. Since noise is random, peak value can't be predicted. The probability of the peak value exceeding a particular value can however be estimated. For a gaussian deistribution:

CF	Probability
>1	32%
>2	4.6%
>3	0.27%
>3.3	0.1%
>4	0.0063%

2. Noise Summation

$$X_n^2 = \frac{1}{T}\int_0^T (x_{n1}(t) + x_{n2}(t))^2\,dt = X_{n1}^2 + X_{n2}^2 + \frac{2}{T}\int_0^T x_{n1}(t)x_{n2}(t)dt$$

The integral is zero for uncorrelated signals. Thus in this case

$$X_n = \sqrt{X_{n1}^2 + X_{n2}^2}$$

3. Noise Spectra

- Noise power X_n^2 is spread over a band of frecuencies.
- Noise power depends on the frequency band width and location.
- Voltage noise power density: $e_n^2(f) = \frac{dE_n^2}{df}$, where E_n^2 is the mean square value of noise.
- Current noise power density: $i_n^2(f) = \frac{dI_n^2}{df}$, where I_n^2 is the mean square value of noise.
- Noise power densities: noise power over a 1Hz frequency band. Plot versus frequency to get a visual description of noise power distribution over a frequency band.
- In the frequency domain,

$$X_n = \sqrt{\int_{f_L}^{f_H} x_n^2(f)df}$$

where x_n represents voltage or current.

3.1. White Noise.

- Uniform spectral density. For white noise

$$X_n = x_{nw}\sqrt{f_H - f_L}$$

and

$$X_n^2 = x_{nw}^2(f_H - f_L)$$

- Noise power is proportional to bandwidth.

3.2. $1/f$ Noise.

- Power density is inversily proportional to frequency.

$$x_n^2 = \frac{K^2}{f}$$

where K is a constant.
- $x_n = K/\sqrt{f} \rightarrow$ bode plot slope will be -10dB/dec.
- Noise rms is

$$X_n = K\sqrt{ln(f_H/f_L)}$$

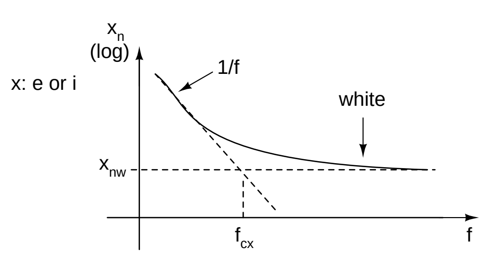
and noise power is

$$X_n^2 = K^2 ln(f_H/f_L)$$

This means that $1/f$ noise will have the same power content in each decade.

3.3. IC Noise.

- Mixture of white and $1/f$ noise.



- x is either voltage or current. $e_n^2 = e_{nw}^2\left(\frac{f_{ce}}{f}+1\right)$; $i_n^2 = i_{nw}^2\left(\frac{f_{ci}}{f}+1\right)$
- For voltage

$$E_n = e_{nw}\sqrt{f_{ce} ln\left(\frac{f_H}{f_L}\right) + f_H - f_L}$$

- Similar expression for current.
- To minimize noise limit bandwidth as necessary.

4. Noise Dynamics

- Noise analysis: find total rms noise at the output of a circuit given the noise density at its input and its frequency response.
- For a voltage amplifier, output noise = gain times input noise

$$e_{no}(f) = |A_n(jf)|e_{ni}(f)$$

- E_{no}^2 =total output rms noise = $\int_0^\infty e_{no}^2(f)df$, or

$$E_{no} = \sqrt{\int_0^\infty |A_n(jf)|^2 e_{ni}^2(f)df}$$

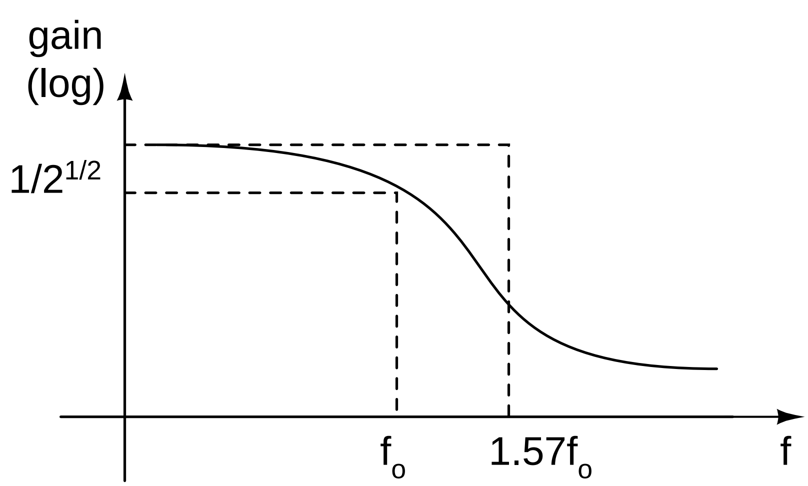
- Similarly, for a transimpedance amplifier

$$E_{no} = \sqrt{\int_0^\infty |Z_n(jf)|^2 e_{ni}^2(f)df}$$

- Noise equivalent bandwidth (NEB): For a gain with a dominant pole,

$$E_{no} = e_{nw}\sqrt{\int_0^\infty \frac{df}{1+(f/f_o)^2}} = e_{nw}\sqrt{\pi f_o/2} = e_{nw}\sqrt{1.57 f_o}$$

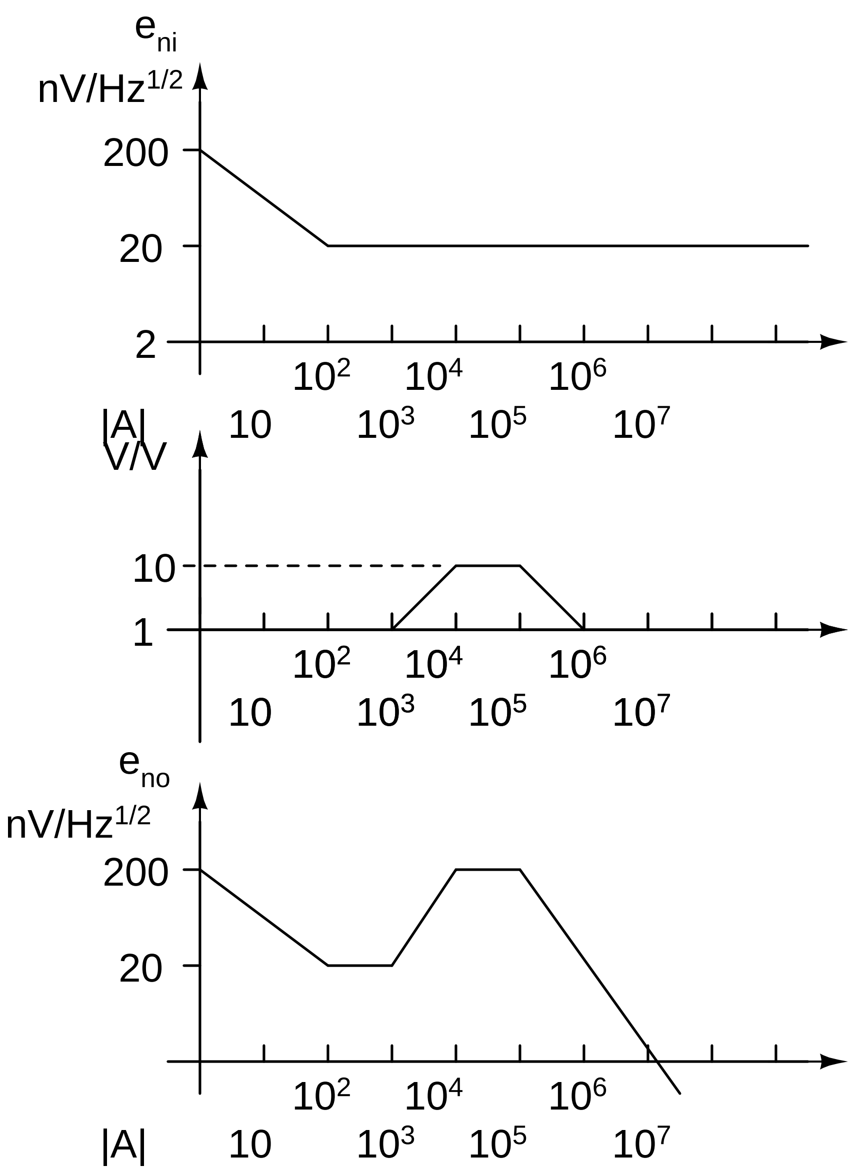
Works as a brick-wall filter with bandwidth $1.57f_o$.



- More generally,

$$NEB = \frac{1}{A_{n_{max}}^2} \int_0^\infty |A_n(jf)|^2 df$$

4.1. **Example 7.3.**



From 1 Hz to 1kHz:

$$E_n = e_{nw}\sqrt{f_{ce} ln(f_H/f_L) + f_H - f_L}$$

with $e_{nw} = 20nV/\sqrt{Hz}$, $f_{ce} = 100Hz$, $f_L = 1Hz$ and $f_H = 1kHz$. The result is $E_{no1} = 0.822\mu V$.

From 1kHz to 10kHz, e_{no} increases with f at a rate of 1dec/dec. So let

$$e_{no}(f) = \left(20nV/\sqrt{Hz}\right) \times (f/10^3) = 2 \times 10^{-11} f$$

and

$$E_{no2} = 2 \times 10^{-11} \sqrt{\int_{10^3}^{10^4} f^2 df} = 11.5\mu V$$

For $f > 10^4 Hz$, we have white noise with $e_{nw} = 200nV/\sqrt{Hz}$ going throught a low-pass filter with corner frequency $f_o = 100kHz$. Using

$$E_{no3} = e_{nw}\sqrt{1.57 f_o}$$

$$E_{no3} = (200nV/\sqrt{Hz}) \times \sqrt{1.57 \times 10^5 - 10^4} = 76.7\mu V$$

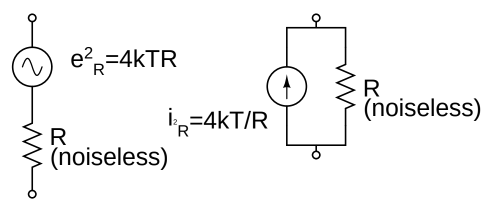
The total is the root of the sum of the square of the three components

$$E_{no} = \sqrt{E_{no1}^2 + E_{no2}^2 + E_{no3}^2} = 77.5\mu V$$

5. Sources of Noise

5.1. Johnson (Thermal) Noise.

- Due to vibrations in solid
- White noise
- Present in all pasive devices - resistors, inductors, capacitors.
- Modelled as a voltage source and Thevenin resistance:



5.2. Shot Noise.

- Coused by random charge crossing across potential barriers (like pn-junctions)
- Uniform power density: $i_n^2 = 2qI$, where I is the current thorugh the junction.

5.3. Flicker Noise.

- Due to traps caused by contamination and crystal dislocations on base-emitter junction.
- Also called *contact noise*
- $1/f$ noise:

$$i_n^2 = K\frac{I^a}{f}$$

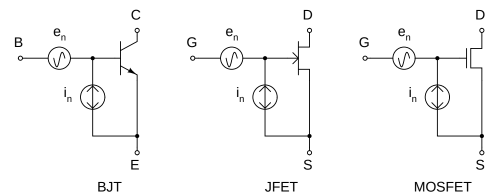
  where K is a device constant, I is the devices' dc current and a is a device constant between $\frac{1}{2}$ and 2.
- Present in active devices.
- Also present in resistors, where it is called *excess noise*.
  - Wire-wound resistors are the quitest.
  - Carbon compisition resistors can be an order of magnitude noisier.

- Carbon- and metal-film are intermediate.
- Important only if resistor's d.c. current is large.

5.4. Avalanche Noise.

- Present in Zaner diodes, where it is very intense and make them very noisy.

6. Transistor Noise



6.1. BJT.

- Exhibit all the above but Avalanche.
- e_n is thermal from r_b at the base + collector current shot noise reflected back to the base: $e_n^2 = 4kT\left(r_b + \frac{1}{2g_m}\right)$
- i_n is base current shot + flicker at base + collector current shot noise reflected to the base: $i_n^2 = 2q\left(I_B + K_1\frac{I_B^a}{f} + \frac{I_C}{|\beta(jf)|^2}\right)$
- r_b is the intrinsic base resistance, I_B and I_C are dc base and collector currents, g_m is the device transconductance, K and a are device constants and β is the current gain as a function of frequency.

6.2. JFET.

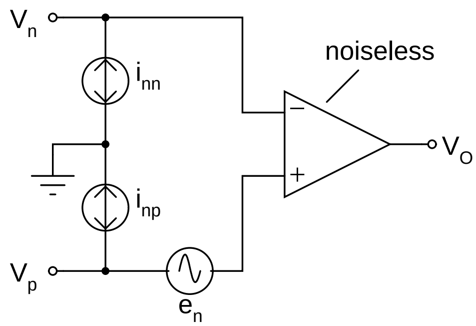
- e_n: channel thermal noise plus drain-current flicker noise
$$e_n^2 = 4kT\left(\frac{2}{3g_m} + K_2\frac{I_D^a/g_m^2}{f}\right)$$
- i_n is negligible at room temperature but increases and might be significant at high temperatures.
$$i_n^2 = 2qI_G + \left(\frac{2\pi f C_{gs}}{g_m}\right)^2\left(4kT\frac{2}{3}g_m + K_3\frac{I_D^a}{f}\right)$$
- g_m is the transconductance; I_D is the dc drain current; I_G is the gate leakage current; K_2, K_3 and a are device constants and C_{gs} is the gate-to-source capacitance.
- Due to low g_m FET's e_n is ussually higher then BJT's, but i_n is smaller.

6.3. MOSFET.

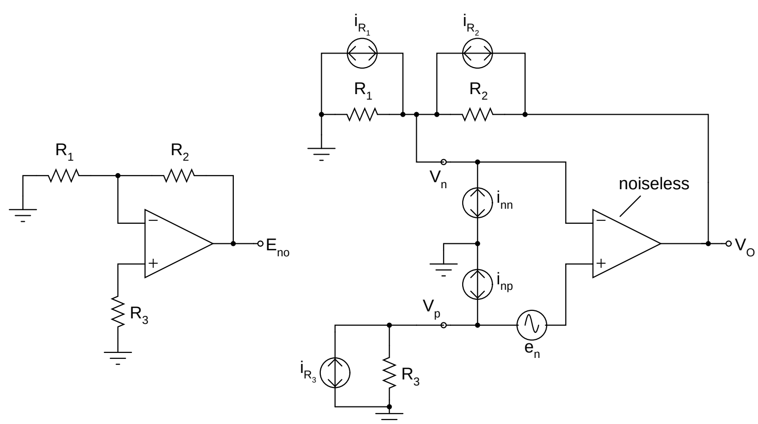
- e_n: thermal noise from channels resistance and drain-current Flicker noise
$$e_n^2 = 4kT\frac{2}{3g_m} + K_4\frac{1}{WLf}$$
- Flicker noise is inversely proportional to transistor area $W \times L$ so it is reduced by using large geometries.
- $i_n^2 = 2qI_G$ is negligible at room temperature; increases with temperature.
- g_m is transconductance, $K_4$is a device constant, W and L are channel width and length.

7. Op Amp Noise

- Modelled like offsets: with two current sources and one voltage source.



- i_{nn} and i_{np} are equal in voltage-mode op amps, different in current-feedback amps (CFAs).
- In low-noise applications it is important to estimate the level of output RMS noise, E_{no}.
- Example:



- Superposition:

$$e_{ni}^2 = e_n^2 + i_{np}^2 R_3^2 + i_{R_3}^2 R_3^2 + (i_{nn}^2 + i_{R_1}^2 + i_{R_2}^2)(R_1||R_2)^2$$

using $i_R^2 = \frac{4kT}{R}$,

$$e_{ni}^2 = e_n^2 + i_{np}^2 R_3^2 + 4kTR_3 + (i_{nn}^2 + \frac{4kT}{R_1} + \frac{4kT}{R_2})(R_1||R_2)^2$$

$$e_{ni}^2 = e_n^2 + i_{np}^2 R_3^2 + 4kTR_3 + i_{nn}^2 (R_1||R_2)^2 + \frac{4kT(R_1+R_2)}{R_1R_2}(\frac{R_1R_2}{R_1+R_2})^2 = e_n^2 + i_{np}^2 R_3^2 + i_{nn}^2 (R_1||R_2)^2 + 4kT\,(R_3 + R_1||$$

- For $i_{np} = i_{nn} = i_n$,

$$e_{ni}^2 = e_n^2 + i_n^2 R_{s2}^2 + 4kTR_s$$

where $R_s = R_3 + R_1||R_2$, $R_{s2}^2 = R_3^2 + (R_1||R_2)^2$.

- Setting $R_3 = 0$ reduces noise.
- e_n dominates for low values of R_s: it is called the *short-circuit noise.*
- For $R_s \to \infty$, $e_{ni} \approx i_n^2 R_{s2}^2$: i_n is called the *open-circuit noise.*
- The output noise is amplified by:

$$A_v = \frac{1 + \frac{R_2}{R_1}}{\sqrt{1 + (f/f_A)^2}} = \frac{A_0}{\sqrt{1 + (f/f_A)^2}}$$

- The total output rms noise is

$$E_{no} = A_0 \times \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2}$$

where

$$E_1^2 = e_{nw}^2 \left(f_{ce} ln \frac{f_A}{f_L} + 1.57 f_A - f_L \right)$$

$$E_2^2 = R_3^2 i_{npw}^2 \left(f_{cip} ln \frac{f_A}{f_L} + 1.57 f_A - f_L \right)$$

$$E_3^2 = (R_1 || R_2)^2 i_{nnw}^2 \left(f_{cin} ln \frac{f_A}{f_L} + 1.57 f_A - f_L \right)$$

and

$$E_4^2 = 4kT(R_3 + R_1||R_2)(1.57 f_A - f_L)$$

$f_L$is $1/T_{obs}$ where T_{obs} is the averaging time used to measure the output.
- For low-noise designs, use op amps with low e_{nw} and low corner frequencies f_{ci} and f_{cn}.
- The *total rms input noise* can be obtained by dividing by the signal dc gain A_{s0}

$$E_{ni} = \frac{E_{no}}{|A_{s0}|}$$

and the signal-to-noise ratio from

$$SNR = 20 log_{10} \frac{V_{i(rms)}}{E_{ni}}$$