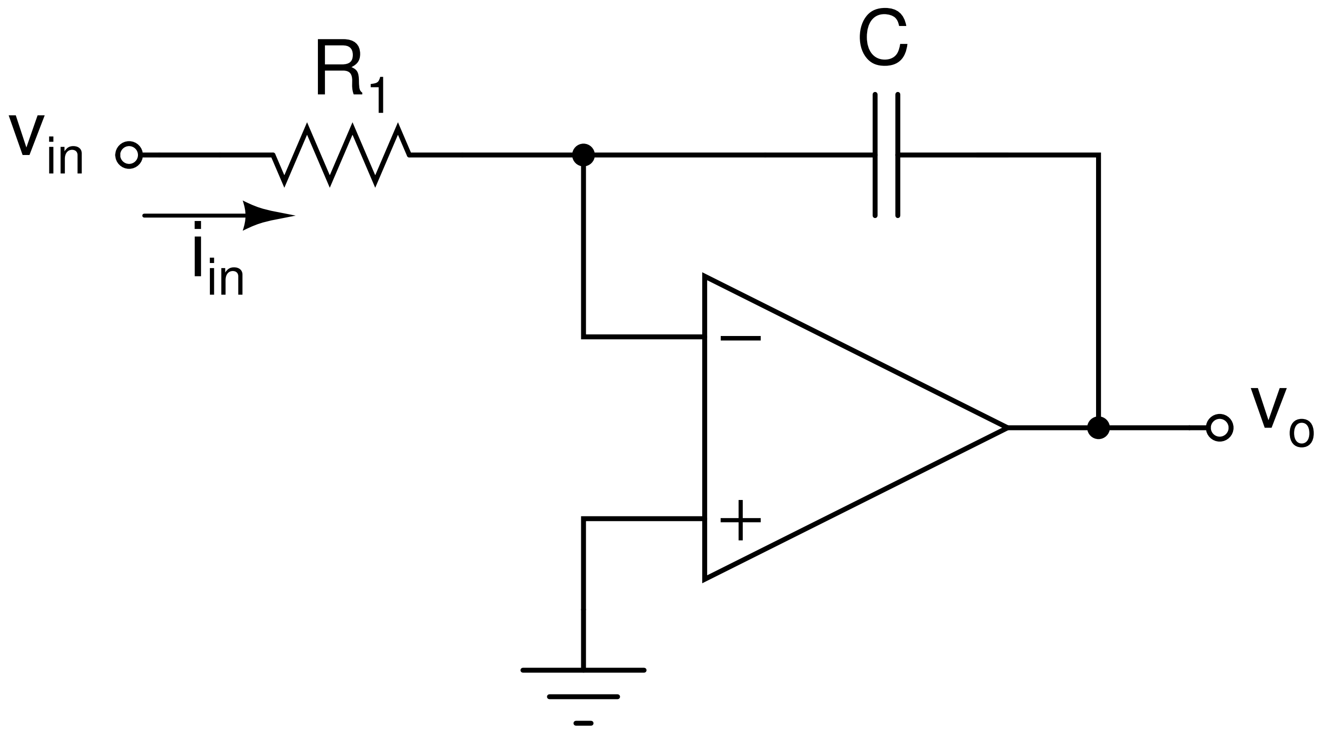
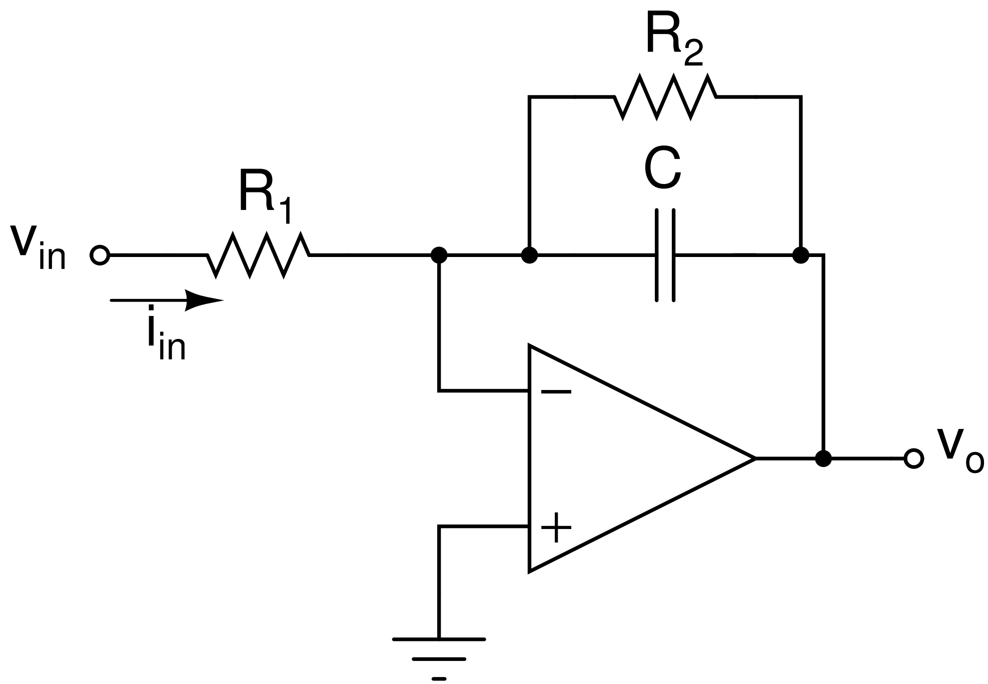
Filters

INEL 5207 - Spring 2011

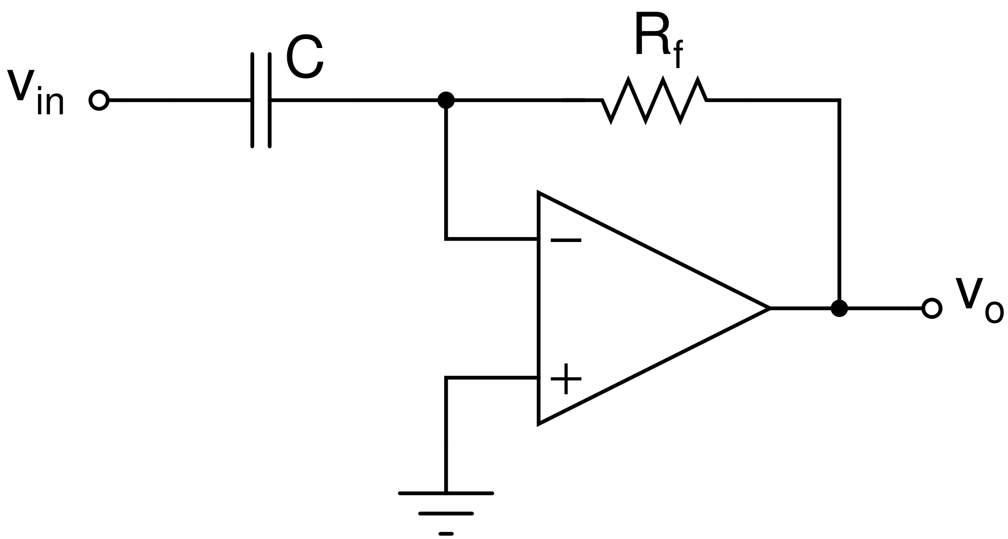
Integrator



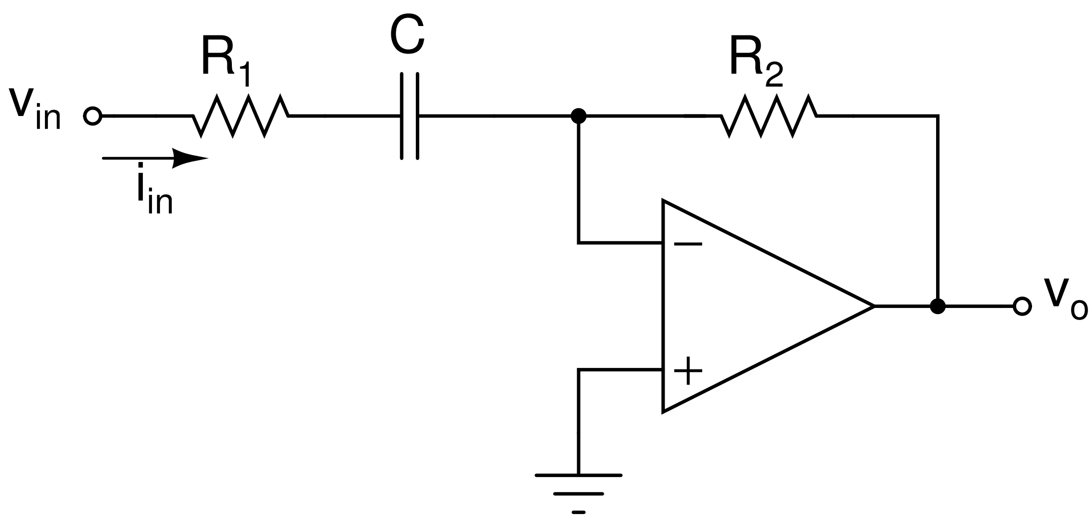
Low-pass filter

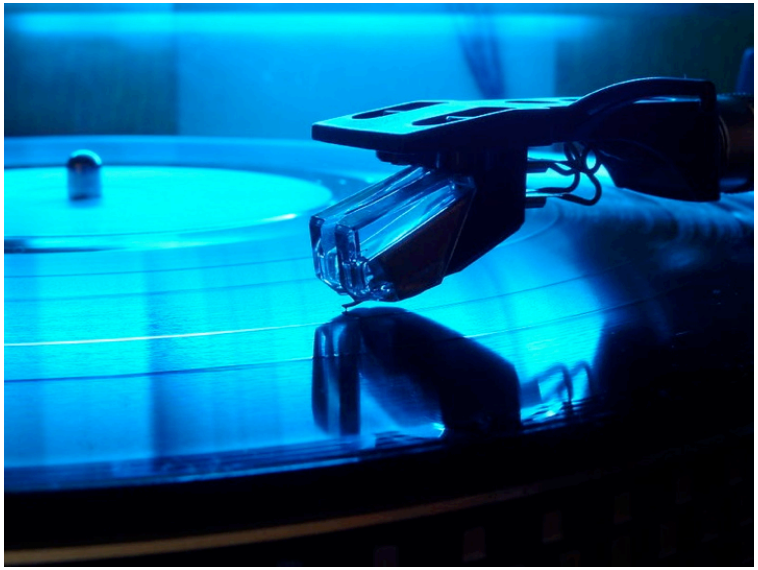


Differentiator



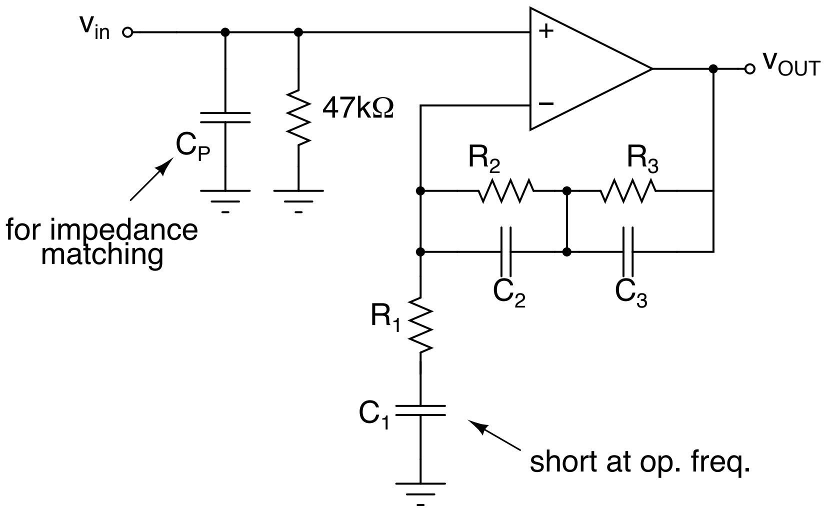
high-pass filter

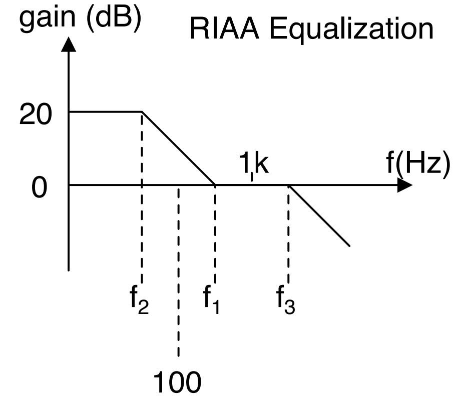




Phono pre-amp

$f_1 = 500\text{Hz},\ f_2 = 50\text{Hz},\ f_3 = 2122\text{Hz}$





shift by 30-40dB for moving magnet cartridge, 50-60dB for moving-coil cartridge

$$H(jf) \simeq 1 + \frac{R_2 + R_3}{R_1} \frac{1 + jf/f_1}{(1 + jf/f_2)(1 + jf/f_3)}$$

$$f_1 = \frac{1}{2\pi(R_2 \| R_3)(C_2 + C_3)}$$

$$f_2 = \frac{1}{2\pi R_2 C_2} \qquad f_3 = \frac{1}{2\pi R_3 C_3}$$

2^{nd} order filters

$$H(s) = \frac{N(s)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

ω_0 = undamped natural frecuency
ζ = damping ratio

$\zeta > 1$	Overdamp: Real, neg. poles. Response shows decaying exponentials
$0 < \zeta < 1$	Underdamped: poles are comples conj. Response is a damped sinusoid
$\zeta = 0$	Undamped: Poles on imag. axis. Response shows sustained oscillations.
$\zeta < 0$	Unstable: rhp poles. Filters must have $\zeta < 0$.

Let $s \to j\omega$,

$$H(j\omega) = \frac{N(j\omega)}{1 - j(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

$$Q \equiv \frac{1}{2\zeta}$$

Low-pass

$$H_{LP}(j\omega) = \frac{1}{1 - j(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

- For $\omega << \omega_0$: $|H_{LP}| = 1 = 0dB$.
- For $\omega >> \omega_0$: $|H_{LP}| = \frac{1}{(\omega/\omega_0)^2)} = -40dB/dec$.
- For $\omega = \omega_0$: $|H_{LP}| = Q$

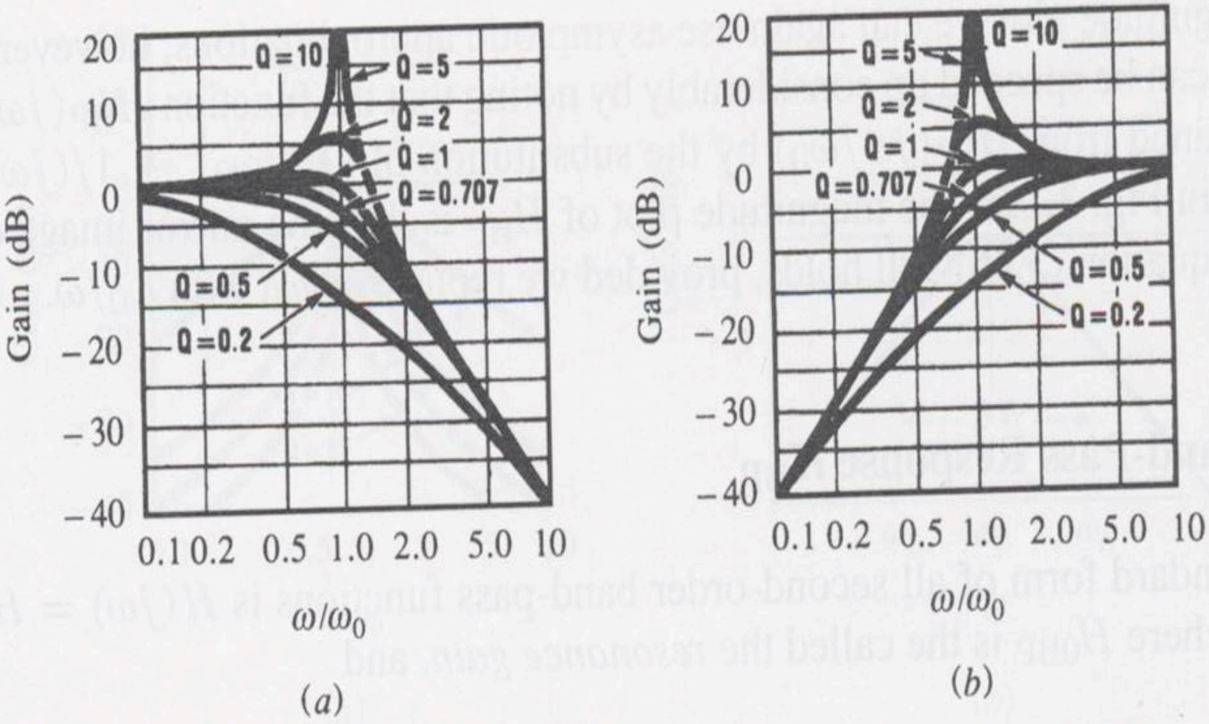
Largest Q before the unset of peaking: $\frac{1}{\sqrt{2}}$

For $Q > \frac{1}{\sqrt{2}}$

$$\frac{\omega_{peak}}{\omega_0} = \sqrt{1 - \frac{1}{2Q^2}}$$

$$|H_{LP}|_{max} = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}$$

2^{nd} order Response



Lowpass and highpass

High-pass

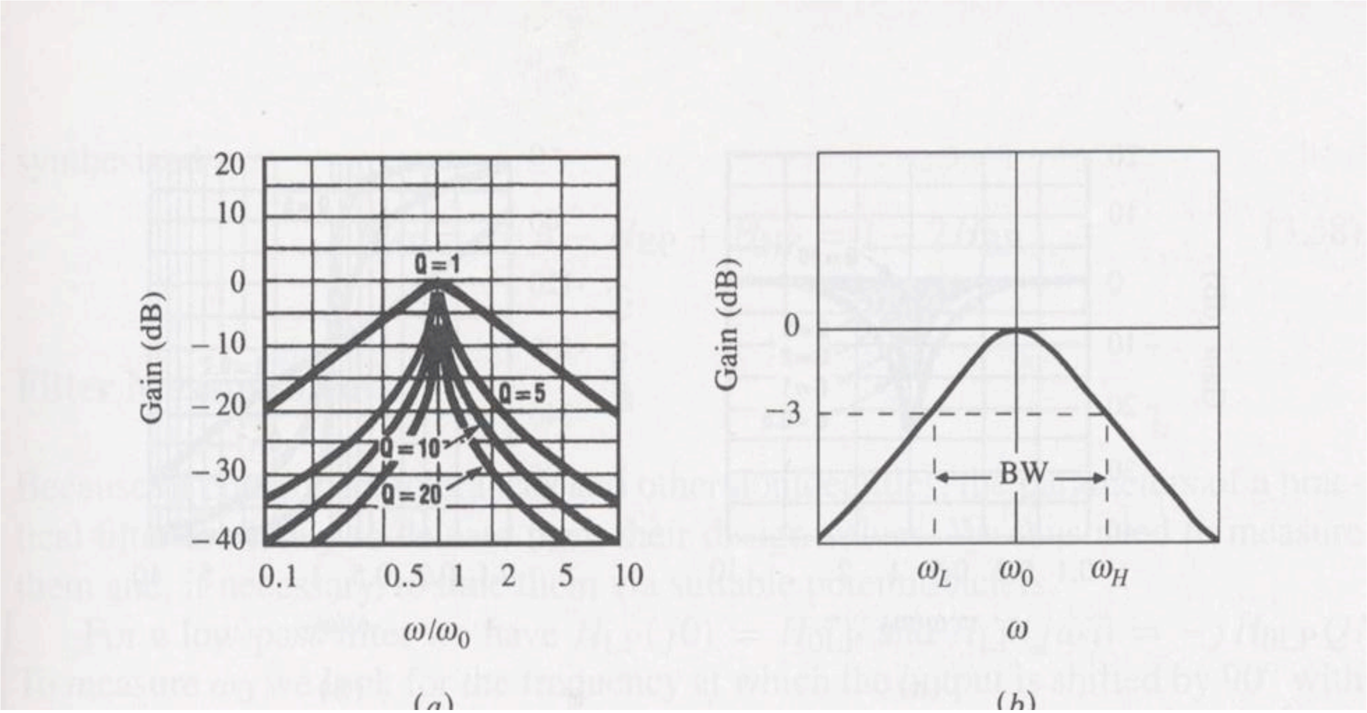
$$H_{HP}(j\omega) = \frac{-(\omega/\omega_0)^2}{1 - j(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

Band-pass

$$H_{BP}(j\omega) = \frac{j(\omega/\omega_0)/Q}{1 - j(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

- $j(\omega/\omega_0)/Q \rightarrow$ zero at origin.
- For $\omega << \omega_0$: $|H_{BP}| \simeq (\omega/\omega_0)/Q = (\frac{\omega}{\omega_0})_{dB} - Q_{dB}$.
- For $\omega >> \omega_0$: $|H_{BP}| \simeq -(\frac{\omega}{\omega_0})_{dB} - Q_{dB}$.
- For $\omega = \omega_0$: $|H_{LP}| = 0$

$$\begin{aligned} BW &= \omega_H - \omega_L \\ &= \omega_0 \left(\sqrt{1 - \frac{1}{4Q^2}} - \frac{1}{2Q} \right) - \omega_0 \left(\sqrt{1 - \frac{1}{4Q^2}} + \frac{1}{2Q} \right) \\ \omega_0 &= \sqrt{\omega_L \omega_H} \\ Q &= \omega_0 / BW \end{aligned}$$



20
10
0
−10
−20
−30
−40
Gain (dB)
Q = 1
Q = 5
Q = 10
Q = 20
0.1
0.5
1
2
5
10
ω/ω0
(a)
Gain (dB)
0
−3
BW
ωL
ω0
ωH
ω
(b)

Bandpass

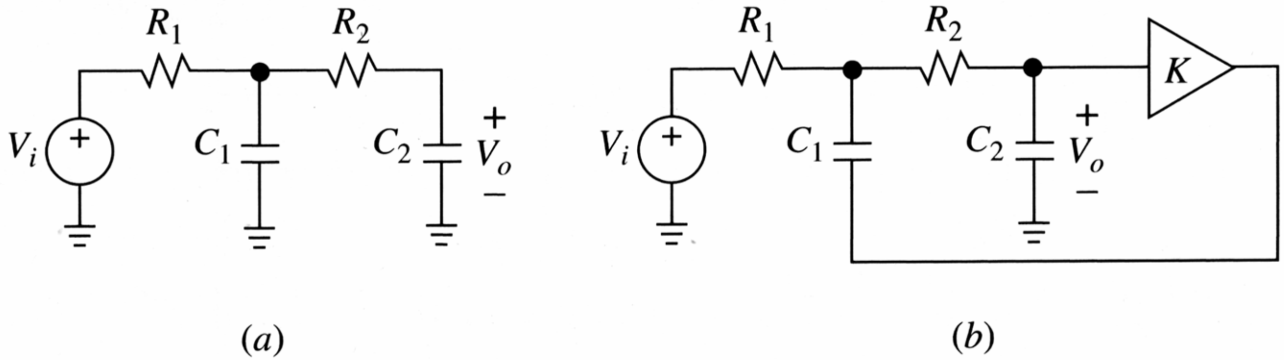
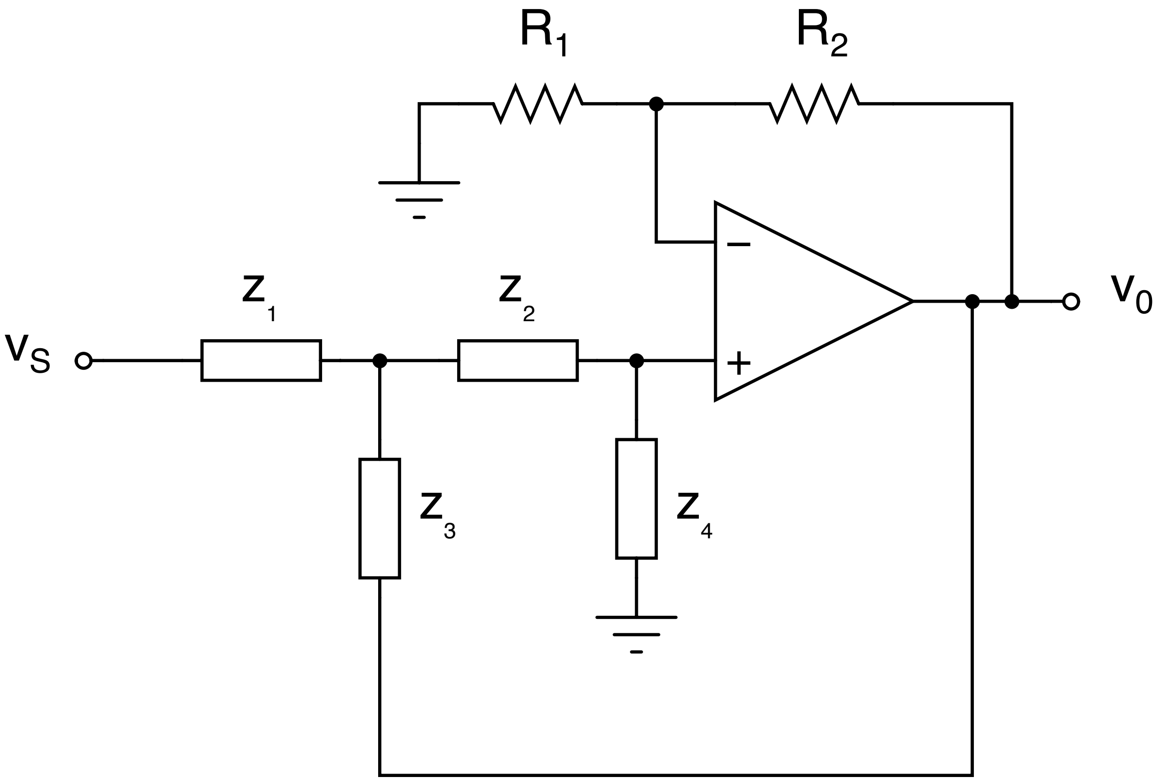


FIGURE 3.22
(a) Passive and (b) active realization of a second-order low-pass filter.

For (a) $Q \leq 0.5$

Sallen-Key (KRC) Filter

Textbook
$Z_1 = R_1$
$Z_2 = R_2$
$Z_3 = 1/sC_1$
$Z_4 = 1/sC_2$
$K = A_{v0}$



$$v_+ = \frac{z_4}{z_2 + z_4}\left(\frac{z_3(z_2+z_4)}{z_1(z_2+z_3+z_4)+z_3(z_2+z_4)}v_S + \frac{z_1(z_2+z_4)}{z_3(z_1+z_2+z_4)+z_1(z_2+z_4)}v_O\right)$$

Setting $v_+ = V_O/A_{v0}$ and rearranging gives

$$\frac{v_O}{A_{V0}} = \frac{z_3 z_4}{z_1(z_2+z_3+z_4)+z_3(z_2+z_4)} v_S + \frac{z_1 z_4}{z_3(z_1+z_2+z_4)+z_1(z_2+z_4)} v_O$$

or

$$\left(1 - \frac{z_1 z_4}{z_3(z_1+z_2+z_4)+z_1(z_2+z_4)}\right) v_O = \frac{z_3 z_4}{z_1(z_2+z_3+z_4)+z_3(z_2+z_4)} v_S$$

which, after rearranging yields

$$\frac{v_O}{v_S} = \frac{z_3 z_4}{z_3(z_1+z_2+z_4)+z_1 z_2+z_1 z_4(1-A_{v0})} A_{v0}$$

where $A_{v0} = 1 + R_2/R_1$.

Component choices are for the *equal component* design

To obtain a Sallen-Key filter, let $z_1 = z_2 = R$ and $z_3 = z_4 = \frac{1}{sC}$. Then

$$\frac{v_O}{v_S} = \frac{\frac{1}{s^2C^2}}{\frac{1}{sC}(2R + \frac{1}{sC}) + R^2 + \frac{R}{sC}(1 - A_{v0})} A_{v0}$$

which can be rearranged to obtain

$$\frac{v_O}{v_S} = \frac{A_{v0}}{s^2(RC)^2 + (3 - A_{v0})(RC)s + 1}$$

Letting RCs represent a scaled frequency $s' = j\omega'$,

$$\frac{v_O}{v_S} = \frac{A_{v0}}{s'^2 + (3 - A_{v0})s' + 1}$$

whose denominator is of the form

$$s^2 + 2\zeta s + 1$$

$$Q = 3 - A_{v0}$$

Example: Design an equal-component low-pass filter with $f_0 = 1kHz$ and $Q = 5$.
Example: Modify the design to get a dc gain of 0dB.

Unity-gain option

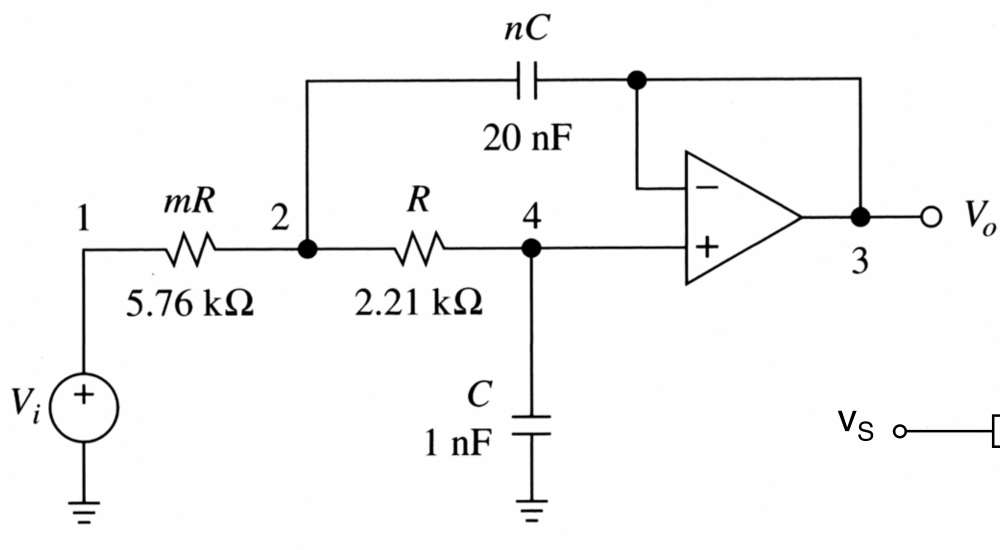
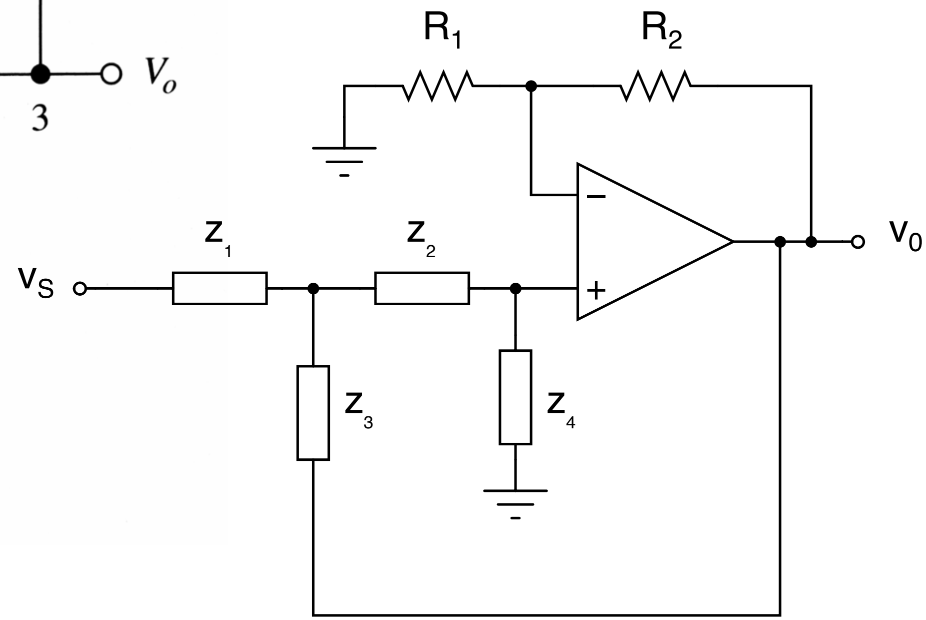


FIGURE 3.25
Filter of Example 3.10.



$$\frac{v_O}{v_S} = \frac{z_3 z_4}{z_3(z_1 + z_2 + z_4) + z_1 z_2 + z_1 z_4(1 - A_{v0})} A_{v0}$$

$$z_1 = mR \qquad z_2 = R \qquad z_3 = 1/nsC \qquad z_4 = 1/sC$$

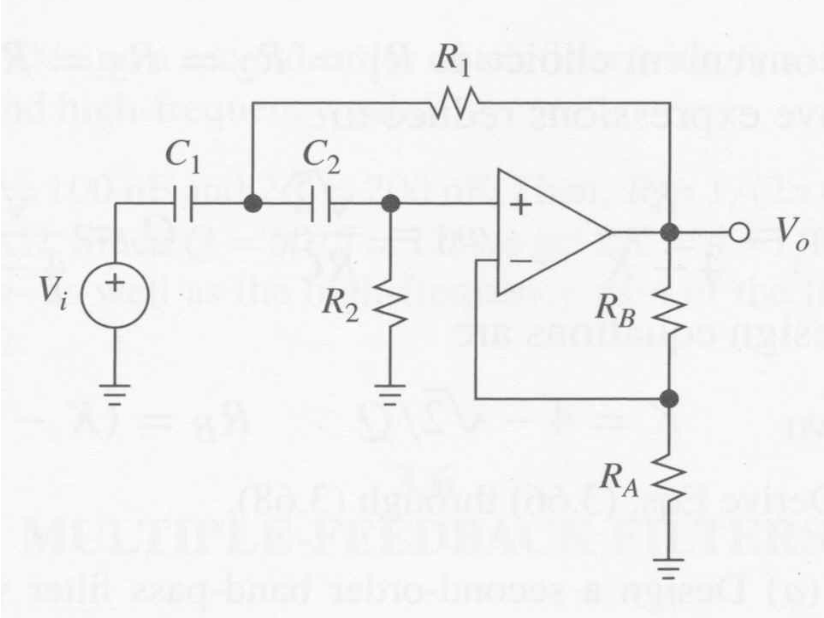
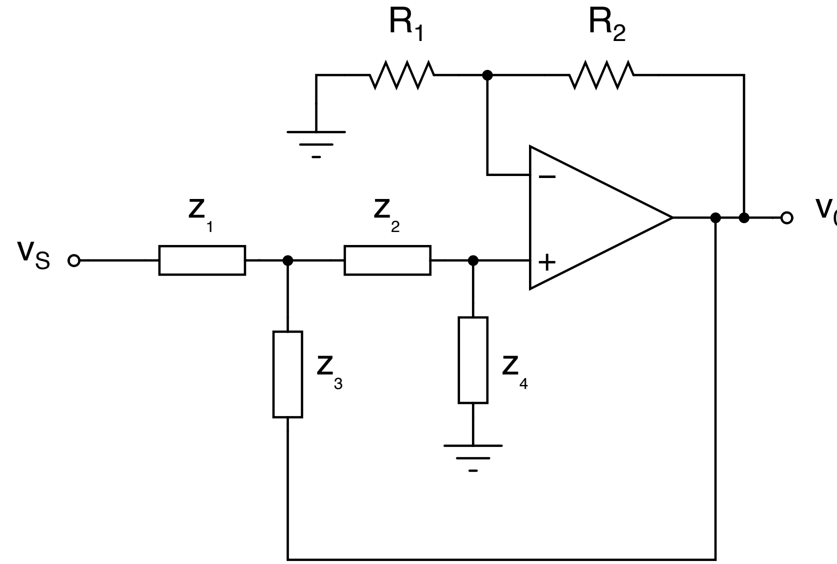


FIGURE 3.27
High-pass *KRC* filter.



$$\frac{v_O}{v_S} = \frac{z_3 z_4}{z_3(z_1 + z_2 + z_4) + z_1 z_2 + z_1 z_4 (1 - A_{v0})} A_{v0}$$

$$z_1 = 1/sC_1 \qquad z_2 = 1/sC_2 \qquad z_3 = R_1 \qquad z_4 = R_2$$

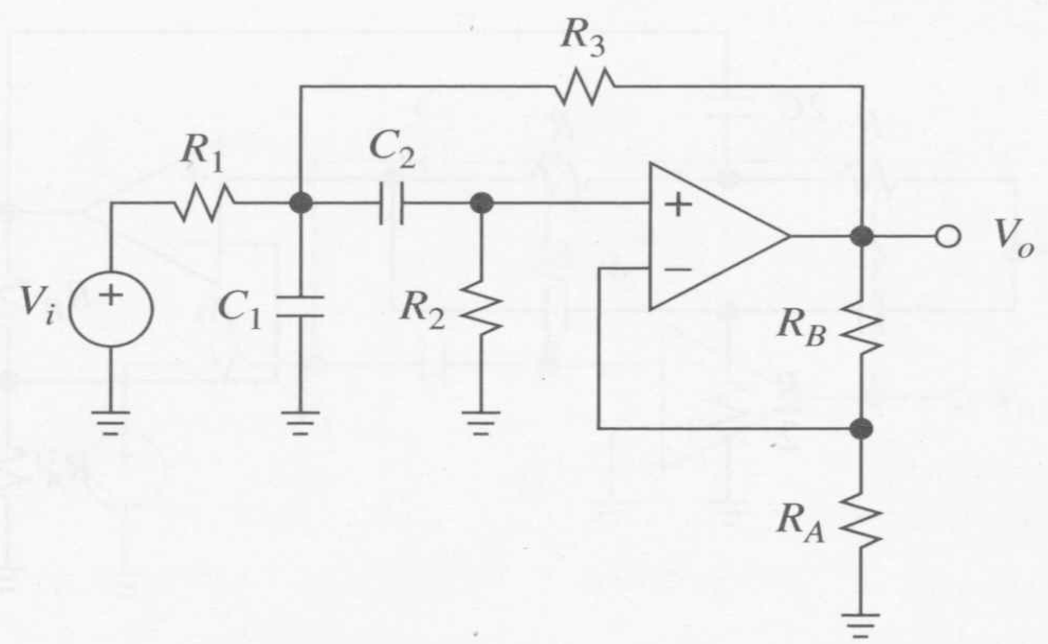


FIGURE 3.28
Band-pass *KRC* filter.

Band-Pass KRC: $V_O/V_i = H_{0BP}H_{BP}$ where H_{BP} is given in slide 12 and, for $Q > \sqrt{2}/3$, $R_1 = R_2 = R_3 = R$ and $C_1 = C_2 = C$,

$$H_{0BP} = \frac{K}{4-K} \qquad \omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4-K}$$

Design equations:

$$RC = \sqrt{2}/\omega_0 \qquad K = 4 - \sqrt{2}/Q \qquad R_B = (K-1)R_A$$

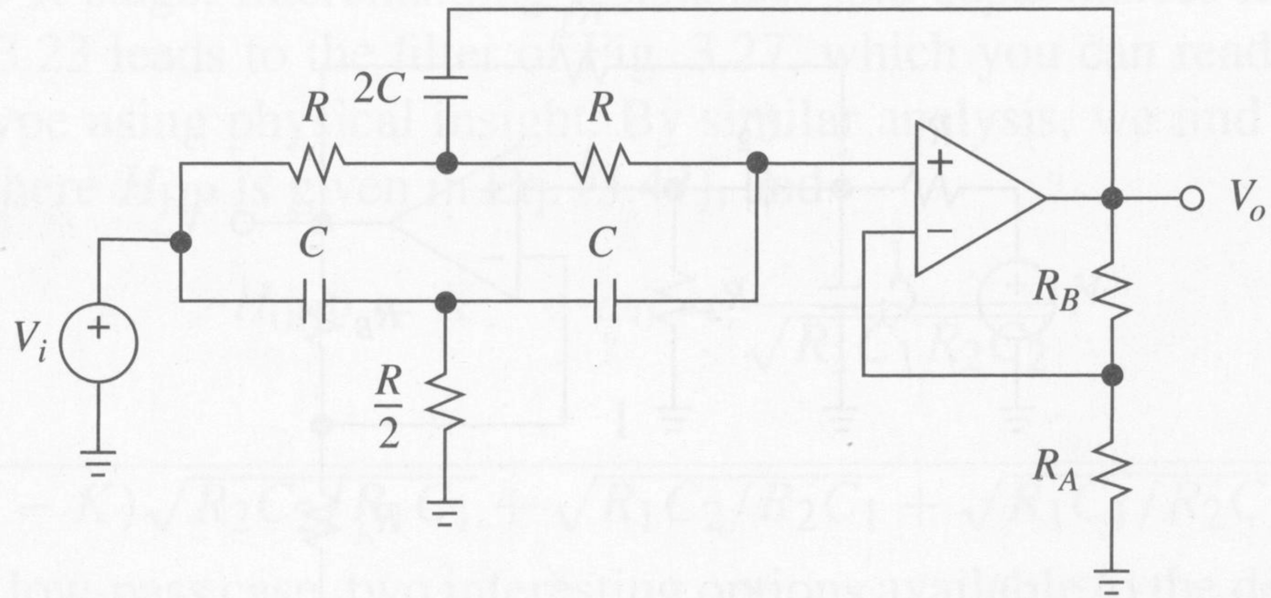


FIGURE 3.29
Band-reject *KRC* filter.

Band Reject (Notch) Filter

$$H_N(j\omega) = \frac{1-(\omega/\omega_0)^2}{1-(\omega/\omega_0)^2 + j(\omega/\omega_0)/Q}$$

$$V_O/V_i = H_{0N} H_N \qquad H_{0N} = K \qquad \omega_0 = \frac{1}{RC} \qquad Q = \frac{1}{4-2K}$$