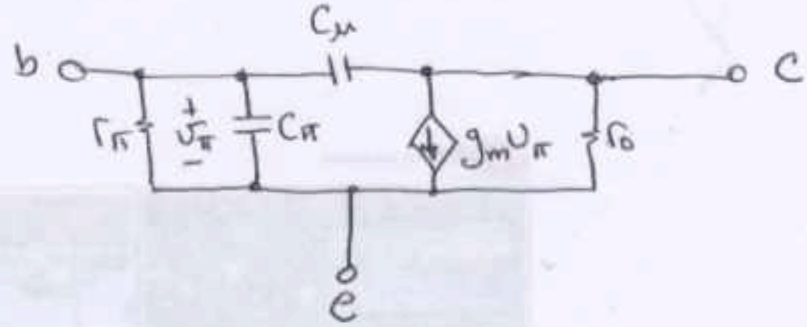
InEl 4202 - Clase 3

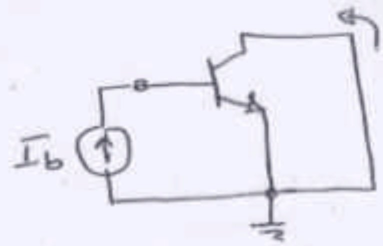
High-frequency response

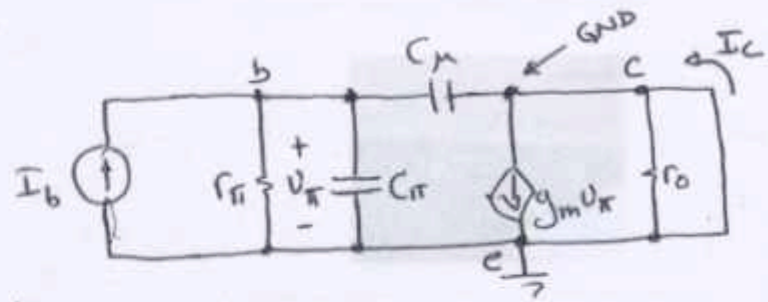
BJT → hybrid-π equivalent circuit is expanded to include capacitive effects on the transistor's junctions.



C_π & C_μ reduce the gain at higher frequencies

Short-circuit current gain





$v_{C_\mu} = v_\pi$

KCL at b

$$I_b = \frac{v_\pi}{r_\pi} + \frac{v_\pi}{1/sC_\pi} + \frac{v_\pi}{1/sC_\mu}$$

$$I_b = v_\pi\left(1/r_\pi + sC_\pi + sC_\mu\right)$$

$$v_\pi = \frac{r_\pi I_b}{1 + s\, r_\pi (C_\pi + C_\mu)}$$

KCL at c

$$I_c = g_m v_\pi - \frac{v_\pi}{1/sC_\mu} = (g_m - sC_\mu)\, v_\pi$$

For all frequencies of interest, $g_m >> sC_\mu$

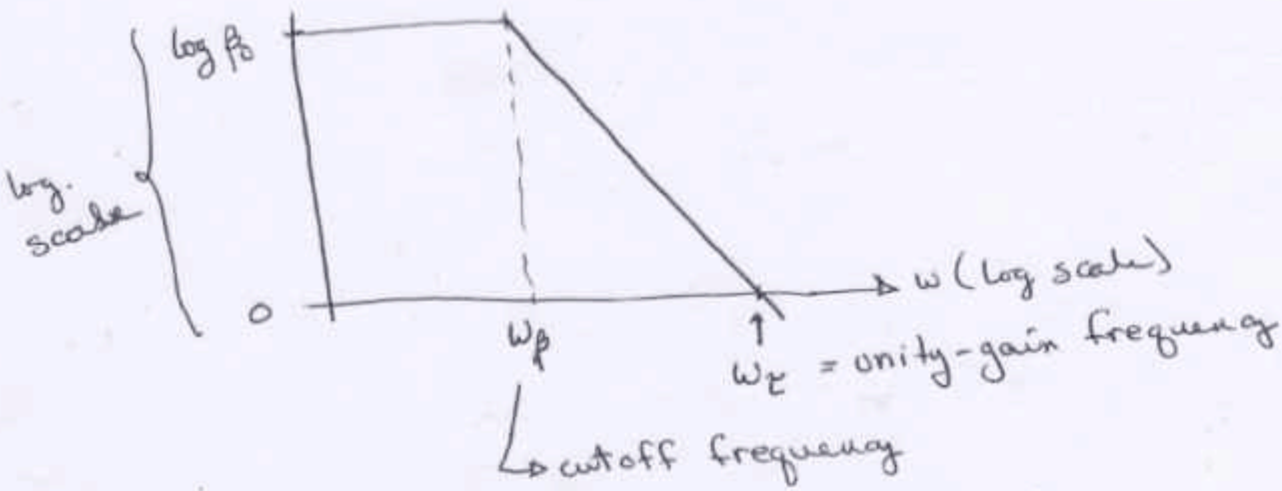
$$\therefore \quad I_C \doteq g_m v_\pi = \frac{g_m r_\pi}{1 + s r_\pi (C_\pi + C_\mu)} I_b$$

This provides an expression for $I_c / I_b = \beta$ as a function of s,

$$\beta(s) = \frac{I_c}{I_b} = \frac{\beta_0}{1 + s r_\pi (C_\pi + C_\mu)}$$

$$\beta_0 = g_m r_\pi = \text{low-frequency } I_c / I_b$$

$$\text{pole at } \omega_\beta = \frac{1}{r_\pi (C_\pi + C_\mu)}$$



At ω_T, $\beta(s) = 1$ by definition

$$\left| \frac{\beta_0}{1 + j\omega_T r_\pi (C_\pi + C_\mu)} \right| = \frac{\beta_0}{\sqrt{1 + \omega_T^2 r_\pi^2 (C_\pi + C_\mu)^2}} = 1$$

$$\omega_T = \frac{\sqrt{\beta_0^2 - 1}}{r_\pi (C_\pi + C_\mu)} \simeq \boxed{\frac{\beta_0}{r_\pi (C_\pi + C_\mu)} \doteq \omega_T}$$

$\beta_0 \rightarrow$ also called h_{fe0}

$\beta(s) \rightarrow$ " " $h_{fe}(s)$

$$\omega_t \cong \frac{\beta_0}{r_\pi (C_\pi + C_\mu)} = \frac{g_m \cancel{r_\pi}}{\cancel{r_\pi}(C_\pi + C_\mu)} = \frac{g_m}{C_\pi + C_\mu}$$

Note: book uses ω_T (f_T) for ω_t (f_t).

For transistor 2N2222A $\rightarrow$ $f_T \cong 300\,MHz$.

* ——————— *

P. 7.39 $\quad I_{CQ} = 0.5\,mA$, $C_\mu = 0.15\,pF$, $f_T = 5\,GHz$, $\beta_0 = 150$

Find C_π, f_β

$$2\pi(5)10^9 = \cancel{20\pi}\ 10\pi \times 10^9 = \frac{g_m}{C_\pi + 0.15\,pF}$$

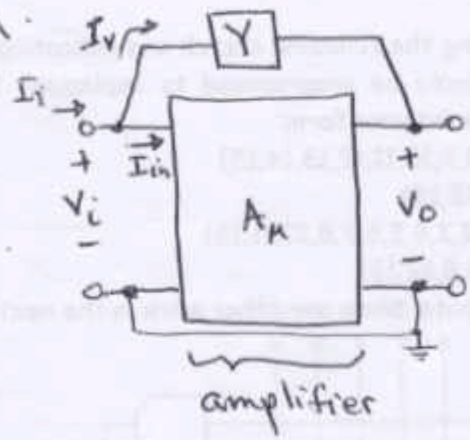
$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.5\,mA}{25\,mV} = 20\,mA/V$$

$$C_\pi = \frac{20\,mA/V}{\cancel{20\pi}\ 10\pi \times 10^9} - 0.15\,pF = \boxed{\cancel{0.168}\ 0.49\,pF = C_\pi}$$

$$f_\beta \cong f_t/\beta_0 = \frac{5000\,MHz}{150} = \boxed{33.3\,MHz = f_\beta}$$

Miller's effect

Consider:



Y = admittance

if component is a cap $Y = SC$

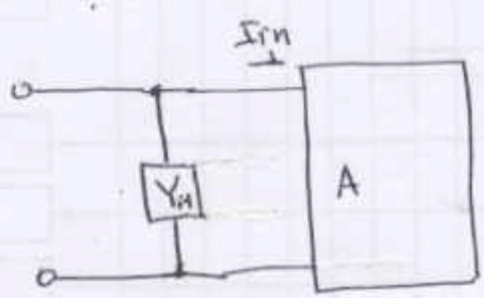
A_μ = amplifier's gain

Approx. → assume A_μ does not depend on Y

Then $I_i = I_{in} + I_Y$

$$I_Y \simeq Y(V_i - V_o) = Y(1 - A_\mu)V_i$$

$$I_i = I_{in} + Y(1 - A_\mu)V_i = I_{in} + Y_M V_i$$



$$Y_M = (1 - A)Y$$

If $Y = SC_\mu$

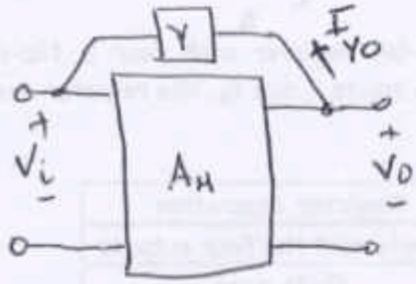
$$Y_M = \underbrace{(1 - A_M)C_\mu}\, S$$

C_M → augmented C_μ; if $A_\mu \ll -1$ (large & neg like in the CE)

For $A_M \ll -1$

C_μ → looks like a much larger capacitor from the input port.

Output port

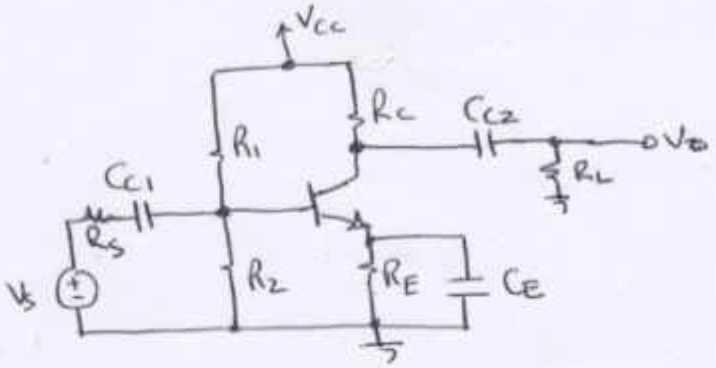


$$I_{Yo} = Y\left(V_o - V_i\right)$$
$$= Y\left(V_o - \frac{V_o}{A_M}\right)$$
$$= Y\left(1 - \frac{1}{A_M}\right) V_o$$
$$Y_o = \frac{I_{Yo}}{V_o} = Y\left(1 - 1/A_M\right)$$

if $A_M \ll -1$

$$\boxed{Y_o \doteq Y}$$

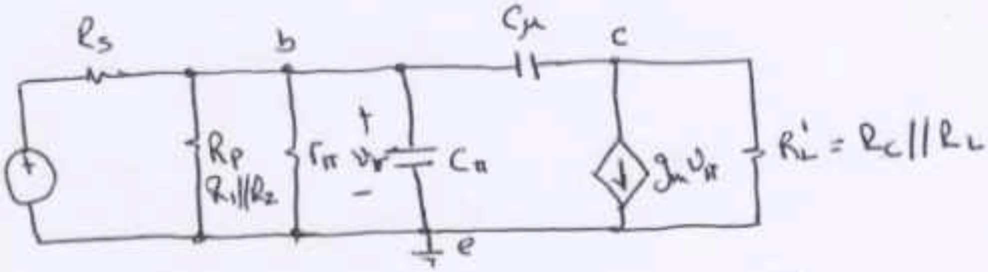
Common-emitter



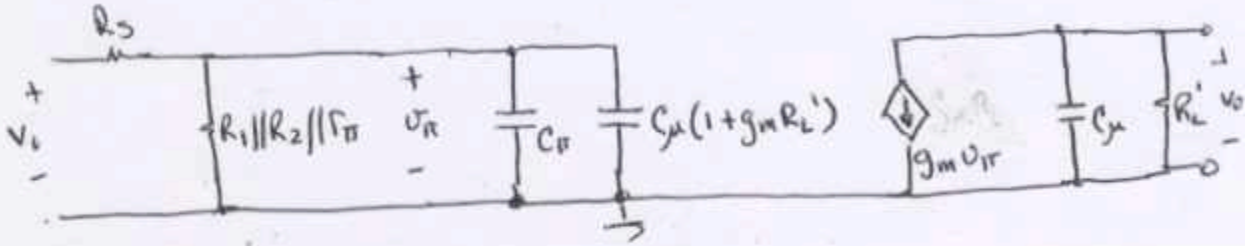
At high-freqs.

C_{C1}, C_{C2} & C_E are shorts

$V_{CC} \to$ ground



$$A_M = \frac{v_c}{v_b} \text{ (ignoring } C_\mu\text{)} = -g_m R_L'$$



In the input part of the circuit, we see an equivalent capacitance

$$C_{eq} = C_\pi + C_\mu(1 + g_m R_L')$$

Notice that, according our analysis of the RC circuits in lecture 1, this capacitance produces a pole at

$$\omega_{H1} = \frac{1}{(R_S || r_\pi || R_1 || R_2)\, C_{eq}}$$

The output segment also exhibits a pole at

$$\omega_{H2} = \frac{1}{C_\mu R_L'}$$

Because typically R_S is smaller than R_L', and also $C_{eq} \gg C_\mu$, $\omega_{H1} \ll \omega_{H2}$ and ω_{H1} dominates the high-frequency response of the CE.

Example $R_1 || R_2 = 200K\Omega$, $R_S = 5K\Omega$, $r_\pi = 2.6K\Omega$

$R_C || R_L = R_L' = 2K\Omega$, $C_\pi = 4pF$, $C_\mu = 0.2pF$

and $g_m = 38.5\ mA/V$.

$$C_M = (1 + g_m R_L')C_\mu = 78\, C_\mu = 15.4\ pF$$

equivalent ~~cap.~~ at base $= C_{eq} = 19.4\ pF$

$$\omega_{H1} = \frac{1}{(5K || 2.5K || 200K)(19.4pF)} = \frac{1}{(1.65K\Omega)(19.4pF)}$$

$$= \boxed{31.2\ \cancel{MHz}}\ \text{Mrps}$$

$$\omega_{H2} = \frac{1}{(2K\Omega)(.2pF)} = 2.5\ \cancel{GHz}\ \text{Grps}$$

Since ω_{H2} is larger than ω_{H1} by more than a decade, ω_{H1} dominates the high-freq. response; it is the <u>dominant</u> high-freq. pole.

If there is no dominant high-frequency pole because two or more poles are closer than one decade, we can estimate the high-frequency pole using

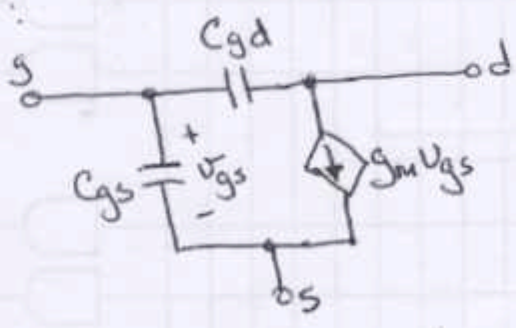
$$f_{h_{eff}} \simeq \frac{1}{\sum_{i=1}^{n} \frac{1}{f_i}}$$

<u>example</u>: 3 poles at 10MHz, 20MHz and 30MHz produce an "effective" h-f. pole at

$$f_h = \frac{1}{\frac{1}{10} + \frac{1}{20} + \frac{1}{30}} \cong 5.5\,\text{MHz}$$

Field-effect Transistors.

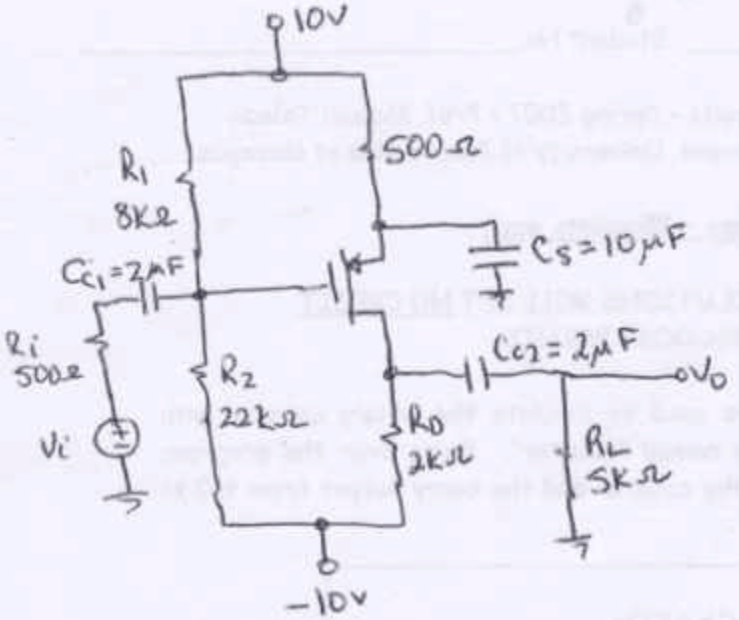
HF model:



$$f_T = \frac{g_m}{2\pi (C_{gs} + C_{gd})}$$

Miller's theorem applies to MOSFET circuits in the same way than it applies to BJT circuits.

Example : Problem 7.58



$V_{TP} = -2\,V$

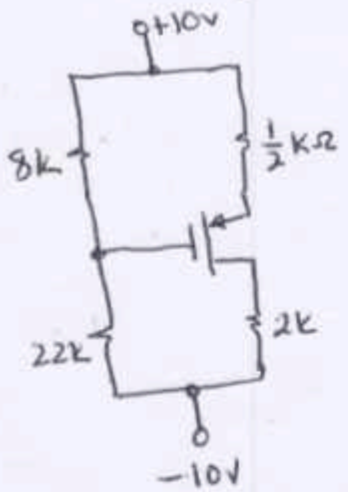
$K_p = 1\,mA/V^2$

$\lambda = 0$

$C_{gs} = 15\,pF$

$C_{gd} = 3\,pF$

D.C. analysis



$$V_G = -10 + \frac{22}{30}\,20 = 4.67\,V$$

$$I_D = K_p\left(V_{SG} + V_{TP}\right)^2 = \frac{10 - V_{SG} - 4.67}{\frac{1}{2}\,k\Omega}$$

$$\left(\tfrac{1}{2}\,V^{-1}\right)\left(V_{SG}^2 - 4V_{SG} + 4\right) = 5.33 - V_{SG}$$

$$V_{SG}^2 - 2V_{SG} - 6.67 = 0$$

$$V_{SG} = \underline{\underline{3.77\,V}}\ ,\ -1.77\,V$$

$$g_m = 2K_p\left(V_{SG} - 2V\right) = 2\,mA/V^2\,(3.77 - 2)$$

$$\boxed{g_m = 3.54\,mA/V}$$

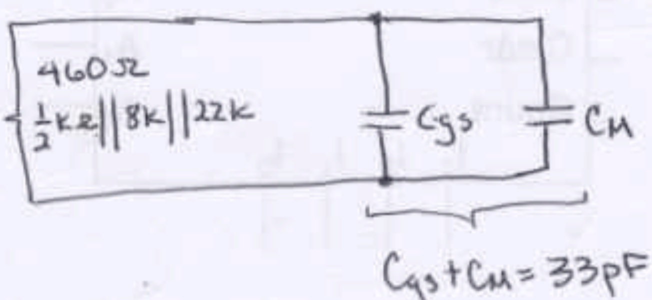
midband gain

$$A_{MB} = -\frac{22||8}{22||8 + .5}\left(3.54\,\frac{mA}{V}\right)\left(2k\,||\,5k\right)$$

$$= -\frac{5.87}{6.37}(5) = \boxed{-4.6\,V/V \;= A_{MB}}$$

Miller gain $= A_M = -g_m R_L' = -5\,V/V$

$$C_M = C_{gd}\,(1+5) = 3pF\,(6) = \boxed{18pF \;= \text{Miller's cap.}}$$



$$\omega_{H1} = \frac{1}{33pF \times 460\Omega}$$

$$\boxed{\omega_{H1} = 65.8\ Mrps}$$

10.5 MHz

The other pole is at

$$\omega_{H2} = \frac{1}{(5k||2k)\,3pF} = 233\ Mrps = 37\ MHz$$

No dominant pole. Effective pole at $\omega_H \cong \underline{8.2\ MHz}$

We can also calculate the <u>low-freq. poles</u>.

$$Req_{CC1} = \tfrac{1}{2}k\Omega + 8k||22k = 6.37k\Omega \Rightarrow \omega_{LP1} = \frac{1}{2\mu F \times 6.37k\Omega}$$

$$= 78.5\ rps = 12.5\ Hz$$

$$Req_{CC2} = 7k\Omega\ ;\ \omega_{LP2} = \frac{1}{2\mu F \times 7k\Omega} = 71.4\ rps = 11.4\ Hz$$

$$Req_{CS} = 500\Omega\,||\,\frac{1}{g_m} = 500||282\Omega = 180.3\Omega\ ;\ \omega_{LPS} = \frac{1}{(10\mu F)(180.3\Omega)}$$

$$= 555\ rps = 88.3\ Hz$$

The zero due to the bypass cap. is located at

$$\omega_z = \frac{1}{10\mu F \times 500\Omega} = 200 \text{ rps} = 31.8\,Hz$$

We get an effective low-freq. pole at

$$\omega_L \sim \sum \omega_i = 12.5 + 11.4 + 88.3 - 31.8\,Hz$$
$$\approx 80.4\,Hz$$