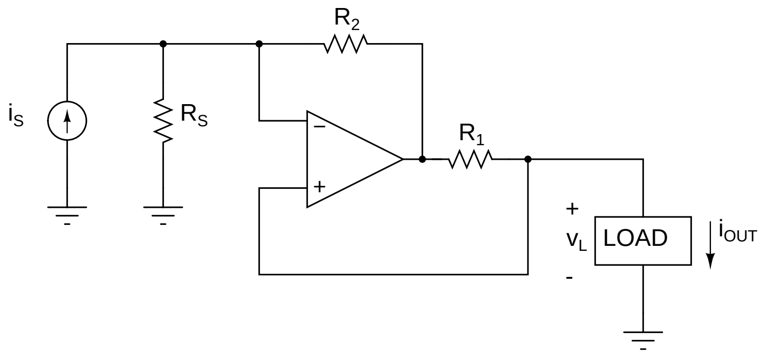
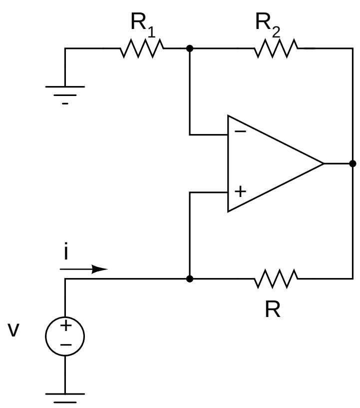
INEL 5207 - Solutions to Practice Problem Set 2 - Exam 1

1. Show that $i_{OUT} = Ai_S - \frac{1}{R_O}v_L$, where $A = -\frac{R_2}{R_1}$and $R_O = -\frac{R_1}{R_2}R_S$, for the following circuit.



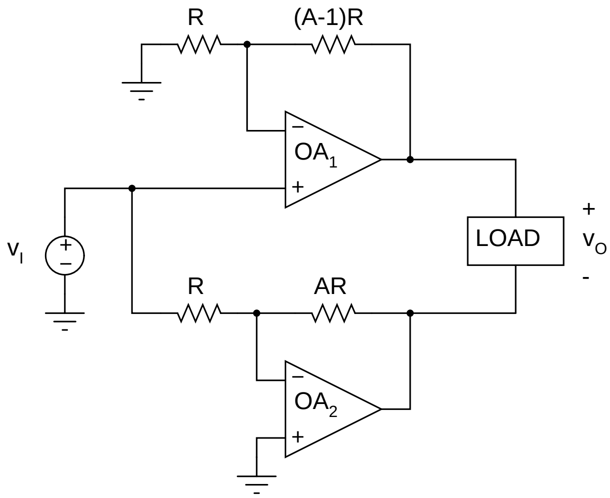
ANSWER: Observe that $v_L = v_+ = v_-$. Thus $i_{R_S} = \frac{v_L}{R_S}$ flowing towards ground, and $i_{R_2} = i_S - \frac{v_L}{R_S}$. The op amp output voltage is $v_O = v_L - R_2 i_{R_2} = v_L + \frac{R_2}{R_S}v_L - R_2 i_S$. Since no current flows into the op amp input, $i_{R_1} = i_L = \frac{v_O - v_L}{R_1} = -\frac{R_2}{R_1}i_S + \frac{R_2}{R_1}\frac{1}{R_S}v_L$.

2. Show that $R_{EQ} = \frac{v}{i} = -\frac{R_1}{R_2}R$ for the following circuit.



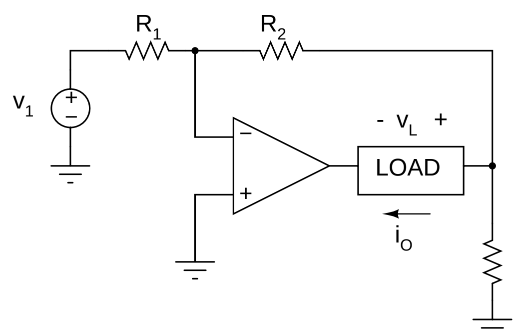
ANSWER: Since $v_+ = v$, the op amp's output voltage is $v_O = \left(1 + \frac{R_2}{R_1}\right)v = v - iR$. Solving for $\frac{v}{i}$ yield the desired expression.

3. The circuit in the figure below, a *bridge amplifier*, allows one to double the linear output range as compared with a single op amp. (a) Show that if the resistances are in the ratios shown, then $v_O/v_I = 2A$. (b) If the individual op amps saturate at $\pm 13V$, what is the maximum peak-to-peak output voltage that the circuit can provide without distortion?



ANSWER: Let v_{O1} and v_{O2} denote OA_1 and OA_2's output voltages, respectively. Observe that $v_{O1} = \left(1 + \frac{(A-1)R}{R}\right) v_i = Av_i$ and $v_{O2} = -\frac{AR}{R} v_i = -Av_i$. Thus, $v_O = 2Av_i$. Since each op amp output can swing $\pm$13V, the v_O can be up to $26V_{peak}$, or $52V_{peak-peak}$.

4. (a) Show that the floating-load V-I converter shown below yields $i_O = v_I/(R_1/k)$, $k = 1 + R_2/R_3$, (b) Specify standard 5% resistances for a sensitivity (gain) of 1mA/V and $R_i = 1M\Omega$, where R_i is the resistance seen by the input source.

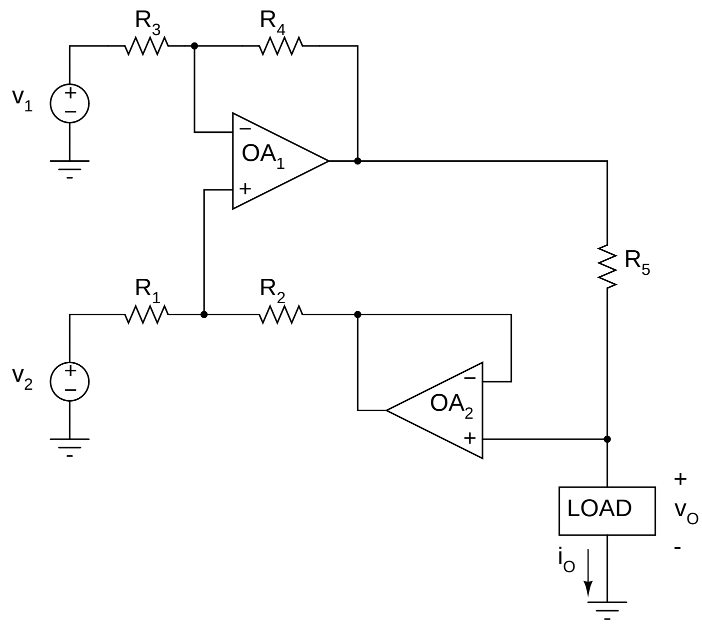


ANSWER:

(a) $i_o = i_{R_2} - i_{R_3} = i_{R_1} - i_{R_3} = \frac{v_i}{R_1} + \left(\frac{R_2}{R_1} v_i\right) \frac{1}{R_3} = \frac{v_i}{R}$; $R = \frac{R_1}{1+\frac{R_2}{R_3}}$.

(b) $R_i = R_1$; $R_1 = 1M\Omega$. The sensitivity is $1\frac{mA}{V} = \frac{1}{R}$so $R = 1k\Omega = \frac{1M\Omega}{1+\frac{R_2}{R_3}}$so $\frac{R_2}{R_3} = 999 \approx 1k\Omega$. You can select $R_2 = 1M\Omega$ and $R_3 = 1k\Omega$.

5. (a) For the circuit shown below, given that $i_O = av_I - (1/R_O)v_L$, $v_I = v_2 - v_1$, find expressions for A and R_O, as well as the condition among resistances that yields $R_O = \infty$.



ANSWER: Let v_{O1} and v_{O2} be OA_1 and OA_2's output voltages. Observe that $v_{O2} = v_O$, and that $v_{+OA_1} = v_{p1} = v_2 - \frac{v_2 - v_O}{R_1+R_2} R_1 = \frac{1}{\frac{R_1}{R_2}+1} v_2 + \frac{1}{1+\frac{R_2}{R_1}} v_O$.

For OA_1, $v_{O1} = \left(1 + \frac{R_4}{R_3}\right) v_{p1} - \frac{R_4}{R_3} v_1 = v_{p1} + \frac{R_4}{R_3}(v_{p1} - v_1)$. Since no current flows into OA_2,

$$
\begin{aligned}
i_O &= \frac{v_{O1} - v_O}{R_5} \\
&= \frac{(1 + \frac{R_4}{R_3}) v_{p1} - \frac{R_4}{R_3} v_1 - v_O}{R_5}
\end{aligned}
$$

Using $(1 + \frac{R_4}{R_3}) v_{p1} = \frac{1+\frac{R_4}{R_3}}{1+\frac{R_1}{R_2}} v_2 + \frac{1+\frac{R_4}{R_3}}{1+\frac{R_2}{R_1}} v_O$, we get

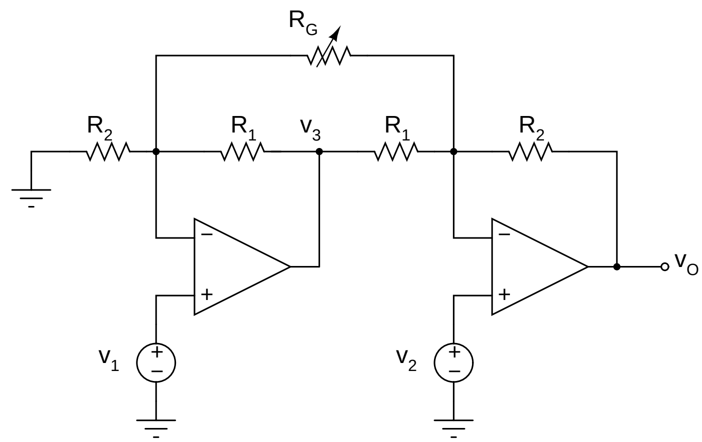
$$
\begin{aligned}
i_O &= \frac{1}{R_5}\left(\frac{1 + \frac{R_4}{R_3}}{1 + \frac{R_1}{R_2}} v_2 - \frac{R_4}{R_3} v_1 + \left(\frac{1 + \frac{R_4}{R_3}}{1 + \frac{R_2}{R_1}} - 1\right) v_O\right) \\
&= \frac{1}{R_5}\left(\frac{v_2 - \frac{R_1 R_4}{R_2 R_3} v_1 + \frac{R_4}{R_3}(v_2 - v_1)}{1 + \frac{R_1}{R_2}} + \frac{\frac{R_4}{R_3} - \frac{R_2}{R_1}}{1 + \frac{R_2}{R_1}} v_O\right)
\end{aligned}
$$

Thus if $\frac{R_1 R_4}{R_2 R_3} = 1$, $\frac{1}{R_O} = \frac{\frac{R_4}{R_3} - \frac{R_2}{R_1}}{1+\frac{R_2}{R_1}} = 0 = \frac{1}{\infty}$, and

$$
\begin{aligned}
i_O &= \frac{1}{R_5} \frac{1 + \frac{R_2}{R_1}}{1 + \frac{R_1}{R_2}} (v_2 - v_1) \\
&= \frac{1}{R_5} \frac{R_2}{R_1} v_I
\end{aligned}
$$

In summary, selecting $\frac{R_4}{R_3} = \frac{R_2}{R_1}$ makes $R_O = \infty$ and the gain $a = \frac{i_O}{v_2 - v_1} = \frac{R_2}{R_1 R_5}$.

6. The following sketch shows a variable-gain, dual-op amp instrumentation amplifier. Show that $A = \frac{v_O}{v_2 - v_1} = 1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G}$.



ANSWER: By superposition

$$
\begin{aligned}
v &= \left(1 + \frac{R_1}{R_2||R_G}\right) v_1 - \frac{R_1}{R_G} v_2 \\
v_O &= -\frac{R_2}{R_1} v - \frac{R_2}{R_G} v_1 + \left(1 + \frac{R_2}{R_G||R_1}\right) v_2
\end{aligned}
$$

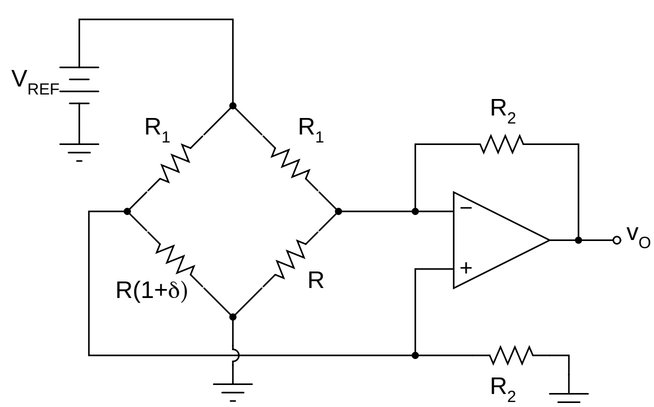
This can be re-written as

$$
\begin{aligned}
v_O &= -\frac{R_2}{R_1} v_1 - \frac{R_2}{R_2||R_G} v_1 + \frac{R_2}{R_G} v_2 - \frac{R_2}{R_G} v_1 + v_2 + \frac{R_2}{R_G||R_1} v_2 \\
&= \left(1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G}\right)(v_2 - v_1)
\end{aligned}
$$

which is the desired relationship.

7. A single op amp transducer bridge amplifier is shown below. Show that

$$
v_O = \frac{R_2}{R} \times \frac{\delta}{R_1/R + (1 + R_1/R_2)(1+\delta)} \times V_{REF}
$$



ANSWER: KCL at the input nodes yield

$$
\begin{aligned}
\frac{V_{REF} - v_n}{R_1} &= \frac{v_n}{R} + \frac{v_n - v}{R_2} \\
\frac{V_{REF} - v_p}{R_1} &= \frac{v_p}{R(1+\delta)} + \frac{v_p}{R_2}
\end{aligned}
$$

Using $v_p = v_n$ and rearranging

$$\begin{aligned} -\frac{v}{R_2} &= \frac{V_{REF}}{R_1} - \left(\frac{1}{R_1} + \frac{1}{R} + \frac{1}{R_2}\right) v_n \\ v_n &= v_p = \frac{V_{REF}}{R_1} \times \frac{1}{\frac{1}{R_1} + \frac{1}{R(1+\delta)} + \frac{1}{R_2}} \end{aligned}$$

Thus

$$v = -V_{REF} \times \frac{R_2}{R_1}\left(1 - \frac{\frac{1}{R_1} + \frac{1}{R} + \frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R(1+\delta)} + \frac{1}{R_2}}\right) = -V_{REF} \times \frac{R_2}{R_1} \times \frac{\frac{1}{R(1+\delta)} - \frac{1}{R}}{\frac{1}{R_1} + \frac{1}{R(1+\delta)} + \frac{1}{R_2}}$$

$$v = V_{REF} \times \frac{R_2}{R} \times \frac{\delta}{(1+\delta)\left(1 + \frac{R_1}{R_2}\right) + \frac{R_1}{R}}$$

which is the desired relationship.