Feedback Amplifiers

INEL 4202 - Electronics II - Fall 2012

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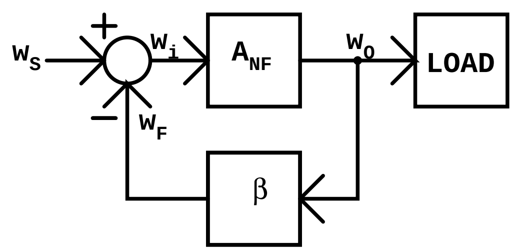
1 Basic Concepts



Signals (w) can be a voltages or currents.
Definitions:

- The output signal is w_O.
- The feedback network, β is normaly pasive resistor two-port network.
- Fundamental feedback equation:

$$w_O = A_{NF} w_i = A_{NF}(w_S - \beta w_O)$$

$$w_O = \frac{A_{NF}}{1 + \beta A_{NF}} w_S = A_F w_S = \frac{A_{NF}}{D} \tag{1}$$

- A_{NF} is the non-feedback gain, but **must include loading due to the feedback network**.
- The *loop-gain or return ratio* is defined as the product βA_{NF}
- The quantity D is called **improvement factor** or **return difference**.
- For $A_{NF} \gg 1$,

$$A_F \approx \frac{1}{\beta}$$

  Since β network is sometimes implemented as a resistor voltage or current divider,

  - when the loop-gain is much larger than unity the feedback amplifier gain is independent of amplifier params
  - resistance errors tend to cancel.
  - variations with temperature cancel

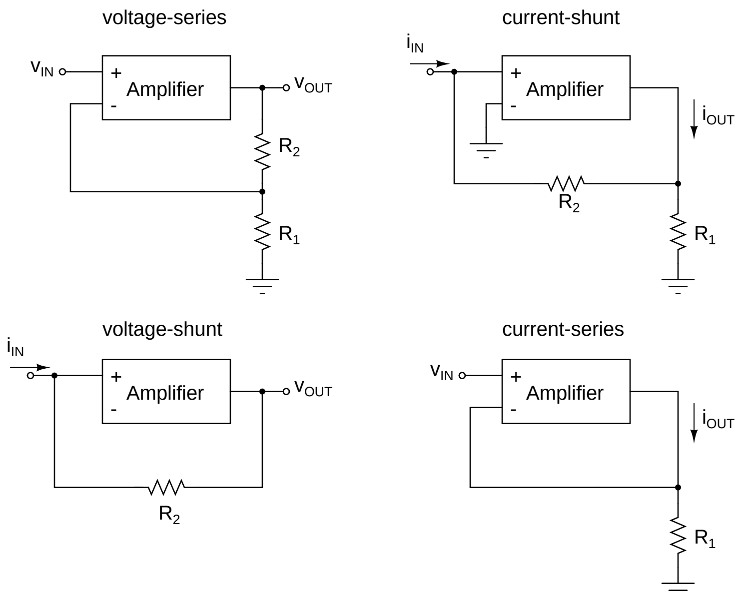


Figure 1: Four feedback topologies.

Sedra's naming	series-shunt	series-series	shunt-series	shunt-shunt
Handout naming	voltage-series	current-series	current-shunt	voltage-shunt
Output (sampled) signal	voltage	current	current	voltage
Input (feedback) signal	voltage	voltage	current	current
model	h	z	g	y
β	$\frac{v_1}{v_2}\Big\vert_{i_1=0}$	$\frac{v_1}{i_2}\Big\vert_{i_1=0}$	$\frac{i_1}{i_2}\Big\vert_{v_1=0}$	$\frac{i_1}{v_2}\Big\vert_{v_1=0}$
A_f	$\frac{A_V}{1+\beta A_V}$	$\frac{G_M}{1+\beta G_M}$	$\frac{A_I}{1+\beta A_I}$	$\frac{R_M}{1+\beta R_M}$
R_{if}	$R_i(1+\beta A_V)$	$R_i(1+\beta G_M)$	$\frac{R_i}{1+\beta A_I}$	$\frac{R_i}{1+\beta R_M}$
R_{of}	$\frac{R_o}{1+\beta A_V}$	$R_o(1+\beta G_M)$	$R_o(1+\beta A_I)$	$\frac{R_o}{1+\beta R_M}$

Table 1: Feedback amplifier formulae.

2 Feedback Topologies

There are four feedback topologies, identified by the output signal being sampled (voltage or current) and the signal subtracted at the input.

Examples of the four topologies are shown in figure 1.

2.1 Summary of Feedback Formulae

For each topology, a different gain must be used when applying the basic feedback formulas. These are summarized in tables 1 and 2, which are equivalent to the textbook's table 8.1.

2.2 Analysis Method

The following steps can be followed to apply the feedback method to analyze an amplifier.

1. Identify the type of feedback being used.
2. Draw a diagram of the feedback network.
3. Find the R_{11}, R_{22} and β parameters that correspond to the type of feedback.

Sedra's naming	series-shunt	series-series	shunt-series	shunt-shunt
model	h	z	g	y
input source	Thevenin	Thevenin	Norton	Norton
output source	Norton	Thevenin	Thevenin	Norton
R_{11}	$\frac{v_1}{i_1}\|_{v_2=0}$	$\frac{v_1}{i_1}\|_{i_2=0}$	$\frac{v_1}{i_1}\|_{i_2=0}$	$\frac{v_1}{i_1}\|_{v_2=0}$
β	$\frac{v_1}{v_2}\|_{i_1=0}$	$\frac{v_1}{i_2}\|_{i_1=0}$	$\frac{i_1}{i_2}\|_{v_1=0}$	$\frac{i_1}{v_2}\|_{v_1=0}$
R_{22}	$\frac{v_2}{i_2}\|_{i_1=0}$	$\frac{v_2}{i_2}\|_{i_1=0}$	$\frac{v_2}{i_2}\|_{v_1=0}$	$\frac{v_2}{i_2}\|_{v_1=0}$

Table 2: Summary of textbook feedback formulae.

4. Draw a diagram of the non-feedback amplifier, including R_{11} and R_{22}, R_{sig} (the source's Thevenin or Norton resistance) and R_{LOAD}. Assume caps are shorts. Remember to use the correct type of input source (current source for current-mixing topology, and voltage source for voltage mixing topology).

5. Find the non-feedback amplifier gain that correspond to the type of feedback being used (i.e. voltage gain $A_v = v_{out}/v_{in}$, current gain $A_i = i_{out}/i_{in}$, trans-conductance $G_M = i_{out}/v_{in}$ or trans-resistance $R_M = v_{out}/i_{in}$).

6. Find the non-feedback amplifier input and output resistance, R_i and R_o.

7. Find the feedback amplifier gain, input and output resistance.

8. Find the feedback amplifier voltage and current gains, $A_v = \frac{v_o}{v_s}$ and $A_i = \frac{i_o}{i_s}$, respectively, where $v_s = R_{sig} i_s$ and R_{sig} represents the source's Thevenin/Norton resistance.

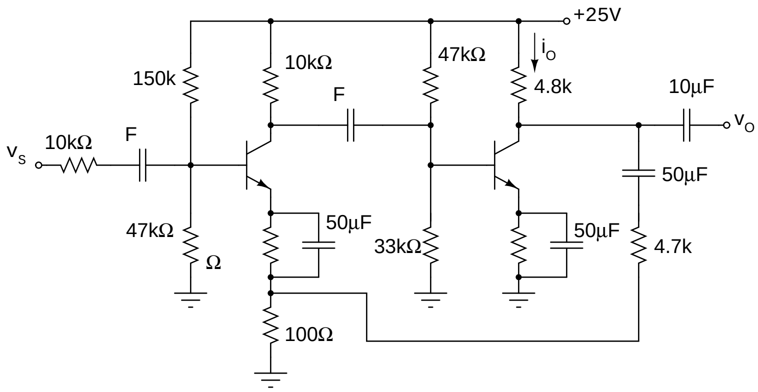
9. If the problem asks for the resistance seen by the load, subtract R_{LOAD} from R_{of}. For current-sampling topologies this generally means to subtract the resistance in parallel (i.e. $R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_{LOAD}}}$ where R_{out} is the resistance "seen" by the load.)

10. If the problem asks for the resistance seen by the load, subtract R_{LOAD} from R_{of}. For current-sampling topologies this generally means to subtract the resistance in parallel (i.e. $R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_{LOAD}}}$ where R_{out} is the resistance "seen" by the load. For voltage-sampling configurations, generally $R_{out} = R_{of} - R_{LOAD}$.

11. If the problem asks for the resistance seen by the source and its resistance, subtract R_{sig} from R_{if}. For current-mixing topologies this generally means to subtract the resistance in parallel (i.e. $R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_{sig}}}$ where R_{in} is the resistance "seen" by the source and its Norton resistance. For voltage-mixing configurations, generally $R_{in} = R_{if} - R_{sig}$.

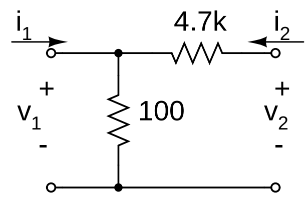
The following four circuits are examples of feedback amplifiers that use discrete components. Use $r_\pi = 1100\Omega$ and $\beta = 50$.

2.2.1 Amplifier 1



ANSWER:

1. Feedback is voltage-sampling, voltage-mixing.
2. The feedback network is:



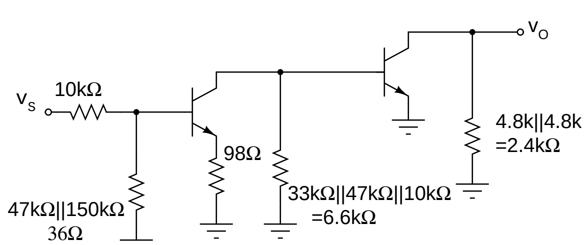
3. For voltage-sampling, voltage-mixing,

$$R_{11} = \frac{v_1}{i_1} \mid_{v_2=0} = 4.7k\Omega \mid\mid 100\Omega = 98\Omega$$

$$R_{22} = \frac{v_2}{i_2} \mid_{i_1=0} = 4.7k\Omega + 100\Omega = 4.8k\Omega$$

$$\beta = \frac{v_1}{v_2} \mid_{i_1=0} = \frac{100\Omega}{4.7k\Omega + 100\Omega} = \frac{1}{48} V/V$$

4. The non-feedback amplifier is:



5. The voltage gain is

$$\begin{aligned} A_v &= \frac{10k\Omega}{10k\Omega + R_{in}} \times A_{v1} \times A_{v2} \\ A_{v2} &= -g_{m2}R_{c2} = -\frac{50}{1100\Omega} 2.4k\Omega = -109.1V/V \\ R_{c1} &= 6.6k\Omega \mid\mid r_{\pi 2} = 6.6k\Omega \mid\mid 1.1k\Omega = 943\Omega \end{aligned}$$

$$
\begin{aligned}
A_{v1} &= -\frac{g_{m1}R_{c1}}{1+g_{m1}R_{e1}} \\
&= -\frac{\frac{50}{1100\Omega}943\Omega}{1+\frac{50}{1100\Omega}98\Omega} \\
&= -7.86V/V \\
R_{in} &= 36k\Omega \;||\; (r_{\pi 1} + (h_{fe1}+1)R_{e1}) \\
&= 36k\Omega \;||\; (1.1k\Omega + 51 \times 98\Omega) \\
&= 5214\Omega \\
A_v &= \frac{5214}{15214} \times -7.86 \times -109.1V/V = 294V/V
\end{aligned}
$$

6.

$$
\begin{aligned}
R_i &= 10k\Omega + R_{in} = 15214\Omega \\
R_o &\approx 2.4k\Omega
\end{aligned}
$$

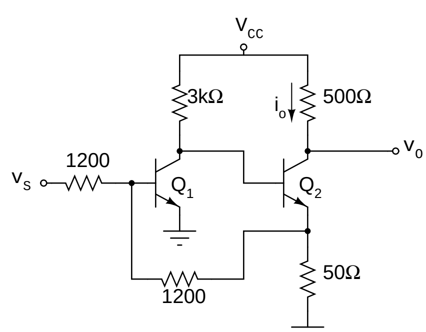
7.

$$
\begin{aligned}
A_{vf} &= \frac{A_v}{1+\beta A_v} \\
&= \frac{294}{1+\frac{1}{48}294} = \frac{294}{7.12} = 41.3V/V \\
R_{if} &= R_i(1+\beta A_v) = 7.12 \times 15214\Omega = 108.3k\Omega \\
R_{of} &= \frac{R_o}{1+\beta A_v} = \frac{2.4k\Omega}{7.12} = 337\Omega
\end{aligned}
$$

8. The voltage gain has been found. The current gain can be found from the voltage gain by observing that $i_o = -v_o/4.7k\Omega$, so that

$$
\begin{aligned}
A_{if} &= \frac{i_o}{i_s} \\
&= -\frac{v_o/4.7k\Omega}{v_s/10k\Omega} \\
&= -\frac{10}{4.7}A_{vf} \\
&= -87.9A/A
\end{aligned}
$$

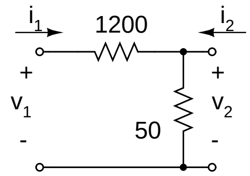
2.2.2 Amplifier 2



ANSWER:

1. The feedback type is current-sampling, current mixing.

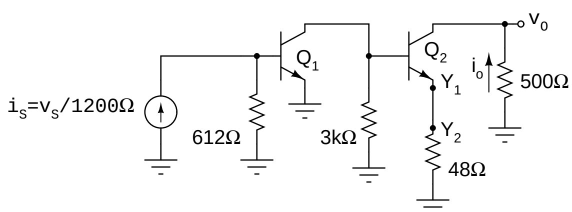
2. The feedback network is:



3.

$$
\begin{aligned}
R_{11} &= \frac{v_1}{i_1}\,|_{i_2=0} = 1250\Omega \\
R_{22} &= \frac{v_2}{i_2}\,|_{v_1=0} = 1200\Omega \,||\, 50\Omega = 48\Omega \\
\beta &= \frac{i_1}{i_2}\,|_{v_1=0} = -\frac{50}{1250} = -\frac{1}{25} A/A
\end{aligned}
$$

4. The small-signal equivalent non-feedback amplifier is:



5. For this amplifier the non-feedback gain is $A_i = \frac{i_o}{i_s}$. Observe that $i_o = i_{c2} = h_{fe} i_{b2}$, Using $R_{in2} = r_{\pi 2} + (h_{fe}+1)R_{e2} = 1.1k\Omega + 51 \times 48\Omega = 3548\Omega$, a current divider on the base of Q_2 yields

$$\frac{i_{b2}}{i_{c1}} = -\frac{3000}{6548} = -0.46$$

where the negative sign accounts for the fact that current flows into the collector. Similarly, for Q_1,

$$i_{c1} = 50 i_{b1}$$

Another current divider at the input gives

$$\frac{i_{b1}}{i_s} = \frac{612}{1712} = 0.36$$

Thus

$$
\begin{aligned}
A_i &= \frac{i_o}{i_s} \\
&= 0.36 \times 50 \times (-0.46) \times 50 \\
&= -414 A/A
\end{aligned}
$$

6. The input resistance is

$$R_i = 1.1k\Omega \,||\, 612\Omega = 393.2\Omega$$

The output resistance that should be used in the formulae is the resistance seen by an ideal load (a short) at the point where the output current is being measured, i.e. by the piece of wire between Y_1 and Y_2.Thus

$$R_o \approx 48\Omega + \frac{1.1k\Omega + 3k\Omega}{51} = 128.4\Omega$$

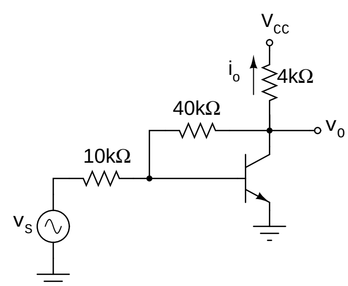
7. Now we can apply the feedback formulae:

$$\begin{aligned}
A_{if} &= \frac{A_i}{1+\beta A_i} \\
&= \frac{-414A/A}{1+\frac{1}{25}414} \\
&= \frac{-414A/A}{17.56} = -23.6A/A \\
R_{if} &= \frac{R_i}{1+\beta A_i} \\
&= \frac{393.2\Omega}{17.56} = 22.4\Omega \\
R_{of} &= R_o(1+\beta A_i) = 17.56 \times 128.4\Omega \\
&= 2255\Omega
\end{aligned}$$

8. The current gain has been found. To find the voltage gain, use $v_o = -500\Omega \times i_o$ and $v_s = 1200\Omega \times i_s$, so that

$$A_{vf} = -\frac{500}{1200} A_{if} = 9.8V/V$$

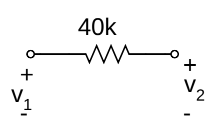
2.2.3 Amplifier 3



ANSWER:

1. The feedback type is voltage-sampling, current-mixing.

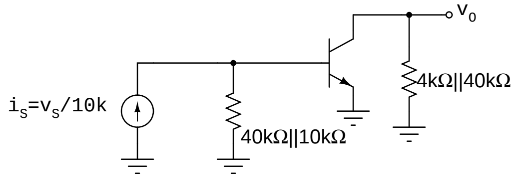
2. The feedback network is just the $40k\Omega$ resistor,



3. The feedback parameters are:

$$\begin{aligned}
R_{11} &= \frac{v_1}{i_1} |_{v_2=0} = 40k\Omega \\
R_{22} &= \frac{v_2}{i_2} |_{v_1=0} = 40k\Omega \\
\beta &= \frac{i_1}{v_2} |_{v_1=0} = -\frac{1}{40k\Omega}
\end{aligned}$$

4. The small-signal equivalent circuit for the non-feedback amplifier is



5. For this amplifier the non-feedback gain is $R_M = \frac{v_o}{i_s}$. Observing that

$$v_o = -3636 \times i_c = -3636 \times 50 \times i_b$$

and that

$$i_b = \frac{8}{9.1} i_s$$

yields

$$R_M = -\frac{8}{9.1} \times 3636 \times 50 \approx -160k\Omega$$

6. The input resistance is

$$R_i = 1.1k\Omega \;||\; 8k\Omega = 967\Omega$$

The output resistance is just about 3636Ω.

7. Now we can apply the feedback formulae:

$$\begin{aligned}
R_{Mf} &= \frac{R_M}{1+\beta R_M} \\
&= \frac{-160k\Omega}{1+\frac{1}{40k\Omega}160k\Omega} \\
&= \frac{-160k\Omega}{5} = -32k\Omega \\
R_{if} &= \frac{R_i}{1+\beta R_M} \\
&= \frac{967\Omega}{5} = 193.4\Omega \\
R_{of} &= \frac{R_o}{1+\beta A_i} \\
&= 727\Omega
\end{aligned}$$

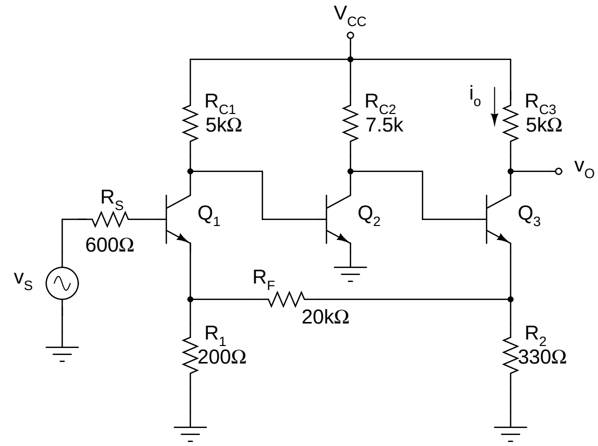
8. To find the voltage and current gains, use $v_o = 4k\Omega \times i_o$ and $v_s = 10k\Omega \times i_s$, so that

$$A_{vf} = \frac{R_{Mf}}{10k\Omega} = -3.2V/V$$

and

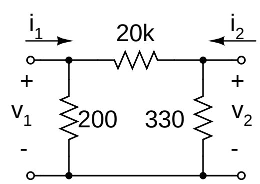
$$A_{if} = \frac{R_{Mf}}{4k\Omega} = -8A/A$$

2.2.4 Amplifier 4



ANSWER:

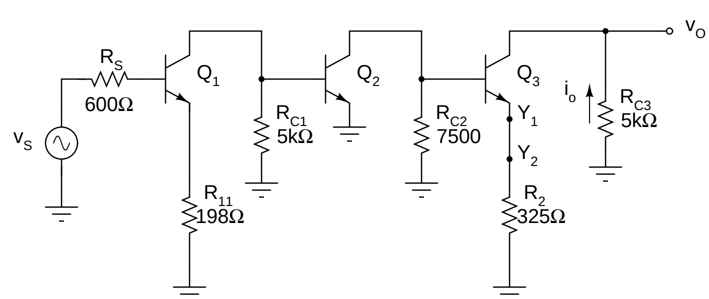
1. The feedback type is current-sampling, voltage-mixing.
2. The feedback network is



3. The feedback parameters are:

$$
\begin{aligned}
R_{11} &= \frac{v_1}{i_1}\,|_{i_2=0} = 200\Omega\ ||\ 20k\Omega \approx 198\Omega \\
R_{22} &= \frac{v_2}{i_2}\,|_{i_1=0} = 330\Omega\ ||\ 20k\Omega = \approx 325\Omega \\
\beta &= \frac{v_1}{i_2}\,|_{i_1=0} = -\frac{200\Omega \times 330\Omega}{20.53k\Omega} = 3.21V/A
\end{aligned}
$$

4. The small-signal equivalent circuit for the non-feedback amplifier is



5. For this amplifier the non-feedback gain is $G_M = \frac{i_o}{v_s}$. Observing that

$$i_o = i_{c3} = h_{fe} i_{b3} = 50 i_{b3}$$

To find the current gain i_o/i_{b1}, we can apply the current divider rule at the bases of Q_2 and Q_3.

$$\begin{aligned}
R_{in3} &= 1.1k\Omega + 51 \times 325\Omega = 17675\Omega \\
\frac{i_{b3}}{i_{c2}} &= -\frac{7500}{25175} = -0.3 \\
R_{in2} &= 1.1k\Omega \\
\frac{i_{b2}}{i_{c1}} &= -\frac{5000}{6100} = -0.82 \\
\frac{i_o}{i_{b1}} &= 50 \times (-0.82) \times 50 \times -0.3 \times 50 = 30750A/A
\end{aligned}$$

Now use

$$i_{b1} = \frac{v_s}{600\Omega + 1100\Omega + 51 \times 198} = \frac{v_s}{11.8k\Omega}$$

gives

$$G_M = \frac{i_o}{v_s} = 2.6A/V$$

6. The input resistance is

$$R_i = 600\Omega + 1100\Omega + 51 \times 198\Omega = 11.8k\Omega$$

The output resistance is that seen by the short between Y_1 and Y_2,

$$R_o = 325\Omega + \frac{1100\Omega + 7500\Omega}{51} = 494\Omega$$

7. Now we can apply the feedback formulae:

$$\begin{aligned}
G_{Mf} &= \frac{G_M}{1 + \beta G_M} \\
&= \frac{2.6A/V}{1 + 3.2V/A \times 2.6A/V} \\
&= \frac{2.6A/V}{9.3} = 0.28A/V \\
R_{if} &= \frac{R_i}{1 + \beta G_M} \\
&= 9.3 \times 11.8k\Omega = 109.7k\Omega \\
R_{of} &= 9.3 \times 494\Omega = 4.6k\Omega
\end{aligned}$$

8. To find the voltage and current gains, use $v_o = -5000\Omega \times i_o$ and $v_s = 600\Omega \times i_s$, so that

$$A_{vf} = -5000\Omega \times G_{Mf} = -1400V/V$$

and

$$A_{if} = 600\Omega \times G_{Mf} = 168A/A$$

3 Benefits of using feedback

Advantages of using feedback:

- Reduced sensitivity to parameters
- Increased bandwidth
- Reduced distortion
- Improved input and output resistance

Cost of using feedback:
reduced gain.

3.1 Feedback and Sensitivity

$$S_P^A = \frac{P}{A}\frac{dA}{dP}$$

For a feedback amplifier,

$$\begin{aligned}
S_P^{A_F} &= \frac{P}{A_F}\frac{dA_F}{dP} \\
&= \frac{P}{A_F}\frac{d\frac{A_{NF}}{1+\beta A_{NF}}}{dP} \\
&= \frac{P}{A_F}\frac{d}{dA_{NF}}\frac{A_{NF}}{1+\beta A_{NF}}\frac{dA_{NF}}{dp} \\
&= \frac{P}{A_F}\frac{dA_{NF}}{dp}\left(\frac{1}{1+\beta A_{NF}} - \frac{\beta A_{NF}}{(1+\beta A_{NF})^2}\right) \\
&= \frac{P}{A_{NF}}\frac{dA_{NF}}{dp}\left(1 - \frac{\beta A_{NF}}{1+\beta A_{NF}}\right) \\
&= \frac{P}{A_{NF}}\frac{dA_{NF}}{dp}\frac{1}{1+\beta A_{NF}} \\
&= S_P^{A_{NF}}\frac{1}{1+\beta A_{NF}}
\end{aligned}$$

Sensitivity to any parameter or factor is reduced by the improvement factor.

3.2 Feedback and Low-frequency Response

non-feedback amplifier gain: approximated by a zero and a dominant pole

Let the zero be at the origin. Then

$$A_{NF}(s) = A_{mid}\frac{s}{s+\omega_L}$$

The feedback gain becomes

$$\begin{aligned}
A_F(s) &= \frac{A_{mid}\frac{s}{s+\omega_L}}{1+\beta A_{mid}\frac{s}{s+\omega_L}} \\
&= \frac{A_{mid}s}{s+\omega_L+\beta A_{mid}s} \\
&= \frac{A_{mid}s}{s(1+\beta A_{mid})+\omega_L} \\
&= \frac{A_{mid}}{1+\beta A_{mid}} \times \frac{s}{s+\frac{\omega_L}{1+\beta A_{mid}}}
\end{aligned}$$

Low-frequency cut-off is reduced by an amount equal to the improvement factor.

3.3 Feedback and High-frequency Response

non-feedback amplifier high-frequency response is represented by a single pole,

$$A_{NF} = A_{mid}\frac{\omega_H}{s+\omega_H}$$

then

$$A_F(s) = \frac{A_{mid}\frac{\omega_H}{s+\omega_H}}{1+\beta A_{mid}\frac{\omega_H}{s+\omega_H}}$$

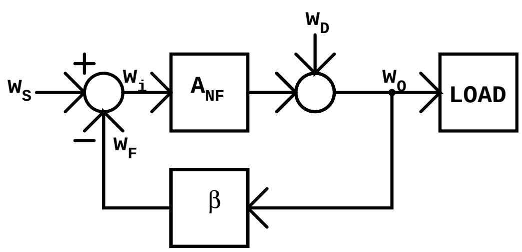
$$\begin{aligned} &= \frac{A_{mid}\omega_H}{s + \omega_H + \beta A_{mid}\omega_H} \\ &= \frac{A_{mid}\omega_H}{s + \omega_H(1 + \beta A_{mid})} \\ &= \frac{A_{mid}}{1 + \beta A_{mid}} \times \frac{\omega_H(1 + \beta A_{mid})}{s + \omega_H(1 + \beta A_{mid})} \end{aligned}$$

high-frequency cut-off: increased by the improvement factor.

3.4 Feedback and Distortion

Non-linearities: cause its input and output signals to differ in shape.

Nonlinearities + distortion: w_D



$$\begin{aligned} w_O &= w_D + A_{NF}w_i \\ &= w_D + A_{NF}(w_S - \beta w_O) \\ &= \frac{w_D}{1 + \beta A_{NF}} + \frac{A_{NF}}{1 + \beta A_{NF}} \end{aligned}$$

Non-linearities: are reduced by improvement factor

Noise: might not reduced because more stages are needed