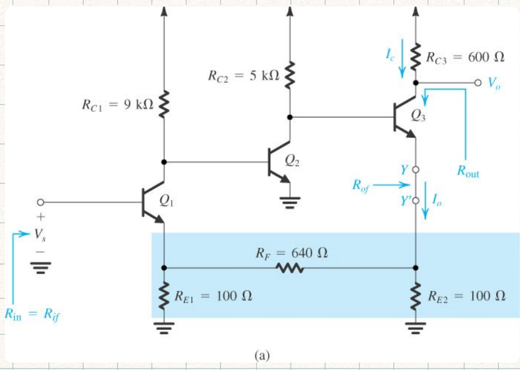
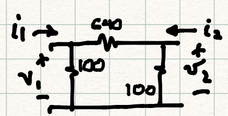
Feedback example



gm1 = 0.6mA/0.025V = 24mA/V
gm2 = 1mA/0.025V = 40mA/V
gm3 = 4mA/0.025V = 160mA/V
rπ1=100/24mA/V = 4.1kOhms
rπ2=100/40mA/V = 2.5kOhms
rπ3=100/160mA/V=625Ohms

IC1 = 0.6mA, IC2 = 1mA, IC3 = 4mA, hfe = 100, rO = ∞

Type of feedback: current-sampling, voltage-mixing (series-series)
Feedback network:

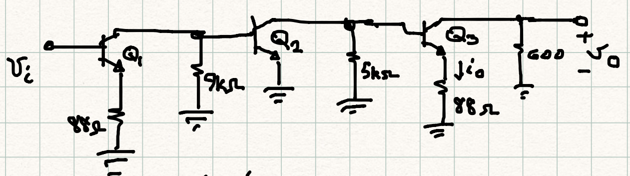


$$R_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = 100 \| 740 = \boxed{88\,\Omega}$$

$$R_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = \boxed{88\,\Omega}$$

$$\beta = \left.\frac{v_1}{i_2}\right|_{i_1=0} = (100\|740)\,\frac{100}{740} = \boxed{11.9\,\Omega}$$

Non-feedback amplifier:



non-feedback gain = transconductance = $i_o/v_i = G$

$$G = \left(\frac{v_{c1}}{v_i}\right)\left(\frac{v_{c2}}{v_{c1}}\right)\left(\frac{v_o}{v_{c2}}\right)\left(\frac{i_o}{v_o}\right)$$

$r_{\pi2} \| 9k\Omega$ $\qquad$ $5k \| [r_{\pi3} + (101)R_{22}] = 5K \| (625 + (101)88) = 3277\,\Omega$ $\qquad$ 600

$$G = \frac{+g_{m1} R_{c1}}{1 + g_{m1} R_{e1}} \times (+g_{m2} R_{c2}) \frac{+g_{m3} \cancel{R_{c3}}}{1 + g_{m3} R_{e3}} \left(\frac{+1}{\cancel{600}}\right)$$

$R_{11} = 88$ $\qquad$ $R_{22} = 88$

$$= \frac{(24\,\text{mA/V})(9k\Omega \| 4.1k\Omega)}{1 + (24\frac{\text{mA}}{\text{V}})(88\,\Omega)} \left(40\frac{\text{mA}}{\text{V}} \times 3277\right) \frac{160\,\text{mA/V}}{1 + (.16\frac{\text{A}}{\text{V}})(88\,\Omega)}$$

$$= (15.1)(131)(0.01061) \simeq 21\,\text{A/V}$$

$$G_f = \frac{21\,A/V}{1+(11.9)(21)} = \frac{21\,A/V}{250.76} = \boxed{83.8\,mA/V = \frac{i_o}{v_o}}$$

To find the voltage gain for the feedback amplifier, observe that

$$v_o = (-600\Omega)(i_o) \Rightarrow \frac{v_o}{v_i} = (-600\Omega)G_f = -600\Omega\left(83.8\,\frac{mA}{V}\right) = \boxed{50\,V/V}$$

Input resistance:

$$R_i = r_{\pi_1} + (101)88\Omega = 13k\Omega \Rightarrow R_{if} = (250.76)(13k\Omega) = \boxed{3.26M\Omega}$$

Output resistance:

$$R_o = \frac{625+5k}{101} + 88\Omega = 144\Omega \Rightarrow R_{of} = 144\Omega \times 250.76 = \boxed{36k\Omega}$$

Ro is the resistance seen by the ideal load, a short between points Y and Y' in the original diagram.

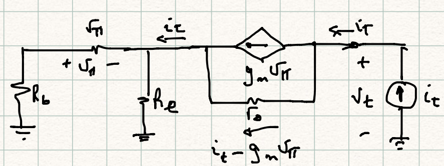
An estimate of Rout can be obtained interpreting Rof as a resistance in the emitter and using the formula

$$R_{out} \simeq r_o(1+g_{m_3}\overset{R_{of}}{R_e}) = 25k\Omega\left[1+\left(160\,\frac{mA}{V}\right)(36k\Omega)\right] = \boxed{144M\Omega}$$

Note: The book uses a different formula

$$R_{out} \simeq r_o\left(1+g_{m_3}\{R_{of}\,||\,r_{\pi_3}\}\right) = 25k\Omega\left(1+\left\{160\,\frac{mA}{V}\right\}\{625\,||\,36k\}\right) = \boxed{2.4M\Omega}$$

The above formula for Rout=ro(1+gm(Re)) is approximate. To obtain a more accurate estimate, consider the following analysis.



$$v_t = i_t\left(R_e\,||\,(r_\pi+R_b)\right) + r_o(i_t - g_m v_\pi)$$

$$v_\pi = -\frac{r_\pi}{r_\pi+R_b}\, i_t\left(R_e\,||\,(r_\pi+R_b)\right)$$

Let $R_A = R_e\,||\,(r_\pi + R_b)$

$$v_t = i_t R_A + i_t r_o + i_t r_o \frac{\beta R_A}{r_\pi + R_b}$$

$$R_{eq} = \frac{v_t}{i_t} = R_A + r_o\left(1 + \frac{\beta R_A}{r_\pi + R_b}\right)$$

$$\frac{\beta R_A}{r_\pi + R_b} = \frac{R_e \cancel{(r_\pi + R_b)}}{R_e + r_\pi + R_b} \quad \frac{1}{\cancel{r_\pi + R_b}}$$

$$\therefore R_{out} = R_A + r_o\left(1 + \frac{\beta R_E}{R_e + r_\pi + R_b}\right) \qquad \text{(eq. 1)}$$

If $r_\pi \gg R_e + R_b$, and $r_o \gg R_A$

$$R_{out} \simeq r_o\,(1 + g_m R_e$$

If this approximation does not hold, use the above eq. 1

Back to the feedback problem,

$$R_A = R_e \| (r_\pi + R_b) = 36k\Omega \| 5625\,\Omega = 4865\,\Omega$$

$$R_{out} = 4865\,\Omega + 25k\Omega\left(1 + \frac{100\,(36k)}{36k + 625 + 5k}\right)$$

$$= 4865\,\Omega + 25k\Omega\,(86.5) \simeq \boxed{2.1\,M\Omega}$$

in good agreement with the book's estimate.