# STABILITY

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## Stability

### Basics

- Basic feedback equation:

$$A_f(s) = \frac{a(s)}{1 + \beta(s)a(s)}$$

  Thus, feedback moves the poles of the amplifier's transfer function.

- Poles of $A_f$ are roots of $1+\beta a$. Thus, feedback moves the poles of the amplifier's transfer function.

- The idea is to determine information about the stability of $A_f$ from the loop gain $T(s) = \beta(s)a(s)$.

Nyquist Theorem

Let $\omega_{180^\circ}$ be the frequency at which the loop gain's phase angle is $-180^\circ$. If

$$| T(j\omega_{180^\circ}) | = | \beta(j\omega_{180^\circ}) A(j\omega_{180^\circ}) | > 1$$

then the amplifier is unstable. Otherwise, it is stable.

Nyquist theorem allows us to answer questions about the stability of $A_f$ by analyzing the loop gain $\beta A$.

## Phase and Gain Margin

- Gain margin: decibels below zero of $\mid T(j\omega_{180^\circ}) \mid$.

- Phase margin: degrees above $-180^\circ$ at the frequency $\omega_{0dB}$ at which $\mid T(j\omega_{0dB}) \mid = 1$, or 0 db.

$$\phi_m = 180 + \angle T(j\omega_{0dB})$$

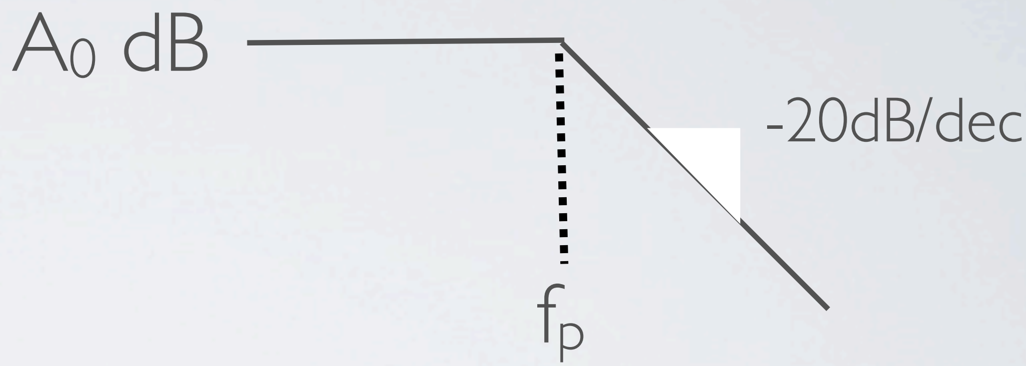
  Note that $\angle T(j\omega_{0dB})$ is usually negative.

- The amplifier is unstable if the gain and phase margins are negative. If the margins are positive or zero the amplifier is *stable* or *marginally stable*, respectively.

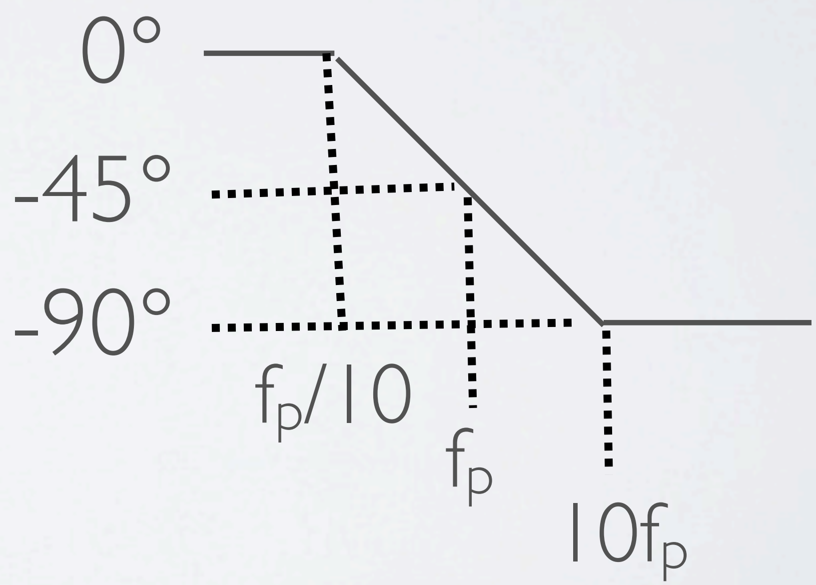
$A_{dB} = 20 \log A$

1 decada = 10*frec.
1 octava = 2*frec

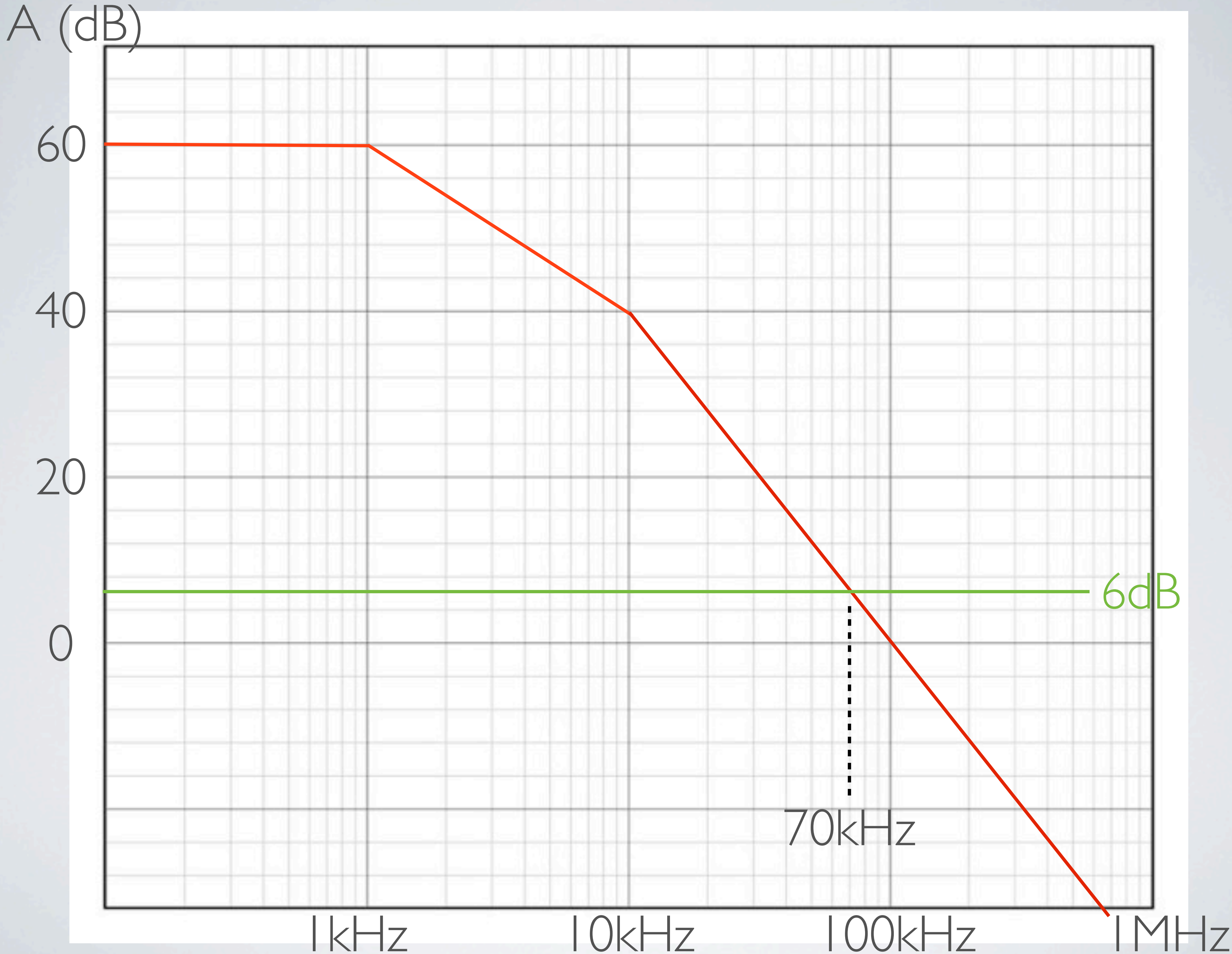
$$\frac{A_0}{1 + j\frac{f}{f_p}}$$




| $f/f_p$ | $\phi$ |
|---|---|
| 0.1 | 5.7° |
| 1 | 45° |
| 10 | 84.3° |



An amplifier with dc gain equal to 1000V/V will be used in a feedback configuration. The amplifier transfer function displays single poles at 1kHz and 10kHz. Find the phase margin $\Phi_m$ if the circuit will use a feedback network with $\beta = 1/2$.


A (dB)
60
40
20
0
6dB
70kHz
1kHz
10kHz
100kHz
1MHz

## Geometrical analysis

- Crossover at ~ 70kHz
- phase = -arctan(70) -arctan(7) = -171 degrees
- Phase margin = 9 degrees

Exact solution: express f in kHz and

$$\text{solve} \quad \frac{1000 \times 0.5}{\sqrt{1+f^2}\,\sqrt{1+\left(\frac{f}{10}\right)^2}} = 1$$

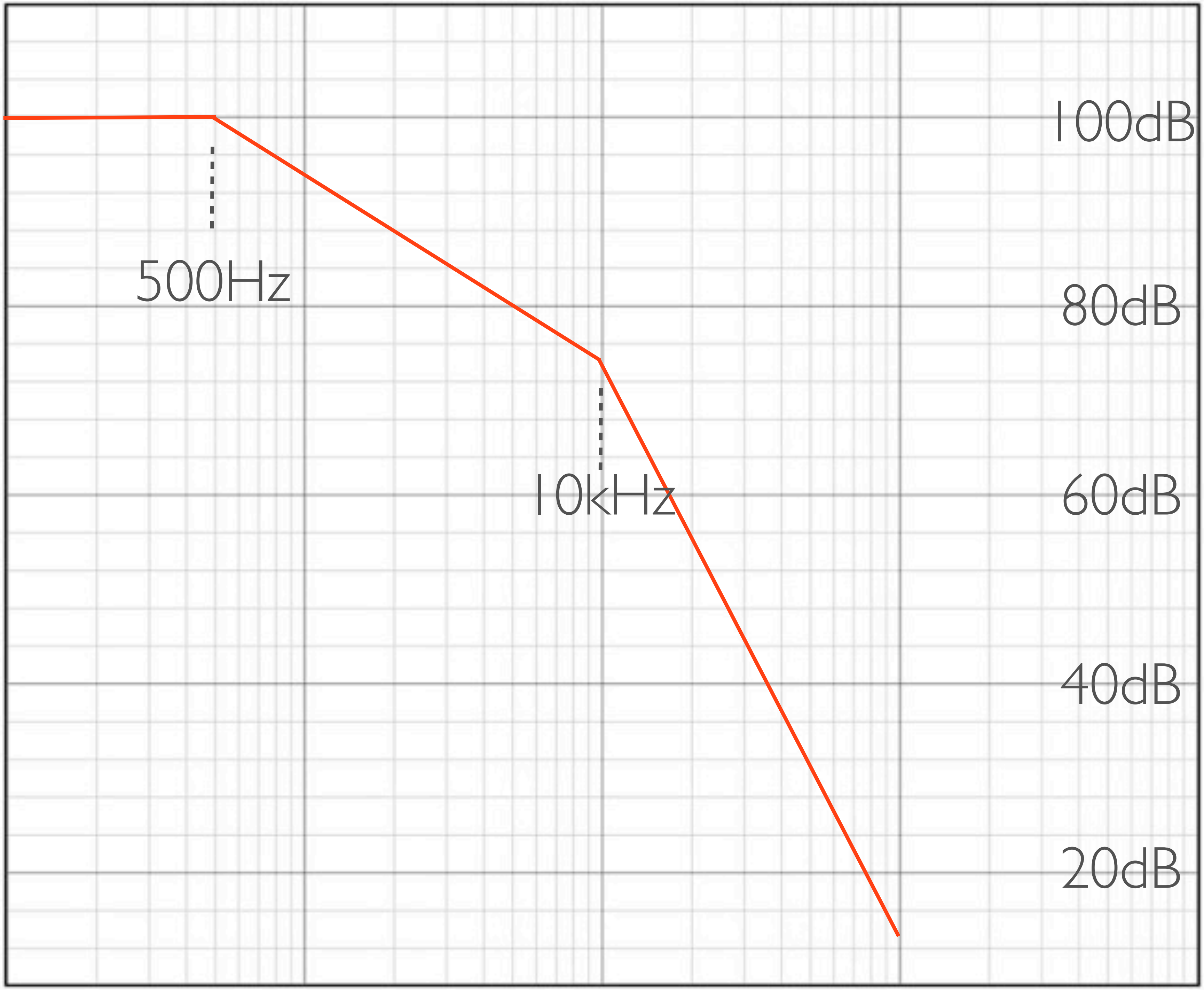
to get f = 70.35kHz. Then proceed as in geometrical analysis.

12.65 A 3-pole amplifier has a loop gain given by

$$T(f) = \frac{10^5 \times \beta}{\left(1 + j\frac{f}{5\times10^2}\right)\left(1 + j\frac{f}{10^4}\right)^2}$$

(a) determine the frequency $f_{180}$ at which the phase is -180 degrees. (b) At $f_{180}$ determine the value of β such that $|T(f_{180})| = 1$.

Extra: c) find the the value of β such that the phase margin is 45 degrees; (d) repeat for a phase margin of 60 degrees; (e) sketch the magnitude and phase bode plots of the amplifier's gain.


100dB
500Hz
80dB
10kHz
60dB
40dB
20dB

- Geometrical reasoning:
  - phase of A is about -180 degrees at the frequency of the double pole (-90 degrees from first pole plus 2 times -45 degrees from double pole), i.e. 10kHz
  - at 10kHz, gain magnitude shown in approximate bode plot is about 74dB, but we know that the real magnitude is about 6dB less (3 from each pole) so use 68dB = 2512V/V and beta = 1/2512 V/V
- Exact approach: express angle as pi radians and

| solve | $-\tan^{-1}\left(\frac{f}{0.5}\right) - 2\tan^{-1}\left(\frac{f}{10}\right) = -\pi$ |
|---|---|

to get f = 10488.1Hz. Then find gain at this frequency:

$$A = \frac{1}{\beta} = \frac{100\,000}{\left(1 + \left(\frac{10\,488.1}{10\,000}\right)^2\right)\sqrt{1 + \left(\frac{10\,488.1}{500}\right)^2}} = 2268\text{ V/V}$$

Notice that the value of *f* was estimated numerically.

Example 1: An op amp with $a_0 = 10^3$ V/V and two pole frequencies at $f_1 = 100$kHz and $f_2 = 2$MHz is connected as a unity-gain voltage follower. Find $\phi_m$.

ANSWER:

$$\text{solve} \quad \frac{1000}{\sqrt{1+\left(\frac{f}{100}\right)^2}\sqrt{1+\left(\frac{f}{2000}\right)^2}} = 1$$

to get f = 14.07MHz and pm = 180 - arctan(14.07/0.1) - arctan(14.07/2) = 8.5 degrees.

Example 2: An amplifier has 3 identical poles at a frequency $f_1$ and is placed in a negative-feedback loop with a frequency independent feedback factor β. Find an expression for $f_{-180°}$ as well as the corresponding value of T.

ANSWER: Phase given by each identical pole is 180/3 = 60 so $f_{-180} = f_p * \tan(60) = 1.732 f_p$. The magnitude of T is $(A_0*\beta)/(1 + 1.732^2)^{3/2} = (A_0*\beta)/8$

Example 3: (a) Verify that the circuit with loop gain $T_0 = 10^2$ and three pole frequencies $f_1 = 100$kHz, $f_2 = 1$MHz and $f_3 = 2$MHz is unstable. (b) Reduce $T_0$ so that $\phi_m$=45°. (c) repeat for $\phi_m$=60°.

$$\text{solve} \quad \frac{100}{\sqrt{1+(\frac{f}{0.1})^2}\sqrt{1+f^2}\sqrt{1+(\frac{f}{2})^2}} = 1$$

To get f = 2.42MHz, and

$$\phi = -\arctan(2.42/0.1) - \arctan(2.42/1) - \arctan(2.42/2)$$
$$= -87.6^\circ - 67.6^\circ - 50.4^\circ = -206^\circ$$

For a margin = 45 degrees, phase should be -135 degrees, which happens at the freq. f obtain from

$$\text{solve} \quad \tan^{-1}\left(\frac{f}{0.1}\right) + \tan^{-1}(f) + \tan^{-1}\left(\frac{f}{2}\right) = \pi \times \frac{135}{180}$$

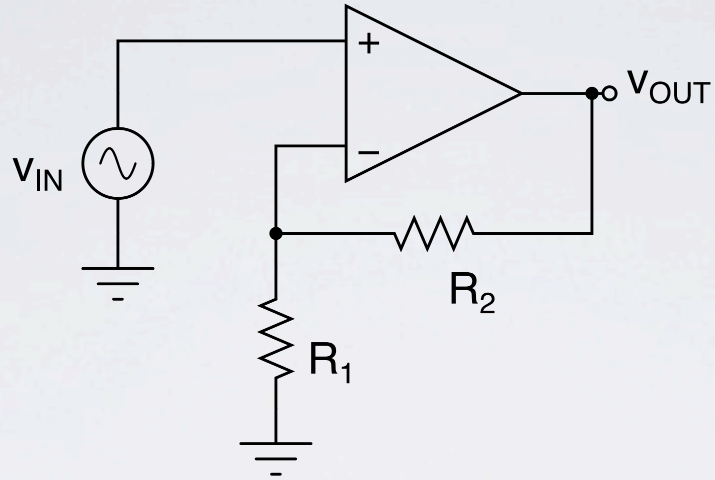
or f = 645kHz. The magnitude at this frequency is

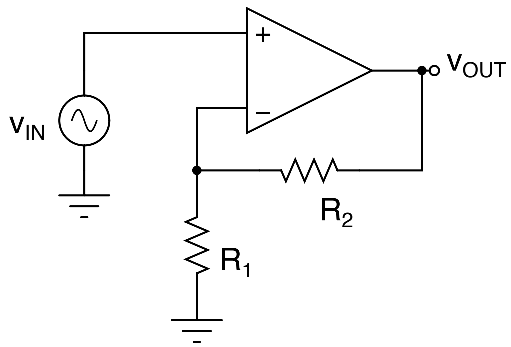
$$|T| = \frac{100}{\sqrt{1+(6.45)^2}\sqrt{1+(0.645)^2}\sqrt{1+(0.645/2)^2}} = 12.25$$

so the gain must be reduced by a factor of 12.25 to have a phase margin of 45 degrees.

For a phase margin of 60 degrees, repeat the procedure but use 120 degrees to find f.

III. (33 pts.) In the following amplifier, the opamp has an open-loop d.c. gain of $10^4$ and 4 poles at 1MHz, 10MHz, 10MHz and 200MHz. Find the smallest ratio $R_2/R_1$ for which the amplifier is stable.






At 10MHz the phase is:

-90 deg from pole at 1MHz

-45 deg from each pole at 10MHz

-0 from pole at 200MHz

So phase at 10MHz = -180 deg. and 1/β must be equal or larger to the gain at 10MHz for the amplifier to be stable

$$|A(10MHz)| = \frac{10^4}{\sqrt{1+\left(\frac{10}{1}\right)^2} \times \left(1+\left(\frac{10}{10}\right)^2\right) \times \sqrt{1+\left(\frac{10}{200}\right)^2}}$$

$$= \frac{10^4}{\sqrt{101} \times 2 \times 1} = 498V/V$$

$$\boxed{\frac{R_2}{R_1} \geq 497V/V}$$

An amplifier has a low-frequency gain of 200 and its transfer function has three poles at $1MHz$, $2MHz$ and $4MHz$.

(a) Sketch the bode plot.

(b) The amplifier is placed in a negative feedback loop with a feedback network with a frequency independent factor $\beta = 0.025$. Is the amplifier stable? Use your estimate of the phase or gain margin to justify your answer.

The response of an op amp can be approximated with a dominant pole frequency $f_1$ and a high-frequency pole $f_2$ to account for higher order roots. (a) Assuming $a_0 = 10^5$ V/V, $f_1 = 10$Hz, and $\beta = 1$ V/V, find the phase margin $\phi_m$ if $f_2 = 1$MHz. (b) Find $f_2$ for $\phi_m = 45°$ and for $\phi_m = 60°$.