High-frequency response

BJT \to hybrid-\pi equivalent circuit is expanded to include capacitive effects on the transistor's junctions.

\[ C_{\pi} \text{ and } C_{\mu} \text{ reduce the gain at higher frequencies} \]

Short-circuit current gain

\[ I_{C} = g_{m} V_{P} - \frac{V_{P}}{1/g_{C_{\mu}}} = (g_{m} - sC_{\mu})V_{P} \]

For all frequencies of interest, \( g_{m} \gg sC_{\mu} \)
\[ I_C = g_m V_{PE} = \frac{g_m \Gamma_{PE}}{1 + s \Gamma_{PE} (C_{PI} + C_{\mu})} I_B \]

This provides an expression for \( \frac{I_C}{I_B} = \beta \) as a function of \( s \),

\[ \beta(s) = \frac{I_C}{I_B} = \frac{\beta_0}{1 + s \Gamma_{PE} (C_{PI} + C_{\mu})} \]

\( \beta_0 = g_m \Gamma_{PE} = \text{low-frequency } \frac{I_C}{I_B} \)

pole at \( \omega_p = \frac{1}{\Gamma_{PE} (C_{PI} + C_{\mu})} \)

\[ \log \beta \]

\[ \log \omega \text{ (log scale)} \]

\( \omega_c = \text{unity-gain frequency} \)

\( \omega_c = \text{cutoff frequency} \)

At \( \omega_c \), \( \beta(s) = 1 \) by definition

\[ \left| \frac{\beta_0}{1 + j \omega_c \Gamma_{PE} (C_{PI} + C_{\mu})} \right| = \frac{\beta_0}{\sqrt{1 + \omega_c^2 \Gamma_{PE}^2 (C_{PI} + C_{\mu})^2}} = 1 \]

\[ \omega_y = \frac{\sqrt{\beta_0^2 - 1}}{\Gamma_{PE} (C_{PI} + C_{\mu})} = \frac{\beta_0 \omega_c}{\Gamma_{PE} (C_{PI} + C_{\mu})} \]

-2-
\[
\beta_0 \rightarrow \text{also called } h_{fe} \\
\rho(s) \rightarrow " h_{fe}(s) \\
W_T = \frac{\rho_0}{\pi (C \pi + C_{\mu})} = \frac{g_m \pi}{\pi (C \pi + C_{\mu})} = \frac{g_m}{C \pi + C_{\mu}}
\]

Note: book uses \( W_T(f_T) \) for \( W_Z(f_Z) \).

For transistor 2N2222A \( \rightarrow f_T = 300 \text{MHz} \).

\[ * \]

p. 7.39 \( I_C = 0.5 \text{mA} \), \( C_{\mu} = 0.15 \text{pF} \), \( f_T = 5 \text{GHz} \), \( \beta_0 = 150 \)

Find \( C_{\pi}, f_p \)

\[
2\pi (s) 10^9 = \frac{20\pi \times 10^9}{10\pi} = \frac{g_m}{C_{\pi} + 0.15 \text{pF}}
\]

\[ g_m = \frac{I_C}{V_T} = \frac{0.5 \text{mA}}{25 \text{mV}} = 20 \text{mA/V} \]

\[
C_{\pi} = \frac{20 \text{mA/V}}{20\pi \times 10^9} - 0.15 \text{pF} = 0.49 \text{pF} = C_{\pi}
\]

\[
f_p = \frac{f_T}{\beta_0} = \frac{5000 \text{MHz}}{150} = 33.3 \text{MHz} = f_p
\]
Miller's effect

Consider $I_v$:

\[ Y = \text{admittance} \]

If component is a cap $Y = SC$

$A_m = \text{amplifier's gain}$

Approx. + assume $A_m$ does not depend on $Y$

Then $I_c = I_{in} + I_y$

$I_y = Y(v_i - v_o) = Y(1 - A_m)v_i$

$I_c = I_{in} + Y(1 - A_m)v_i = I_{in} + Y_m v_i$

\[ Y_m = (1 - A_m)Y \]

If $Y = SC_m$

\[ Y_m = (1 - A_m)C_m S \]

$C_m \rightarrow \text{augmented } C_m$; if $A_m \ll -1$ (large & neg)

For $A_m \ll -1$

$C_m \rightarrow \text{looks like a much larger capacitor from the input port.}$
Output port

\[ I_{yo} = Y(V_o - V_i) = Y(V_o - \frac{V_o}{A_H}) = Y(1 - \frac{1}{A_H}) V_o \]

\[ Y_o = \frac{I_{yo}}{V_o} = Y(1 - \frac{1}{A_H}) \]

if \( A_H << -1 \)

\[ Y_o \approx Y \]

Common-emitter

At high-freqs.

CC1, CC2 & CE are shorts

Vcc \rightarrow ground

\[ A_H = \frac{V_c}{V_b} \text{ (ignoring } C_c) = -g_m R' \]

-5-
In the input part of the circuit, we see an equivalent capacitance

\[ C_{eq} = C_f + C_m (1 + g_m R_L') \]

Notice that, according to our analysis of the RC circuits in lecture 1, this capacitance produces a pole at

\[ \omega_{H1} = \frac{1}{(R_0 || R_f || R_{L1} || R_2) C_{eq}} \]

The output segment also exhibits a pole at

\[ \omega_{H2} = \frac{1}{C_m R_L'} \]

Because typically, \( R_0 \) is smaller than \( R_L' \), and also \( C_{eq} \gg C_m \), \( \omega_{H1} \ll \omega_{H2} \) and \( \omega_{H1} \) dominates the high-frequency response of the CE.

**Example**

\[ R_0 || R_2 = 200 \, \text{k} \Omega, \quad R_0 = 5 \, \text{k} \Omega, \quad R_f = 2.6 \, \text{k} \Omega \]

\[ R_{L1} || R_L = R_L' = 2 \, \text{k} \Omega, \quad C_f = 4 \, \text{pF}, \quad C_m = 0.2 \, \text{pF} \]

and \( g_m = 38.5 \, \text{mA/V} \).

\[ C_H = (1 + g_m R_L') C_m = 78 \, \text{C}_m = 15.4 \, \text{pF} \]

Equivalent capacitance at base = \( C_{eq} = 19.4 \, \text{pF} \)

\[ \omega_{H1} = \frac{1}{(5 || 12.5 || 200) (19.4 \, \text{pF})} = \frac{1}{(1.65 \, \text{k} \Omega) (19.4 \, \text{pF})} = 31.2 \, \text{MHz} \]

\[ \omega_{H2} = \frac{1}{(2 \, \text{k} \Omega)(0.2 \, \text{pF})} = 2.56 \, \text{GHz} \]
Since \( \omega_H \) is larger than \( \omega_m \) by more than a decade, \( \omega_m \) dominates the high-freq. response; it is the dominant high-freq. pole.

If there is no dominant high-frequency pole because two or more poles are closer than one decade, we can estimate the high-frequency pole using

\[
\frac{1}{f_{H_{\text{eff}}}} = \frac{1}{f_i} - \frac{1}{f_{m_i}}
\]

Example: 3 poles at 10MHz, 20MHz and 30MHz produce an "effective" h-f. pole at

\[
\frac{1}{f_{H_{\text{eff}}}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30}
\]

\[f_{H_{\text{eff}}} = 5.5 \text{ MHz}\]

Field-effect Transistors.

HF model:

\[
f_T = \frac{g_m}{2\pi (C_{gs} + C_{gd})}
\]

Miller's theorem applies to MOSFET circuits in the same way that it applies to BJT circuits.
Example: Problem 7.58

\[ V_{TP} = -2 \text{V} \]
\[ K_P = 1 \text{mA/V}^2 \]
\[ \lambda = 0 \]
\[ C_{gs} = 15 \text{pF} \]
\[ C_{gd} = 3 \text{pF} \]

D.C. analysis

\[ V_G = -10 + \frac{22}{30} \times 20 = 4.67 \text{V} \]
\[ I_D = K_P \left( V_{SG} + V_{TP} \right)^2 = \frac{10 - V_{SG} - 4.67}{\frac{1}{2} \text{k}\Omega} \]

\[ \left( \frac{1}{2} V^{-1} \right) \left( V_{SG}^2 - 4 V_{SG} + 4 \right) = 5.33 - V_{SG} \]
\[ V_{SG}^2 - 2 V_{SG} - 6.67 = 0 \]
\[ V_{SG} = 3.77 \text{V} \]
\[ V_{SG} = -1.77 \text{V} \]

\[ g_m = 2 K_P (V_{SG} - 2V) = 2 \text{mA/V}^2 (3.77 - 2) \]

\[ g_m = 3.54 \text{mA/V} \]
midband gain

\[
A_{HB} = - \frac{22\|8}{22\|8 + 5} (3.59 \, \text{mA}) (2\|15 \, \text{k})
\]

\[
= - \frac{5.87}{6.37} (5) = - 4.6 \, \text{V/V} = A_{HB}
\]

Miller gain \( A_m = -g_m R_V \) = -5 \, \text{V/V}

\[
C_m = C_{gd} (1 + 5) = 3pF (6) = 18pF = \text{Miller's cap.}
\]

\[
\frac{460.2}{\frac{1}{2k}\|8k\|22k} = C_{gs} \quad W_{H1} = \frac{1}{33pF \times 460.2}
\]

\[
W_{H1} = 65.8 \, \text{MHz}
\]

\[
C_{gs} + C_m = 33pF
\]

10.5 MHz

The other pole is at

\[
W_{H2} = \frac{1}{(5k\|2k) 3pF} = 233 \, \text{MHz} = 37 \, \text{MHz}
\]

No dominant pole. Effective pole at \( W_H = 8.2 \, \text{MHz} \)

we can also calculate the low-freq. poles.

\[
R_{eqc1} = \frac{1}{2} k_R + 8k\|22k = 6.37 \, \text{k}
\]

\[
\Rightarrow W_{LP1} = \frac{1}{2\mu F \times 6.37 \, \text{k}}
\]

\[
= 78.5 \, \text{rps} = 12.5 \, \text{Hz}
\]

\[
R_{eqc2} = 7 \, \text{k}
\]

\[
\Rightarrow \quad W_{LP2} = \frac{1}{2\mu F \times 7 \, \text{k}}
\]

\[
= 71.4 \, \text{rps} = 11.4 \, \text{Hz}
\]

\[
R_{eqc5} = 500 \| \frac{1}{g_m} = 500/1282 \, \text{r} = 180.3 \, \text{r}
\]

\[
\Rightarrow \quad W_{LP5} = \frac{1}{(10\mu F)(180.3 \, \text{r})}
\]

\[
= 55.5 \, \text{rps} = 88.3 \, \text{Hz}
\]
The zero due to the bypass cap. is located at

\[ \omega_2 = \frac{1}{10 \mu F \times 5000} = 200 \text{ rps} = 31.8 \text{ Hz} \]

We get an effective low-freq. pole at

\[ \omega_L \approx \sum \omega_c = 12.5 + 11.4 + 88.3 - 31.8 \text{ Hz} \]
\[ \approx 80.4 \text{ Hz} \]