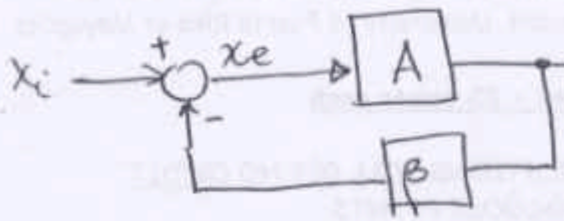
5207 - Clase ~~3~~ 2

Negative feedback



$$x_o = A x_e = A(x_i - \beta x_o)$$

$$\therefore \; x_o(1+\beta A) = A x_i$$

$x \rightarrow$ volt. or curr.

$$\frac{x_o}{x_i} = \frac{A}{1+\beta A} = \frac{A}{\beta A} \frac{1}{1 + 1/\beta A}$$

$$\frac{x_o}{x_i} = \frac{1}{\beta} \frac{1}{1 + 1/T}$$

$T =$ loop-gain $= \beta A$

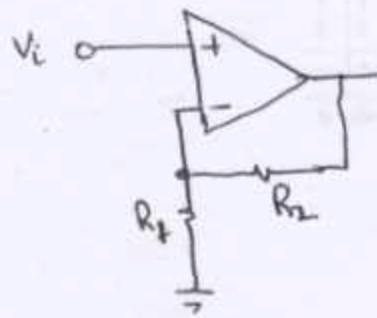
Ideal case: $\beta A \rightarrow \infty \implies \frac{x_o}{x_i} \simeq \frac{1}{\beta}$

$$\therefore \; \frac{x_o}{x_i} = A_{ideal} \frac{1}{1 + 1/T}$$

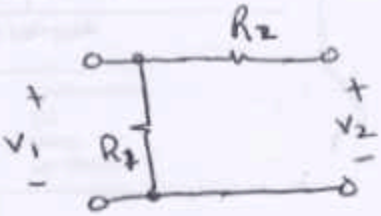
Aideal can be obtained from an "ideal analysis" of the opamp.

$$\text{Error function} = \epsilon = \frac{1}{1 + 1/T}$$

feedback analysis of non-inverting amplifier



feedback network



$$\beta = \frac{R_1}{R_1 + R_2}$$

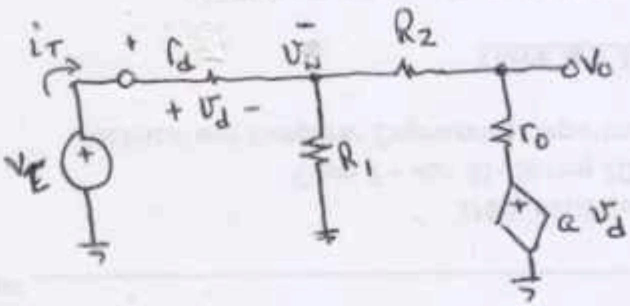
ideal gain : $\frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$

$$A_v = \frac{1}{\beta} \frac{1}{1 + \frac{1}{\beta a}} = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + \left(\frac{R_1 + R_2}{R_1}\right) / a}$$

or $$A_v = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + 1 / \underbrace{\left(a R_1 / R_1 + R_2\right)}_{T}}$$

This is the same result obtained in Lect. 1 using circuit analysis

Input impedance



$$i_t = \frac{v_N}{R_1} + \frac{v_N - a v_d}{R_2 + r_o}$$

$$v_d = i_t r_d$$

$$v_N = v_t - i_t r_d$$

$$i_t = \frac{v_t}{R_1} - i_t\frac{r_d}{R_1} + \frac{v_t}{R_2+r_o} - i_t\frac{r_d}{R_2+r_o} - a\frac{r_d}{R_2+r_o}\, i_t$$

$$i_t\left(1+\frac{r_d}{R_1} + r_d\frac{a+1}{R_2+r_o}\right) = v_t\left(\frac{1}{R_1}+\frac{1}{R_2+r_o}\right)$$

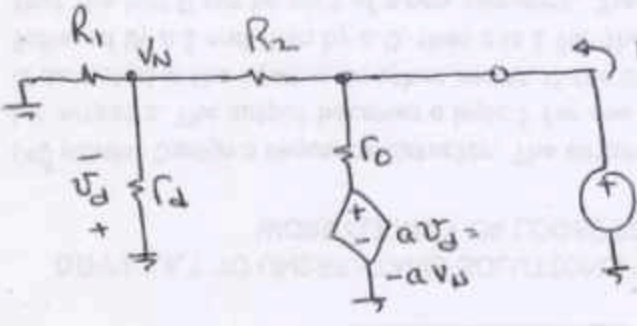
$$R_i = \frac{v_t}{i_t} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2+r_o}} + r_d\,\frac{\frac{1}{R_1}+\frac{1}{R_2+r_o}+\frac{a}{R_2+r_o}}{\frac{1}{R_1}+\frac{1}{R_2+r_o}}$$

$$R_i = \frac{v_t}{i_t} = R_1 \| (R_2+r_o) + r_d\left(1+\frac{a}{1+\frac{R_2+r_o}{R_1}}\right)$$

$$R_i \simeq r_d\left(1+\frac{a}{\frac{R_2}{R_1}+1}\right) = r_d\left(1+ aR_1/R_1+R_2\right)$$

$$\boxed{R_i = r_d\,(1+a\beta)}$$

Output Resistance



$$i_t = \frac{v_t + a v_N}{r_o} + \frac{v_t - v_N}{R_2}$$

$$v_N = v_t\,\frac{r_d\|R_1}{R_2 + r_d\|R_1}$$

$$i_t = \frac{v_t}{r_o} + \frac{a}{r_o}\,\frac{r_d||R_1}{R_2 + r_d||R_1}\,v_t + \frac{v_t}{R_2} + \frac{r_d||R_1}{R_2 + r_d||R_1}\,\frac{1}{R_2}\,v_t$$

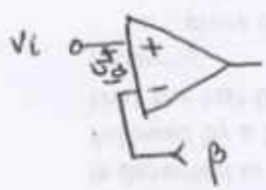
$$r_o\, i_t = \left(1 + \frac{a\,(r_d||R_1)}{R_2 + r_d||R_1} + \frac{r_o}{R_2} + \frac{r_d||R_1}{R_2 + r_d||R_1}\,\frac{r_o}{R_2}\right) v_t$$

$$\simeq \left(1 + \frac{aR_1}{R_2 + R_1} + \frac{R_1}{R_2 + R_1}\,\frac{r_o}{R_2}\right) v_t$$

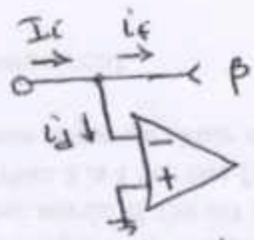
$$\simeq \left(1 + \frac{aR_1}{R_2 + R_1}\right) v_t$$

$$R_o = \frac{v_t}{i_t} \simeq \frac{r_o}{1 + \frac{aR_1}{R_2 + R_1}} = \boxed{\frac{r_o}{1 + a\beta} = \frac{r_o}{1 + T} = R_o}$$

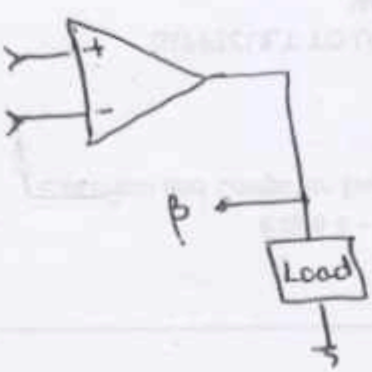
Topologies



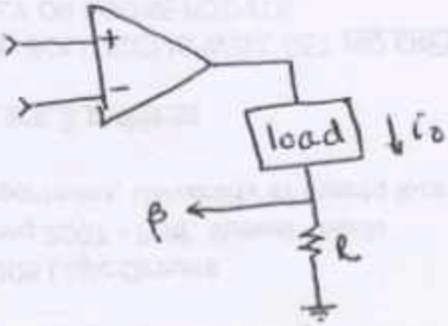
series at input



shunt at input



shunt at output



series at output

For series-input, shunt-output → like non-inv. amp.

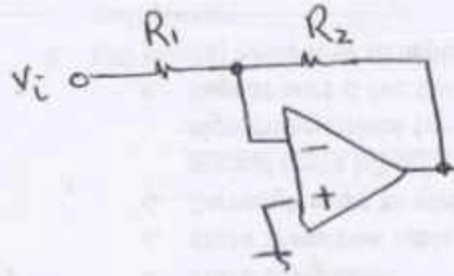
$$\text{gain} = v_o/v_i \cong a$$

$R_{if} = R_{i_{NF}}(1+a\beta)$ → we got $R_i = r_d(1+a\beta)$ in the non-inv. amp. example

↑ non-feedback amp. input impedance

$R_{of} = R_{o_{NF}} / 1+a\beta$ → we got $R_o = \dfrac{r_o}{1+T}$ for non-inv. amp.

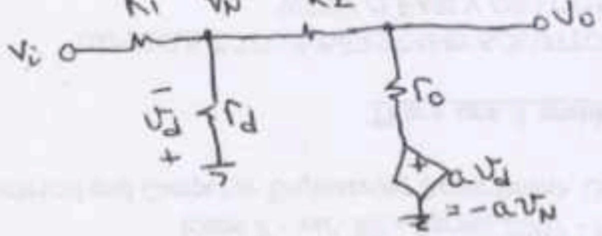
Inverting amplifier



shunt at input, shunt at output

A full feedback analysis (using the feedback method) requires the gain to be $R_M = \dfrac{v_o}{i_i}$

We won't do that. → just use circuit analysis as in textbook



$$\frac{v_i - v_N}{R_1} = \frac{v_N}{r_d} + \frac{v_N - v_o}{R_2}$$

$$\frac{v_N - v_o}{R_2} = \frac{v_o + a v_N}{r_o}$$

$$v_N r_o - v_o r_o = v_o R_2 + a R_2 v_N$$

$$v_N = v_o (R_2 + r_o) / (r_o - a R_2)$$

Neglet $r_o \ll R_2 \ll aR_2$

$$v_N = -\frac{v_o}{a}$$

$$\frac{v_i}{R_1} = -\frac{v_o}{a}\left(\frac{1}{R_1} + \frac{1}{r_d} + \frac{1}{R_2}\right) - \frac{v_o}{R_2}$$

$$v_i = -v_o\left(\frac{1 + R_1/r_d + R_1/R_2}{a} + \frac{R_1}{R_2}\right)$$

$$\simeq -v_o\left(\frac{1 + R_1/R_2}{a} + \frac{R_1}{R_2}\right) = -v_o \frac{1}{R_2}\left(\frac{R_2 + R_1}{a} + R_1\right)$$

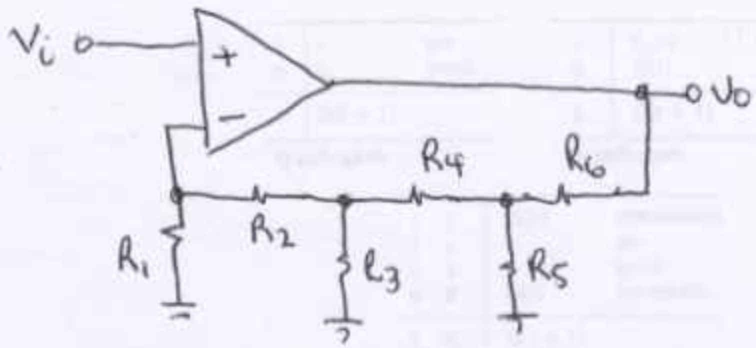
$$\frac{v_o}{v_i} = -R_2 \frac{1}{R_1 + \frac{R_2 + R_1}{a}} = -\frac{R_2}{R_1} \frac{1}{1 + \left(\frac{R_2}{R_1} + 1\right)/a}$$

$-\frac{R_2}{R_1}$: ideal gain — obtain using the ideal op-amp approx

$\frac{1}{1 + \left(\frac{R_2}{R_1} + 1\right)/a} = \frac{1}{1 + 1/a\beta}$ — but the β is the non-feedback amplifier's β !

$\therefore$ In this course we often use the non-inv. amp. β even if we have an ~~[illegible]~~ inverting configuration

Example



(a) find A_{ideal} if all resistors are equal

(b) Assuming $r_d = \infty$, $r_o = 0$, find a_{min} such that the deviation of A from A_{ideal} is less than 0.1%.

(c) repeat (a) for the equivalent inverting configuration