Example: The response of an op-amp can be approximated with a dominant pole frequency f_1 and a high-frequency pole f_2 to account for higher order roots. Assuming $a_0 = 10^5 V/V$, $f_1 = 10Hz$, and $\beta = 1V/V$,

a) find the phase margin ϕ_m if $f_2 = 1MHz$.

ANSWER: The loop gain at frequency f is given by

$$\frac{10^5}{\sqrt{1+(f/10)^2}\sqrt{1+(f/10^6)^2}}$$

We need to determine the frequency f_{0dB} at which the gain equals 1. Observing that $f_{0dB} >> 10$, we can approximate the above equation, and after re-arranging, obtain

$$10^5 = \frac{f_{0dB}}{10}\sqrt{1+\left(\frac{f_{0dB}}{10^6}\right)^2}$$

which can be solved to get $\boxed{f_{0dB} = 7.86 \times 10^5 = 786kHz}$. The phase at this frequency is

$$\phi = -\arctan(78600) - \arctan(.786) \simeq -90^\circ - 38.2^\circ = -128.2^\circ$$

$$\boxed{\phi_m = 180^\circ - 128.2^\circ = 51.8^\circ}$$

b) Find the gain margin G_M if $f_2 = 1MHz$ is a double pole instead of a single pole.

ANSWER: Solve

$$\begin{aligned} \phi &= -\arctan(f_{-180}/10) - 2\arctan(f_{-180}/10^6) \\ &\simeq -90^\circ - 2\arctan(f_{-180}/10^6) = -180^\circ \\ f_{-180} &= 10^6\tan(45^\circ) = 10^6 Hz \end{aligned}$$

Observe that

- a double-pole is used for f_2 because otherwise the phase reaches 180° only asymptotically, and
- geometrical insight could have been used to find out that $\phi = -180^\circ$ at f_2 since at this frequency the first pole contributes -90° and the each of the second poles contributes 45°.

Now find the gain at this frequency

$$A(10^6) = \frac{10^5}{\sqrt{1+(10^5)^2}\,(1+(1)^2)} \simeq \frac{10^5}{10^5(2)} = 0.5 = -6dB$$

So $\boxed{G_M = 6dB}$.

This result can also be determined, without using algebra, by observing that (1) f_2 is 5 decades away from f_1, (2) the magnitude bode plot have a slope of $-20dB/dec$ between the f_1 and f_2 and the gain is reduced by $100dB = 10^5 V/V$, (3) the gain at the second pole would thus be $0dB$, but this figure needs to be corrected by subtracting $6dB$'s because the actual magnitude plot is $3dB$ below the asymptotic at the pole frequency, and there are two poles at f_2.

c) Find f_2 for $\phi_m = 45°$.[1]

ANSWER: Assuming the new $f_2 >> 10Hz$,

$$\phi = -135° = -90° - \arctan\left(f_{0dB}/f_2\right)$$

which indicates that $f_{0dB} = f_2$. The magnitude equation becomes

$$1 \simeq \frac{10^5}{\frac{f_2}{10}\sqrt{2}} \Rightarrow f_2 = \frac{10^6}{\sqrt{2}} = \boxed{707kHz}$$

d) Find f_2 for $\phi_m = 60°$.

ANSWER: Assuming the new $f_2 >> 10Hz$,

$$\phi = -120° = -90° - \arctan\left(f_{0dB}/f_2\right)$$

$$\frac{f_{0dB}}{f_2} = \tan\left(30°\right) = 0.577$$

The magnitude equation becomes

$$1 \simeq \frac{10^5}{\frac{f_{0dB}}{10}\sqrt{1+0.577^2}} \Rightarrow f_{0dB} = \frac{10^6}{1.1547} = 866kHz$$

$$f_2 = 866kHz/.577 = \boxed{1.5MHz}$$

[1] f_2 is a single pole again.