1. For the following circuit

\[ a) \text{ Show that the quiescent operating currents are } I_{CQ1} \approx 486 \mu A \text{ and } I_{CQ2} \approx 1.35 \text{mA,} \]

KCL at \( Q_1 \)'s collector:

\[
500 \mu A = 0.5mA = I_{CQ1} + \frac{I_{CQ2}}{100} \\
50mA = 100I_{CQ1} + I_{CQ2}
\]

KCL at \( Q_2 \)'s emitter:

\[
I_{EQ2} = \frac{101}{100} I_{CQ2} = I_{CQ1}/100 + \frac{0.7V + \frac{I_{CQ2}}{100} \times 36k\Omega}{1k\Omega} \\
101I_{CQ2} = 37I_{CQ1} + 70mA \\
I_{CQ2} = \frac{37I_{CQ1} + 70mA}{101} = 0.366 \times I_{CQ1} + 0.693mA
\]

Now substitute this result into first equation to get

\[
50mA = 100I_{CQ1} + 0.366 \times I_{CQ1} + 0.693mA \\
I_{CQ1} = \frac{50mA - 0.693mA}{100} = \boxed{491.3\mu A} \\
I_{CQ2} = 100(500 - 491.3) = \boxed{872.9\mu A}
\]

You can see that unfortunately there was an error in my original estimate of \( I_{CQ2} \). The feedback, however, reduces the sensitivity of the gain, input resistance and output resistance on the parameters. The change in operating point (calculated one versus the one assumed originally) causes very small changes in the results.

\[ b) \text{ Use the two-port method discussed in the lectures to} \]

1) determine the type of amplifier that is appropriate for the feedback analysis and the feedback network’s parameters \( \beta, R_{11} \) and \( R_{22} \):

ANSWER: The amplifier is shunt-shunt. The feedback network can be taken to be the \( 36k\Omega \) resistor, and

\[
\beta = \left. \frac{i_1}{v_2} \right|_{v_2=0} = -\frac{1}{36k\Omega} \\
R_{11} = \left. \frac{v_1}{i_1} \right|_{v_2=0} = 36k\Omega \\
R_{22} = \left. \frac{v_2}{i_2} \right|_{v_1=0} = 36k\Omega
\]
2) find the voltage gain, \(v_{out}/v_{in}\), the input resistance seen by the source and its 1kΩ input resistance, and the output resistance seen by the load \(R_L\).

ANSWER: The non-feedback amplifier is shown below.

The transistor parameters are (using the original values \(I_{CQ1} = 486\mu A\) and \(I_{CQ2} \simeq 1.35\,mA\): \(g_{m1} = 0.491\,mA/0.025\,V = 19.64\,mA/V\), \(g_{m2} = 0.873\,mA/0.025\,V = 34.92\,mA/V\), \(r_{\pi 1} = 100/19.44\,mA/V = 5092\,\Omega\), \(r_{\pi 2} = 100/35\,mA/V = 2857\,\Omega\), \(r_{O1} = 50\,V/0.491\,mA = 102\,k\Omega\) and \(r_{O2} = 50\,V/0.873\,mA = 57\,k\Omega\). The resistance looking into the base of \(Q_2\) is \(R_{i2} = r_{\pi 2} + 101 \times 806 = 86498\,\Omega\). The non-feedback amplifier gain is

\[
A_z = - \frac{973}{r_{\pi 1} + 973} \times 100 \times \frac{102}{102 + 86.5} \times 100 \times 806 \| 57k \\
= -0.16 \times 100 \times 0.54 \times 100 \times 795 \\
= -687k\Omega
\]

From which we obtain that

\[
A_{zf} = \frac{v_{O}}{v_{in}} = \frac{-687k\Omega}{1 + 687k\Omega/36k\Omega} = -34.2k\Omega
\]

and

\[
v_{out}/v_{in} = -34.2k\Omega/1k\Omega = -34.2V/V
\]

The input resistance is

\[
R_i = 973\Omega\|5092\Omega = 817\Omega \\
R_{if} = 818\Omega/1+687k\Omega/36k\Omega = 40.7\Omega
\]

and the output resistance is

\[
R_o = 806\Omega\|57k\Omega = 795\Omega \\
R_{of} = 795\Omega/1+687k\Omega/36k\Omega = 39.6\Omega
\]

From these we can estimate the resistance seen by the source and its 1kΩ thevenin resistance,

\[
R_i' = \frac{1}{\frac{1}{44} - \frac{1}{1000}} = 43\Omega
\]

and the output resistance seen by the load \(R_L = 4.7k\Omega\),

\[
R_o' = \frac{1}{\frac{1}{39} - \frac{1}{4700}} = 40\Omega
\]

Assume \(h_{fe} = 100\), \(V_{BE} = 0.7V\) and \(V_A = 50V\) for both transistors.
2. Use SPICE to find the voltage gain, \( v_{\text{out}}/v_{\text{in}} \), the input resistance seen by the source and its 1kΩ input resistance, and the output resistance seen by the load \( R_L \). for the previous circuit. Compare the results to those in the previous problem.

**ANSWER:** My SPICE netlist is the following:

```plaintext
*SPICE circuit <pispice> from XCircuit v3.6 rev 127

vin 1 0 dc 0 ac 1mv
vdc 7 0 dc 12v
ibias 7 6 dc 500ua
rs 2 1 1.0k
rl 5 0 4.7k
c1 3 2 82u
c2 5 4 47u
rf 4 3 36.0k
re 4 0 1.0k
q1 6 3 0 mod1
q2 7 6 4 mod2
.model mod1 npn (Is=1.810e-15A Bf=100 VAf=50V)
.model mod2 npn (Is=1.810e-15A Bf=100 VAf=50V)
.control
delete all
ac dec 1000 10 1000Meg
plot (v(5)/v(1)) xlog
.endc
.end
```

This is written for *MacSpice*, a version of Berkeley SPICE for OSX.

The resulting simulation shows a gain of \(-34.4V/V\) in good agreement with our previous result.

To estimate the input resistance seen by SPICE, set \( rs \) to some small value (like 0.1Ω) and repeat the simulation to obtain the gain that correspond to the amplifier with an ideal source, and no input loading. Then repeat with the estimated value of \( rs = R_{i,f} = 40Ω \). if this is indeed the input resistance estimated by SPICE, the gain should become half of the previous result, and such was the case indicating good agreement with the feedback analysis’ results.

To find out the resistance seen by SPICE at the output, repeat first setting \( rl \) to some large value (like 1GΩ) to obtain the gain when no load is present, and repeat with \( rl = R_{o,f} \). If this causes the gain to become half of the one obtain without a load, then SPICE is estimating the same value of the output resistance. If not adjust \( rs \) until the gain is reduced by half. I had to adjust \( rl \) downn to 19Ω to cause this halving in gain, indicating that our result for output resistance is off by a factor of two.

3. For the following amplifier
a) write an expression for the loop gain if $R_1 = 1k\Omega$, $R_2 = 20k\Omega$, $C_C = 0$, and the op amp transfer function is

$$A(s) = \frac{2 \times 10^{11} \pi^2}{(s + 2\pi \times 10^2)(s + 2\pi \times 10^4)}$$

and use it to estimate the phase margin analytically.

**ANSWER:**

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{21}$$

$$T = A\beta = \frac{2 \times 10^{11} \pi^2}{21 (s + 2\pi \times 10^2)(s + 2\pi \times 10^4)}$$

$$|T| = \frac{5 \times 10^4}{21} \frac{1}{\sqrt{100f^2 + 1} \sqrt{.01f^2 + 1}}$$

where $f$ is expressed in kHz. At the crossover frequency $f_x$, $|T(f_x)| = 1$ so

$$\frac{5 \times 10^4}{21} = \sqrt{100f_x^2 + 1} \sqrt{.01f_x^2 + 1}$$

and

$$\frac{25 \times 10^8}{21^2} = (100f_x^2 + 1) (.01f_x^2 + 1)$$

Letting $x = f_x^2$,

$$\frac{25 \times 10^8}{21^2} = 5.67 \times 10^6 = (100x + 1) (.01x + 1) = x^2 + 100.01x + 1$$

Solving this quadratic equation yields $x = 2331$ and $f_x = 48.3kHz$. At this frequency the phase of $T$ is

$$\phi = -\arctan (48.3/1) - \arctan (48.3/10) = -168.2^\circ$$

and

$$\phi_m = 180^\circ - 168.2^\circ = 13.2^\circ$$

b) use MATLAB (or Octave) to make a Bode plot of $T(s)$. What is the phase margin of this circuit?

**ANSWER:**

Since I don’t have MATLAB on my mac, I used the freeware program “ScicosLab”. The commands that I issued are the following:
c) Can compensation capacitor $C_C$ be added to achieve a phase margin of $45^\circ$? If so, what is the value of $C_C$.

**ANSWER:** Since $T = A\beta$, in decibels

$$T_{dB} = A_{dB} + \beta_{dB} = A_{dB} - \left(\frac{1}{\beta}\right)_{dB}$$

The point at which $T_{dB} = 0dB$ is also where $A_{dB}$ and $\frac{1}{\beta_{dB}}$ are equal. The idea is then to use $\frac{1}{\beta}$ to add $45^\circ$ to the phase of the loop gain by adding the capacitor $C$ to the feedback loop. With the capacitor in place,

$$\beta = \frac{R_1}{R_1 + R_2 \frac{1}{sC}}$$

$$\frac{1}{\beta} = \frac{sCR_2 + 1}{sCR_2 + 1}$$
It can be observed that the zero of $1/\beta$, $\omega_z = 1/C(R_1||R_2)$, is at a higher frequency than the pole, $\omega_p = 1/CR_2$. So one possibility is to change the $1/\beta$ so that it intercepts $A$ at $\omega_p$. In this way the phase at the crossover frequency is $\simeq -135^\circ$ (-90 degrees from each of the two poles of $A$ and +45 degrees of the zero of $\beta$. The idea is illustrated in the following diagram.

The dc values for $A$ and $1/\beta$ are $5 \times 10^4$ and 21, respectively, and correspond to $94\,dB$ and $26\,dB$. Since we already know that $|A(f_x)| = |A(48.3\,kHz)| = 21$,

$$C = \frac{1}{2\pi(48.3 \times 10^3\,Hz)(20k\Omega)} = 165pF$$

To verify, we can look at the Bode plot again.

```matlab
-->s=poly(0,'s');
-->a=(5*10^4)*(1 + s/(2*%pi*48.3*10^3))/(21;
-->T=a/((1+s/(2*%pi*1.012*10^6))*(1+s/(2*%pi*10^{-2}))*(1+s/(2*10^{-4}*%pi)));
-->T1=syslin('c',T);
-->bode(T1,10,1000000,.01)
```
The results indicate a $\phi_m \simeq 45^\circ$. 