The transistors in the amplifier shown below have $\beta = 100$. Base currents can be neglected in your analysis.

1. Assuming $R_5 = 50k\Omega$, $R_6 = 10k\Omega$, $R_8 = 0$, and that the area of $Q_6$ equals 5 times the area of $Q_5$, find:

   (a) the differential gain $A_d = \frac{v_{OUT}}{v_1 - v_2}$ (50 points)
   (b) the input resistance $R_{in}$ seen by differential signals between terminals 1 and 2. (25 points)
   (c) the output resistance $R_{out}$ at the output terminal. (25 points)

2. Assuming $I_CQ_6 = 1mA$, select $R_6$ so that $v_{OUT} \approx 0V$ when $v_1 = v_2 = 0V$. (15 points)
1. (a) To find the gain, we need to determine the bias currents.

\[ I_{CE1} = \frac{15 - 2}{85} \cdot \frac{14.3}{56} = \frac{286}{2} \mu A \]

\[ I_{CE2} = 5 \cdot I_{CE1} = 1.43 \text{ mA} \]

\[ I_{C1} = \frac{1}{2} I_{CE2} = 0.715 \text{ mA} \]

\[ V_{BE1} = 15 \cdot 10k \cdot (0.715 \text{ mA}) + 7.85v \]

\[ I_{C3} = \frac{15 - V_{BE3} + 7}{1.4k \cdot 4.4k} = 1.47 \text{ mA} \]

\[ V_{BE3} = -15 \cdot (1.47 \text{ mA})(25) - 15 \cdot (1.47 \text{ mA})(10k) \]

\[ I_{CE4} = \frac{V_{BE4} - 15}{1.5k} = 0.3 \text{ mA} \]

From this we can find the \( g_m \)’s

\[ g_{m1} = \frac{715 \text{ mA}}{25 \text{ mV}} = 0.0286 \text{ A/V} \]

\[ g_{m3} = \frac{1.47 \text{ mA}}{25 \text{ mV}} = 0.0588 \text{ A/V} \]

\[ g_{m4} = \frac{0.3 \text{ mA}}{25 \text{ mV}} = 0.372 \text{ A/V} \]

The load at the collector of \( C_2 \) is

\[ R_c = R_4 \parallel \left( R_3 + (4h) + 4k \right) \]

\[ = 10k \parallel \left( \frac{100 \text{ (25)}}{4.7k} \right) = 10k \Omega \]

\[ V_{CE} = \frac{1}{2} \cdot 3.6 \cdot 10k \Omega = \frac{1}{2} \cdot (0.0286 \text{ A})(0.785 \text{ V}) = 0.190 \text{ V} \]
\[
\frac{V_{c3}}{V_{c2}} = \frac{-g_{m3} R_{e3}}{1 + g_{m3} R_2}
\]

\[
R_{eq} = R_3 || \left( \frac{100}{0.372} + 151.5k \right) = 10k || \left( \frac{100}{0.372} + 151.5k \right) = 10k || (151.5k) = 9.38k
\]

\[
\frac{V_{c3}}{V_{c2}} = \frac{-0.0588 \times 9.38k}{1 + 0.0588 \times 9.38k} = -2.12\text{ V/V}
\]

\[
\frac{V_{eq}}{V_{c3}} = \frac{+g_{m4} R_4}{1 + g_{m4} R_4} = \frac{0.372(1500)}{1 + 0.372 \times 1500} = 0.998
\]

\[
A_d = (+140\%)(-2.12\text{ V/V})(0.998\text{ V/V}) = 2.963\text{ V/V}
\]

(b) \[
R_m = 2R_2 = 2 \times \frac{100}{g_{m2}} = 2 \times \frac{100}{0.0286A/V} = 7.05k
\]

(c) \[
R_{out} = 1.5k || \left( \frac{100}{0.372} + 10k \right) = 1.5k || \left( \frac{100}{0.372} + 10k \right) = 95.25k
\]

(2) if \(I_{c6} = 1mA, I_{c2} = 0.5mA, V_{c2} = 10V\) \(I_{c5} = \frac{15 - 10}{9.4k} = 0.54mA\)

For \(V_{out} = 0\), \(V_{c3} = +0.7V\) \(i_c = \frac{15.7V}{0.98mA} = 16.05k\)

(3) Wdcan Current Source

\[
I_0 = V_t \ln \left( \frac{\text{IREF}}{I_0} \right), \quad \text{IREF} = 82\text{mA} \Rightarrow V_t = \frac{14.3V}{82\text{mA}} \approx 0.173\text{V/mA}
\]

\[
R_c = \frac{25\text{mV}}{1mA} \ln (2) = 17.3\text{ k} \Omega
\]