Stability

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Stability

Basics

- Basic feedback equation:

$$A_f(s) = \frac{a(s)}{1+\beta(s)a(s)}$$

  Thus, feedback moves the poles of the amplifier's transfer function.

- Poles of A_f are roots of $1+\beta a$. Thus, feedback moves the poles of the amplifier's transfer function.

- The idea is to determine information about the stability of A_f from the loop gain $T(s) = \beta(s)a(s)$.

Nyquist Theorem

Let ω_{180° be the frequency at which the loop gain's phase angle is -180°. If

$$\mid T(j\omega_{180^\circ}) \mid = \mid \beta(j\omega_{180^\circ}) A(j\omega_{180^\circ}) \mid > 1$$

then the amplifier is unstable. Otherwise, it is stable.

Nyquist theorem allows us to answer questions about the stability of A_f by analyzing the loop gain βA.

Phase and Gain Margin

- Gain margin: decibels below zero of $\mid T(j\omega_{180^\circ}) \mid$.

- Phase margin: degrees above -180° at the frequency ω_{0dB} at which $\mid T(j\omega_{0dB}) \mid = 1$, or 0 db.

$$\phi_m = 180 + \angle T(j\omega_{0dB})$$

  Note that $\angle T(j\omega_{0dB})$ is usually negative.

- The amplifier is unstable if the gain and phase margins are negative. If the margins are positive or zero the amplifier is *stable* or *marginally stable*, respectively.

12.65 A 3-pole amplifier has a loop gain given by

$$T(f) = \frac{10^5 \times \beta}{\left(1 + j\frac{f}{5\times 10^2}\right)\left(1 + j\frac{f}{10^4}\right)^2}$$

(a) determine the frequency f_{180} at which the phase is -180 degrees. (b) At f_{180} determine the value of β such that $|T(f_{180})| = 1$.

Extra: c) find the the value of β such that the phase margin is 45 degrees; (d) repeat for a phase margin of 60 degrees; (e) sketch the magnitude and phase bode plots of the amplifier's gain.

Example: An op amp with $a_0 = 10^3$ V/V and two pole frequencies at $f_1 = 100$kHz and $f_2 = 2$MHz is connected as a unity-gain voltage follower. Find φ_m.

Example: An amplifier has 3 identical poles at a frequency f_1 and is placed in a negative-feedback loop with a frequency independent feedback factor β. Find an expression for $f_{-180°}$ as well as the corresponding value of T.

Example: (a) Verify that the circuit with loop gain $T_0 = 10^2$ and three pole frequencies $f_1 = 100$kHz, $f_2 = 1$MHz and $f_3 = 2$MHz is unstable. (b) Reduce T_0 so that $\varphi_m=45°$. (c) repeat for $\varphi_m=60°$.

Rate of Closure

- The phase of a transfer function H can be estimated from its magnitude Bode plot. Let Δ_{0dB} represent the slope of the transfer function's magnitude at crossover frequency, expressed in dB/decade. If the roots of H are real and at least a decade apart, then

$$\angle H \approx 4.5 \times \Delta_{0dB}$$

- Since $\mid T(s) \mid = \mid \beta(s) \mid\mid a(s) \mid$, expressed in decibels

$$T_{dB} = a_{dB} + \beta_{dB} = a_{dB} - \left(\frac{1}{\beta}\right)_{dB}$$

  so we can obtain the magnitude Bode plot of $T(s)$ from that of $a(s)$ by subtracting that of $1/\beta$. The *rate of closure* give us the slope of T_{dB} at crossover frequency and can be obtained from

$$ROC = \mid \Delta_a - \Delta_{1/\beta} \mid$$

  where Δ_a and $\Delta_{1/\beta}$ are the slopes of a_{dB} and $(1/\beta)_{dB}$ at the crossover frequency.

- We can use the above formulas to find the phase margin from the rate of closure. Some easy to remember numbers are:

ROC (dB/dec)	ϕ_m(degrees)
20	90
30	45
40	0
over 40	less than 0