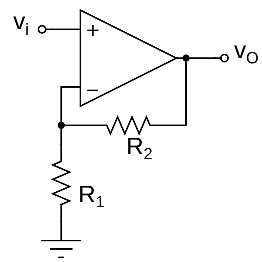
Practice Problems Solutions - Exam 2 - INEL5207 March 19, 2009

1. An otherwise ideal op amp has one pole at 10kHz, a second pole at 200kHz, and a unity-gain frequency of $10MHz$. The op amp will be used to design a one-stage non-inverting amplifier as shown in the following figure.



Determine the following quantities:

(a) opamp open-loop d.c. gain.
ANSWER:

$$\mid a(jf) \mid = \frac{a_0}{\sqrt{1+\left(\frac{f}{10kHz}\right)^2}\sqrt{1+\left(\frac{f}{200kHz}\right)^2}}$$

Since $\mid a(10MHz) \mid = 1$,

$$a_0 = \sqrt{1+\left(\frac{10^7}{10^4}\right)^2}\sqrt{1+\left(\frac{10^7}{2\times 10^5}\right)^2} = \boxed{50000V/V}$$

(b) phase-margin if $R_2 = 10k\Omega$ and $R_1 = 100\Omega$.
ANSWER:

$$\beta = \frac{R_1}{R_2+R_1} = \frac{100}{10100} \rightarrow \frac{1}{\beta} = 101 \simeq 40dB$$

Since there are poles at $10kHz$ and $200kHz$, a's magnitude bode plot will have a slope of $-40dB/dec$ at the unity-gain frequency $f_\tau = 10MHz$, where $\mid a \mid = 0dB$. Thus, one decade below f_τ, $\mid a(1MHz) \mid \simeq 40dB$ and it is equal to $\frac{1}{\beta}$. The phase at $1MHz$ is

$$\phi = -\arctan\left({}^{10^6}/_{10^4}\right) - \arctan\left({}^{10^6}/_{2\times10^5}\right) \simeq -168^\circ$$

and

$$\phi_m \simeq 180^\circ - 168^\circ = \boxed{12^\circ}$$

(c) phase-margin if $R_2 = 100k\Omega$ and $R_1 = 100\Omega$.
ANSWER:
For these resistor values, ${}^1/_\beta = 1001$. Equating $\mid a(jf) \mid$ and ${}^1/_\beta$ at the crossover frequency f_x and rearranging the above equation for a gives

$$\left(1+\left({}^{f_x}/_{10^4}\right)^2\right)\times\left(1+\left({}^{f_x}/_{2\times10^5}\right)^2\right) = \left({}^{50,000}/_{1001}\right)^2 = 2495$$

Now express f_x in MHz and let $x = f_x^2$ to get the quadratic equation

$$(1+10^4x)(1+25x) \simeq (10^4x)(1+25x) = 2495$$

The solution is $x = 0.0819$ and

$$f_x = \sqrt{x} = 0.286MHz = 286kHz$$

At this frequency,

$$\phi = -\arctan(28.6) - \arctan(1.43) \simeq -143°$$

Thus

$$\phi_m \simeq 180° - 143° = \boxed{37°}$$

(d) R_1/R_2 so that the phase margin is 45°.
ANSWER:
For $\phi_m = 45°$, we need

$$\phi = -\arctan\left({}^{f_x}/_{10^4}\right) - \arctan\left({}^{f_x}/_{2\times10^5}\right) = -135°$$

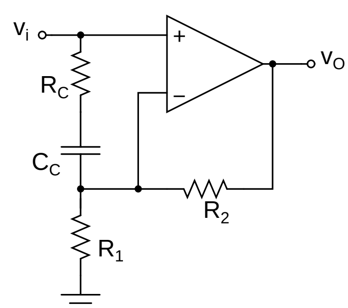
At the second pole, with $f_x = 200kHz$, one gets $\phi = -132°$. Trying a few values, one finds that at $220kHz$ the phase is quite close to $-135°$. For $\phi_m = 45°$ the value of $1/\beta$ must be equal to $\mid a \mid$ at this frequency, so

$$1/\beta = 1 + R_2/R_1 = \mid a(j220kHz) \mid = \frac{50000}{\sqrt{1+22^2}\sqrt{1+1.1^2}} \simeq \frac{50000}{22\times1.5} = 1527$$

and

$$\boxed{\frac{R_2}{R_1} \simeq 1526}$$

2. The op amp described in problem 1 is used as shown below to design a single-stage non-inverting amplifier with a d.c. gain of $100V/V$. Select R_C and C_C to get a phase margin of 45°.



ANSWER:
Let $R_2 = 99k\Omega$ and $R_1 = 1k\Omega$, so thet the d.c gain is $100V/V$. In the limit of $f = \infty$, using our results from problem 1(d),

$$1/\beta_\infty = 1 + \frac{99k\Omega}{1k\Omega \parallel R_C} = 1527$$

This yields

$$\boxed{R_C = 69.4\Omega}$$

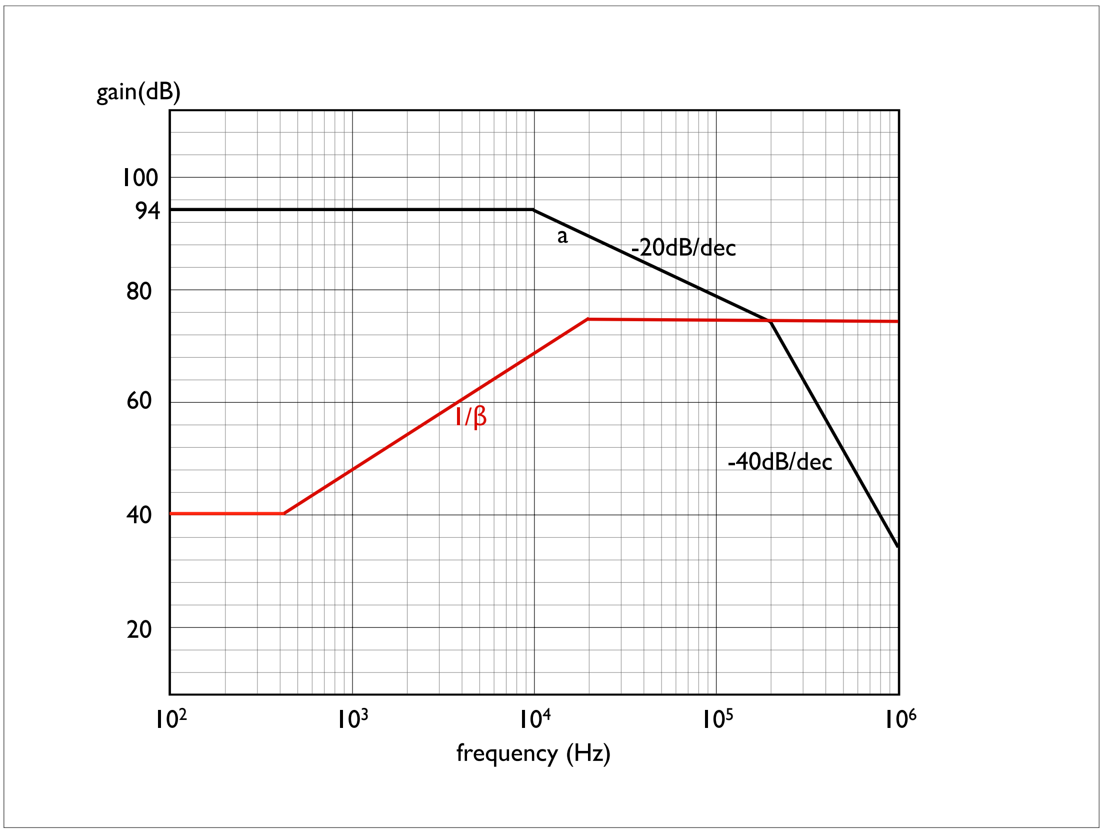
From the textbook's discussion for the input-lag compensation method, we know that the frequecy of the pole for $1/\beta$ should be one decade below the crossover frequency. Thus

$$f_p = \frac{1}{2\pi R_C C_C} = \frac{1}{10} \times 220kHz$$

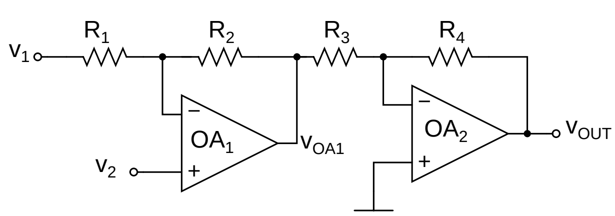
and

$$\boxed{C_C = \frac{5}{\pi \times 69.4\Omega \times 220kHz} = 100nF}$$

An approximated solution can also be obtained graphically, as shown below.



3. For the following circuit,



let v_2 be connected to ground and v_1 be the input signal. If $R_1 = R_3 = 1k\Omega$, $R_2 = R_4 = 5k\Omega$ and the opamp parameters are: open-loop d.c. gain $a_0 = 10^4$, unity-gain frequency $f_\tau = 1MHz$, and slew-rate $S.R. = 0.5V/\mu s$, find

(a) the bandwidth if the slew-rate limitation is ignored, (15 pts)
ANSWER: For each stage $\beta = \frac{R_1}{R_1+R_2} = \frac{1}{6}$. Since there are two stages,

$$f_{-3db} = \beta f_\tau \sqrt{2^{1/2} - 1} \approx \boxed{107kHz}$$

(b) the maximum frequency of an input sinusoid that produces an undistorted output signal with peak value equal to $10V$. (10 pts)

$$f = \frac{SR}{2\pi V_{om}} = \frac{0.5V/\mu s}{2\pi 10V} \approx \boxed{8kHz}$$

4. For the circuit in the previous problem, let v_1 be connected to ground and v_2 be the input signal, and assume that each stage have a voltage gain of ± 3. The opamp's unity-gain frequency is $f_\tau = 1MHz$. Find

(a) the expression for the gain as a function of frequency for OA_1, $A_{OA1}(s) = \frac{v_{OA1}}{v_1}$.
ANSWER: The pole of OA_1 is at $\beta_{ni} f_\tau = 333kHz$, so

$$\boxed{A_{OA1}(s) = \frac{3}{1+\frac{s}{2\pi 333kHz}} = \frac{3}{1+\frac{s}{2.1\times 10^6 rps}}}$$

(b) the expression for the gain as a function of frequency for OA_2, $A_{OA2}(s) = \frac{v_{OUT}}{v_{OA1}}$.
ANSWER: The pole of OA_2 is at $\beta_{ni} f_\tau = \frac{1MHz}{A_{inv}+1} = 250kHz$, so

$$\boxed{A_{OA2}(s) = \frac{-3}{1+\frac{s}{2\pi 250kHz}} = \frac{-3}{1+\frac{s}{1.6\times 10^6 rps}}}$$

(c) the exact bandwidth of the circuit, by combining the expressions found in parts (a) and (b), equating the overall gain's magnitude to $\frac{A_0}{\sqrt{2}}$ (where A_0 represents the overall d.c. gain) and solving for the frequency. Show all work.
ANSWER:

$$\mid A \mid = \frac{3}{\sqrt{1+\left(\frac{f}{333kHz}\right)^2}} \frac{3}{\sqrt{1+\left(\frac{f}{250kHz}\right)^2}} = \frac{9}{\sqrt{2}}$$

or

$$\left(1+\left(\frac{f}{333kHz}\right)^2\right)\left(1+\left(\frac{f}{250kHz}\right)^2\right) = 2$$

If we express frequency in MHz,

$$\begin{aligned}\left(1+(3f)^2\right)\left(1+(4f)^2\right) &= 2\\ (1+9x)(1+16x) &= 2\\ 144x^2+25x-1 &= 0\end{aligned}$$

where $x = f^2$. The solutions are $x = 0.0335$ and -0.2. Selecting the first solution yields

$$f_{-3db} = \sqrt{0.0335} = .183MHz = \boxed{183kHz}$$

5. An otherwise ideal op amp that has two poles at 10kHz, a third pole at 50kHz, and a unity-gain frequency of $1MHz$. Use this amplifier to design a single-stage non-inverting amplifier with a d.c. gain of $100V/V$. Design the circuit so that the phase margin is $60°$. Explain your design procedure in detail.

   Answer:

   To find the frequency at which the phase of a is $120°$, solve

$$120° = -2\arctan\frac{f}{10kHz} - \arctan\frac{f}{50kHz}$$

   either numerically, graphically or by trial and error. Graphically, I got $f \approx 13kHz$. At this frequency, the magnitude of a is

$$\mid a(j13kHz) \mid = \frac{a_0}{\left(1 + \left(\frac{13}{10}\right)^2\right)\sqrt{1 + \left(\frac{13}{50}\right)^2}}$$

   We need to find a_0. A graphical analysis reveals that, since $f_\tau = 1MHz$ and the magnitude bode is droping at $-60dB/dec$, at $100kHz$ the magnitude of a is $60dB$. To get the gain at $50kHz$ (one octave below $100kHz$), add $3\times 6dB/oct$ (because the curve's slope is $60dB/dec$) to get $78dB$. The gain at $10kHz$ is $12dB$ below $40dB + 78dB$, or $106dB$, and equals a_0 because we are using asymptotes. You can then proceed as in problem 2 to synthesize a solution.