The amplifier shown in the above sketch has a single coupling capacitor $C_C$. The transistor is build such that the capacitance $C_\pi$ is very small and can be neglected. Thus, the high frequency pole of the circuit is determined by the transistor’s capacitance $C_\mu$.

The drawing on the right displays the frequency response for the amplifier’s gain, $A = v_{out}/v_s$. You should neglect $r_o$ and $r_x$ in your work. You must apply the time-constant methods that we discussed in class, show all steps and justify your answers clearly.

Assuming that at the operating point $i_C = 0.5mA$ and $\beta = 75$, find

1. $R_C$.

   ANSWER: From the bode plot, the midband gain magnitude is $40dB = 100$. This must be equal to the common-emitter gain times the input loading factor. Using the formula sheet,

   $100 = \frac{\beta R_C}{r_\pi + (\beta + 1)R_e} \times \frac{20k\Omega \| [r_\pi + (\beta + 1)R_e]}{20k\Omega \| [r_\pi + (\beta + 1)R_e] + 10k\Omega}$

   Using $R_e = 100\Omega$, $r_\pi = \beta V_T/i_C = 75 \times 25mV/0.5mA = 3750\Omega$, and solving for $R_C$ yields

   $R_C = 36k\Omega$

2. $C_C$.

   ANSWER: From the bode plot, the low-frequency pole is at $1000Hz = 6280rps$. Since the coupling capacitor $C_c$ “sees” a resistance

   $R_{eq} = 10k\Omega + 20k\Omega \| [r_\pi + (\beta + 1)R_e] = 10k\Omega + 7.24k\Omega = 17.24k\Omega$

   Thus,

   $$C_c = \frac{1}{R_{eq}\omega_L} = \frac{1}{6280 \times 17.24k\Omega} = 9pF$$
3. $C_\mu$ if Miller’s Theorem can be applied and the output capacitor can be neglected. Use $R_C = 2k\Omega$, not the value you found in part (a).

ANSWER:

Miller’s gain would be

$$K = v_c/v_b = \frac{\beta R_C}{r_\pi + (\beta + 1)R_e} = \frac{-75 \times 2k\Omega}{3.75k\Omega + 76 \times 100\Omega} = -13.3\,V/V$$

Thus applying Miller’s theorem would yield a capacitor $C_{IN} = 14.3 \times C_\mu$ from base to ground. Such capacitor would “see” an equivalent resistance

$$R_{eq} = 10k\Omega \parallel 20k\Omega \parallel [r_\pi + (\beta + 1)R_e] = 4.2k\Omega$$

Since this capacitor will cause the high-frequency pole at $10^7Hz = 62.8\,Mrps$,

$$C_{mu} = \frac{1}{(1 - K)R_{eq}\omega_H} = 0.27\,pF$$

4. $C_\mu$ without applying Miller’s Theorem, and by replacing the transistor with its model and analyzing the resulting circuit to find out the resistance seen by $C_\mu$. Use $R_C = 2k\Omega$, not the value you found in part (a).

ANSWER:

The high-frequency pole at $\omega_H = 62.8\,Mrps$ is associated with $C_\mu$ and thus $C_\mu = \frac{1}{R_{eq}\omega_H}$. After replacing the transistor with it’s model and placing a test source in place of $C_\mu$ to find $R_{eq}$, the following diagram results

A KVL in the left loop yields

$$v_1 = i_b(3750 + 76 \times 100) = 11350\Omega \times i_b$$

Thus

$$i_1 = \frac{v_1}{6.7k\Omega} = 1.69i_b$$
A KCL on the top-left node gives

\[ i_t = i_1 + i_b = 2.69i_b \]

A KCL on the bottom node yields,

\[ i_2 = i_c + i_1 = 76i_b + 1.69i_b = 77.69i_b = 28.9i_t \]

Finally, a KCL in the outer loop give

\[ v_t = v_1 + 2k \times i_2 = (\frac{11350\Omega}{2.69} + 2000\Omega \times 28.9)i_t \]

and

\[ R_{eq} = \frac{v_t}{i_t} = 62k\Omega \]

\[ C_{\mu} = \frac{1}{62 \times 10^3 \Omega \times 6.28 \times 10^7} = 0.26pF \]
\[ R_{\text{IN}} = R_b \parallel (r_\pi + (\beta + 1)R_h) \]

\[ R_{\text{EQ}} = R_c \parallel \{ r_\pi (1+g_m R_e) \} \]

\[ v_c / v_b = -g_m R_e / (1+g_m R_e) \]

\[ v_c / v_e = +g_m R_c \]

\[ v_a / v_b = 1 \]

\[ R_{\text{IN}} = R_g \]

\[ R_{\text{EQ}} = R_d \parallel \{ r_\pi (1+g_m R_h) \} \]

\[ v_d / v_g = -g_m R_d / (1+g_m R_h) \]

\[ v_d / v_s = +g_m R_d \]

\[ v_a / v_g = 1 \]

small-signal incremental model