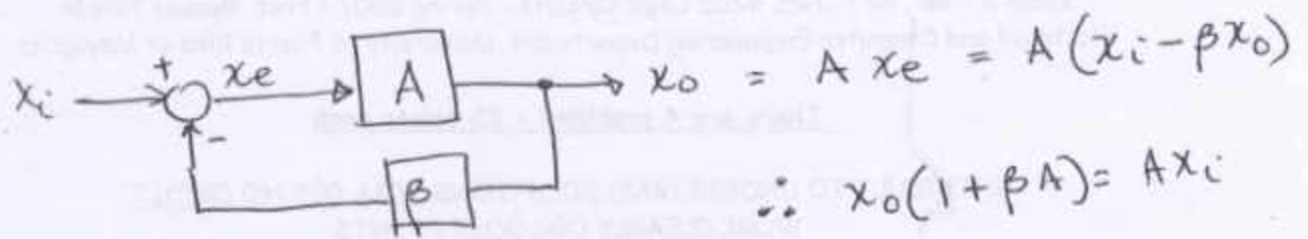


Negative feedback

$$\therefore x_o(1 + \beta A) = A x_i$$

$x \rightarrow$ volt. or. cur.

$$\frac{x_o}{x_i} = \frac{A}{1 + \beta A} = \frac{A}{\beta A} \frac{1}{1 + 1/\beta A}$$

$$\frac{x_o}{x_i} = \frac{1}{\beta} \frac{1}{1 + 1/T}$$

$$T = \text{loop-gain} = \beta A$$

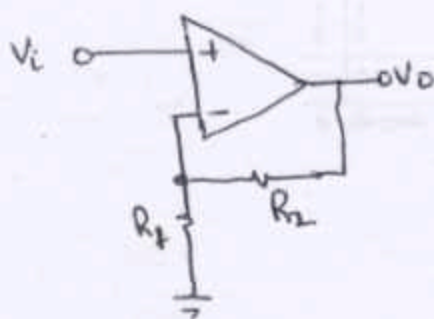
$$\text{Ideal case: } \beta A \rightarrow \infty \Rightarrow \frac{x_o}{x_i} \approx \frac{1}{\beta}$$

$$\therefore \frac{x_o}{x_i} = A_{\text{ideal}} \frac{1}{1 + 1/T}$$

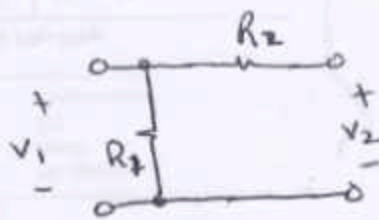
A_{ideal} can be obtained from an "ideal analysis" of the opamp.

$$\text{Error function} = \epsilon = \frac{1}{1 + 1/T}$$

feedback analysis of non-inverting amplifier



feedback network



$$\beta = \frac{R_1}{R_1 + R_2}$$

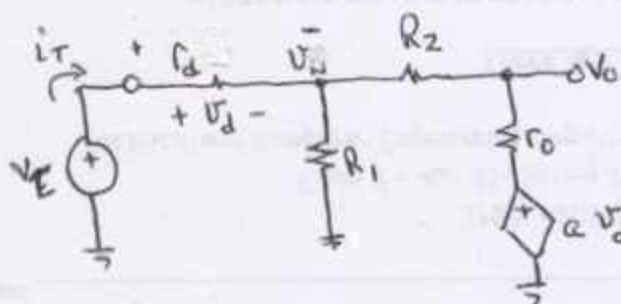
ideal gain: $\frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$

$$A_v = \frac{1}{\beta} \frac{1}{1 + \frac{1}{\beta A}} = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + \left(\frac{R_1 + R_2}{R_1}\right) \frac{1}{A}}$$

$$\approx A_v = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + \frac{1}{A \left(\frac{R_1}{R_1 + R_2}\right)}}$$

This is the same result ~~that was~~ obtained in lect. 1 using circuit analysis

Input impedance



$$i_E = \frac{v_N}{R_1} + \frac{v_N - \alpha v_D}{R_2 + r_o}$$

$$v_D = i_E r_o$$

$$v_N = v_E - i_E r_o$$

$$i_c = \frac{v_c}{R_1} - i_c \frac{r_d}{R_1} + \frac{v_c}{R_2 + r_o} - i_c \frac{r_d}{R_2 + r_o} - a \frac{r_d}{R_2 + r_o} i_c$$

$$i_c \left(1 + \frac{r_d}{R_1} + r_d \frac{a+1}{R_2 + r_o} \right) = v_c \left(\frac{1}{R_1} + \frac{1}{R_2 + r_o} \right)$$

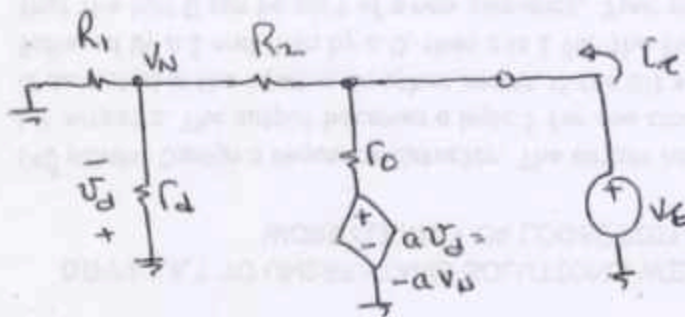
$$R_i = \frac{v_c}{i_c} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + r_o}} + r_d \frac{\frac{1}{R_1} + \frac{1}{R_2 + r_o} + \frac{a}{R_2 + r_o}}{\frac{1}{R_1} + \frac{1}{R_2 + r_o}}$$

$$R_i = \frac{v_c}{i_c} = R_1 \parallel (R_2 + r_o) + r_d \left(1 + \frac{a}{1 + \frac{R_2 + r_o}{R_1}} \right)$$

$$R_i \approx r_d \left(1 + \frac{a}{\frac{R_2 + r_o}{R_1} + 1} \right) = r_d \left(1 + a R_1 / (R_1 + R_2) \right)$$

$$\boxed{R_i = r_d (1 + a\beta)}$$

Output Resistance



$$i_c = \frac{v_c + a v_N}{r_o} + \frac{v_c - v_N}{R_2}$$

$$v_N = v_c \frac{r_d \parallel R_1}{R_2 + r_d \parallel R_1}$$

$$i_E = \frac{v_E}{r_o} + \frac{a}{r_o} \frac{r_d // R_1}{R_2 + r_d // R_1} v_t + \frac{v_E}{R_2} + \frac{r_d // R_1}{R_2 + r_d // R_1} \frac{1}{R_2} v_t$$

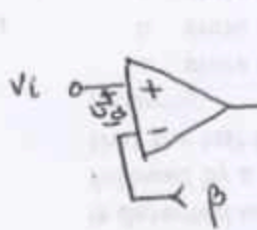
$$r_o i_E = \left(1 + \frac{a (r_d // R_1)}{R_2 + r_d // R_1} + \frac{r_o}{R_2} + \frac{r_d // R_1}{R_2 + r_d // R_1} \frac{r_o}{R_2} \right) v_t$$

$$\approx \left(1 + \frac{a R_1}{R_2 + R_1} + \frac{R_1}{R_2 + R_1} \frac{r_o}{R_2} \right) v_t$$

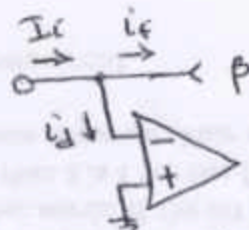
$$\approx \left(1 + \frac{a R_1}{R_2 + R_1} \right) v_t$$

$$R_o = \frac{v_o}{i_t} \approx \frac{r_o}{1 + \frac{a R_1}{R_2 + R_1}} = \boxed{\frac{r_o}{1 + a\beta} = \frac{r_o}{1 + T} = R_o}$$

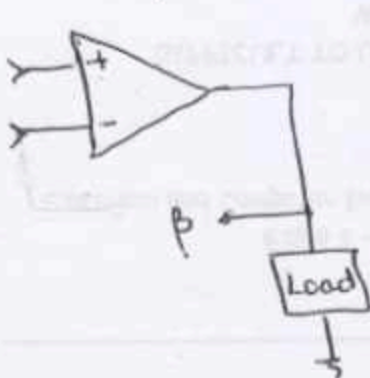
Topologies



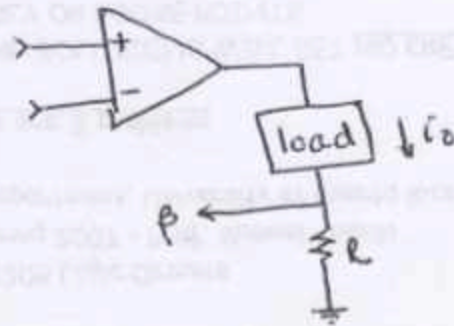
series at input



shunt at input



shunt at output



series at output

For series-input, shunt-output \rightarrow like non-inv. amp.

$$\text{gain} = v_o/v_i \approx a$$

$$R_{if} = R_{iNF} (1 + a\beta) \rightarrow \text{we get } R_i = r_d (1 + a\beta)$$

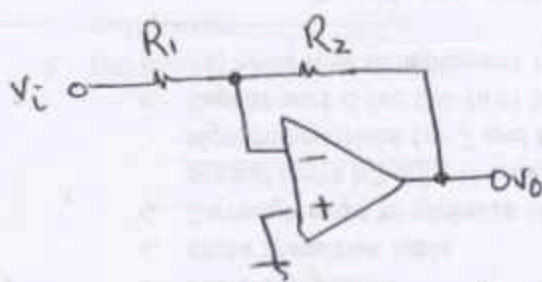
↑
non-feedback
amp. input
impedance

in the non-inv. amp. example

$$R_{of} = R_{oNF} / (1 + a\beta) \rightarrow \text{we get } R_o = \frac{r_o}{1 + T}$$

for non-inv. amp.

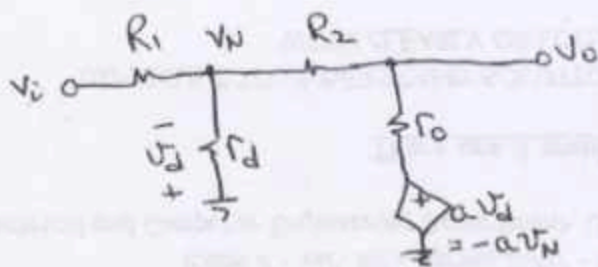
Inverting amplifier



shunt at input, shunt at output

A full feedback analysis (using the feedback method) requires the gain to be $R_M = \frac{v_o}{i_i}$

We won't do that. \rightarrow just use circuit analysis as in textbook



$$\frac{v_i - v_N}{R_1} = \frac{v_N}{r_d} + \frac{v_N - v_o}{R_2}$$

$$\frac{v_N - v_o}{R_2} = \frac{v_o + a v_N}{r_o}$$

$$v_N r_o - v_o r_o = v_o R_2 + a R_2 v_N$$

$$v_N = v_o (R_2 + r_o) / (r_o - a R_2)$$

Neglect $r_o \ll R_2 \ll aR_2$

$$V_N = -\frac{V_o}{a}$$

$$\frac{V_i}{R_1} = -\frac{V_o}{a} \left(\frac{1}{R_1} + \frac{1}{r_o} + \frac{1}{R_2} \right) - \frac{V_o}{R_2}$$

$$V_i = -V_o \left(\frac{1 + R_1/r_o + R_1/R_2}{a} + \frac{R_1}{R_2} \right)$$

$$\approx -V_o \left(\frac{1 + R_1/R_2}{a} + \frac{R_1}{R_2} \right) = -V_o \frac{1}{R_2} \left(\frac{R_2 + R_1}{a} + R_1 \right)$$

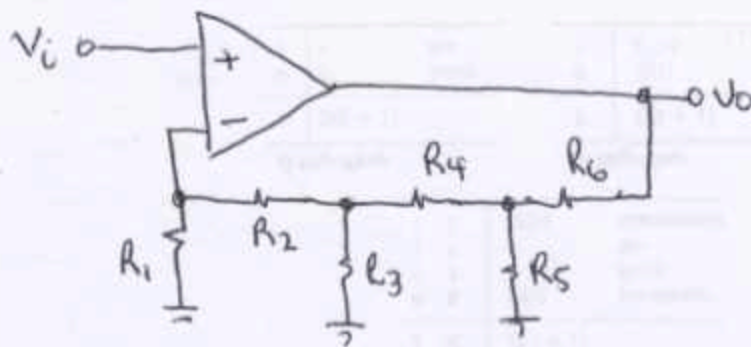
$$\frac{V_o}{V_i} = -R_2 \frac{1}{R_1 + \frac{R_2 + R_1}{a}} = -\frac{R_2}{R_1} \frac{1}{1 + \frac{(R_2 + R_1)/a}{R_1}}$$

obtain using
the ideal op-amp
approx

but the β is
the non-feedback
amplifier's β !

\therefore In this course we ^{often} use the non-inv. amp. β even
if we have an ~~inverting~~ inverting configuration

Example



- (a) Find A_{ideal} if all resistors are equal
- (b) Assuming $r_d = \infty$, $r_o = 0$, find A_{min} such that the deviation of A from A_{ideal} is less than 0.1%.
- (c) repeat (a) for the equivalent inverting configuration