NOISE: Extraneous current & voltage fluctuations that limit the minimum signal level that a circuit can process with acceptable quality

Noise also limits the maximum gain in amplifier circuits as A → grows the output noise also grows taking the circuit either to cutoff or saturation.

- Noise is a random process: Its instantaneous value cannot be predicted.
- Noise analysis is made using the statistical properties of the noise manifestation
- Properties which can be statistically analyzed:
  - Average Power (Assumes stationarity)

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} \frac{v(t)\cdot v^*(t)}{R_L}\,dt \qquad (1)$$

Equation (1) implies

- Square the signal, integrate it, and normalize it over T
- Only possible with stationary signals.

A common definition for $P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} x^2(t)\,dt$, in V^2 (2)

Simplifies computation of avg. noise power over a load R_L as $\frac{P_{av}}{R_L}$

RMS value of noise $P_{RMS} = \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} x^2(t)\,dt}$

Most noise sources are stationary

Noise Spectrum : Frequency content of Noise. Also called the PSD

PSD = Power Spectral Density $S_x(f)$

Shows how much power a signal carries at each frequency.

$S_x(f)$: The average power carried by $x(t)$ in a one-hertz bandwidth around f

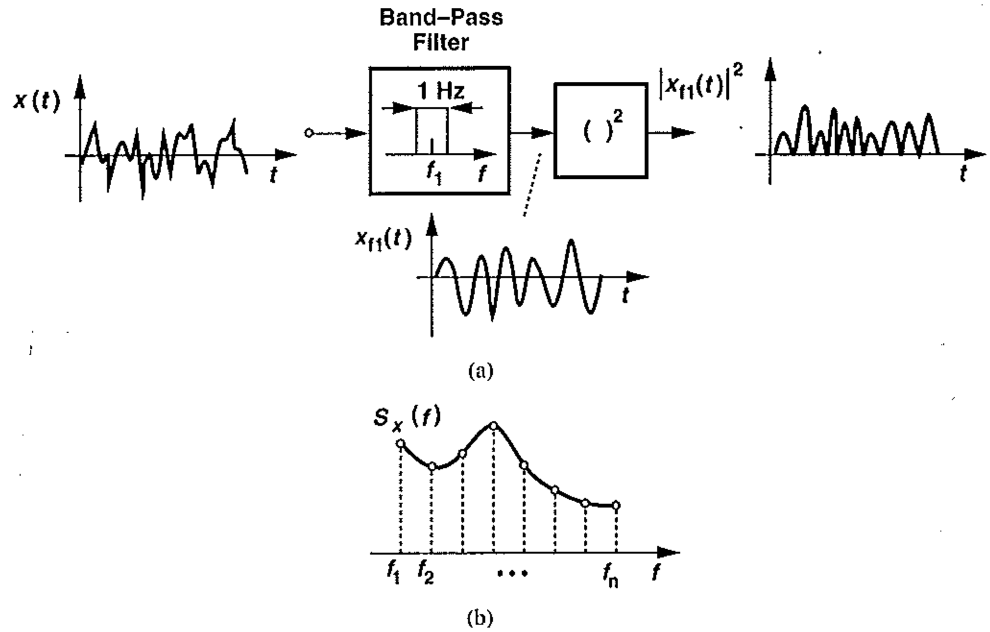


Figure 7.4 Calculation of noise spectrum.

1) Apply $x(t)$ to a bandpass filter with center frequency f_1 and BW = 1Hz
2) Square the output
3) Calculate the average over a long time to obtain $S_x(f_1)$
4) Repeat 1) thru 3) for $f_2, f_3, \ldots$ to obtain the overall shape of $S_x(f)$

This is equivalent to obtaining the Fourier transform of the autocorrelation function of the noise.

$S_x(f)$ is expressed in V^2/Hz (Recall that R_L is eliminated from P_{av})

$S_x(f)$ can also be expressed in $V/\sqrt{Hz}$ $\left(\sqrt{S_x(f)}\right) \rightarrow$ RMS value

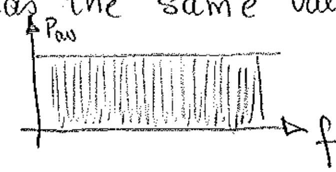
For example

The input noise voltage of an OpAmp at 100MHz is $3nV/\sqrt{Hz}$. implies that

$$P_{av} \text{ in } 1Hz \text{ BW } @ 100MHz = (3\times10^{-9})^2 V^2$$

White noise: Has a "white spectrum" PSD has the same value at all frequencies

$S_n(f)$



WHITE SPECTRUM

- Strictly speaking, white noise does not exist. Note that the total power carried by the noise (area under the PSD) is infinite.

Pink (1/f) 3dB/oct, Brown (red) 1/f², Blue (f) inc 3dB/oct, Violet f², Grey, Other

Theorem: If a signal with spectrum $S_X(f)$ is applied to a LTI system whose transfer function is $H(s)$, then the output spectrum is given by

$$S_Y(f) = S_X(f)\,|H(f)|^2,$$

where $H(f) = H(s = 2\pi j f)$.

This implies that the spectrum of the noise is shaped by the circuit.

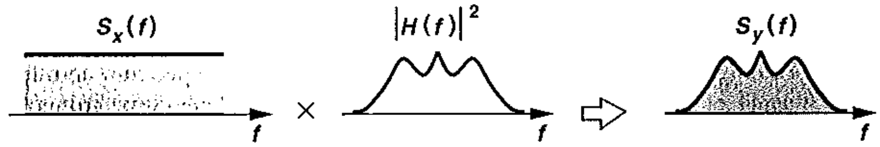


Figure 7.6 Noise shaping by a transfer function.

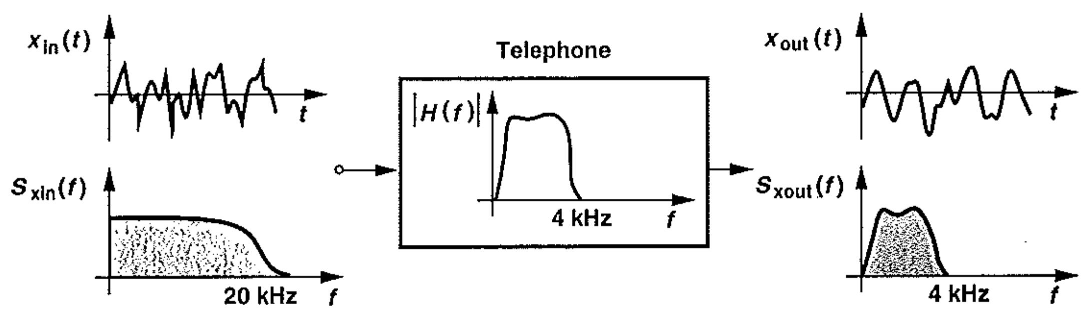


Figure 7.7 Spectral shaping by telephone bandwidth.

Proof: If system is LTI $\Rightarrow$ $y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda$

Thus $Y(f) = X(f)H(f)$

$$S_y(f) \triangleq \lim_{T\to\infty} \frac{1}{T}|Y_T(f)|^2 = \lim_{T\to\infty} \frac{1}{T}\left|X_T(f)H(f)\right|^2$$

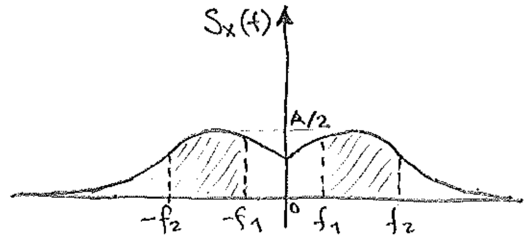
$$= |H(f)|^2 \lim_{T\to\infty} \frac{1}{T}|X_T(f)|^2 = |H(f)|^2 \cdot S_x(f)$$

QED

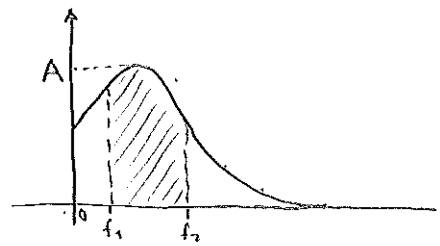
• Note that $S_x(f)$ is even for $x(t)$ real. Thus given a frequency range $[f_1, f_2]$

$$P_{f_1, f_2} = \int_{-f_2}^{-f_1} S_x(f)\, df + \int_{+f_1}^{+f_2} S_x(f)\, df = 2\int_{f_1}^{f_2} S_x(f)\, df$$

Measured Power



TWO-SIDED SPECTRA



ONE-SIDED SPECTRA

What does it like for a white spectrum?

AMPLITUDE DISTRIBUTION (Instead of instantaneous amplitude)

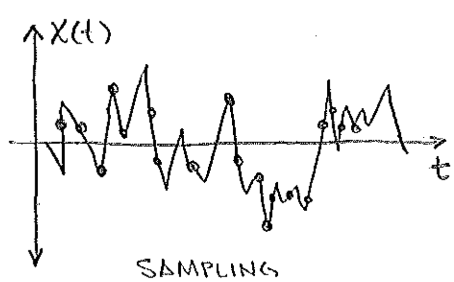
Refresh: CDF = Commulative Distribution Function
• Given RV x,

$$F(a) \triangleq P(x \le a) \equiv \lim_{n \to \infty} \left(\frac{n_{x \le a}}{n} \right)$$

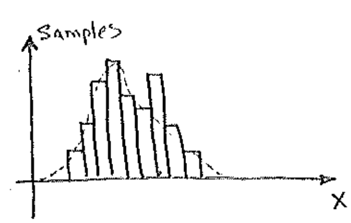
PDF = Probability Density Function
• Given x, a RV

$$f_x(x) = \left. \frac{dF(a)}{da} \right|_{a=x} = \left. \frac{dP(x \le a)}{da} \right|_{a=x} = \lim_{\substack{n \to \infty \\ \Delta x \to 0}} \left[\frac{1}{\Delta x} \left(\frac{n_{\Delta x}}{n} \right) \right]$$

PDF: Practical meaning: How often each value occurs in a Random process.



SAMPLING



DISTRIBUTION

• Binomial
• Poisson
• Uniform
• Gaussian
• Others

Central Limit Theorem: "The sum of a number of independent random variables with arbitrary PDFs approaches a Gaussian (Normal) distribution."

Normal PDF: $$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

σ = std deviation

μ = mean

In noise analysis we often need to add the effect of multiple sources.

to find the total noise. Here superposition does not apply. Instead we search the average noise power.

Consider two noise sources $x_1(t)$ and $x_2(t)$

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} [x_1(t) + x_2(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x_1^2(t)\,dt + \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x_2^2(t)\,dt + \lim_{T\to\infty} \int_{-T/2}^{T/2} 2x_1(t)x_2(t)\,dt$$

$$= \underbrace{P_{av_1} + P_{av_2}}_{\text{Average power of } x_1(t) \text{ and } x_2(t)} + \underbrace{\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 2x_1(t)x_2(t)\,dt}_{\text{Correlation between } x_1(t) \text{ and } x_2(t)}$$

If $x_1(t)$ and $x_2(t)$ are independent sources, their correlation will be zero.
Superposition applies only to uncorrelated sources.

Elements in a noise problem:

- Noise source:
- Transmission media;
- Noise receptor:

<u>Noise sources</u>:

- Electronic Noise: Caused by the device operation itself
- Environmental Noise: Disturbances picked-up by a circuit through the substrate or supply/ground lines.

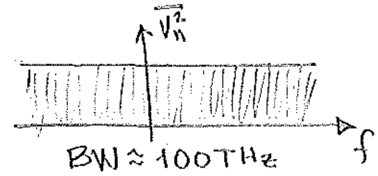
<u>Sources of Electronic Noise</u>:

- <u>Thermal Noise</u>: Caused by the random thermal motion of electrons in a resistive medium.
  Directly proportional to absolute T

The one-sided PSD of thermal noise is $S_v(f) = 4KTR$, $f \geq 0$,

where: K = Boltzmann constant = 1.38×10^{-23} J/°K
T = Absolute temperature in °K
R = Resistance in Ω

Models: $S_v(f) = \frac{\overline{V_n^2}}{\Delta f}$ $\therefore$ $\overline{V_n^2} = 4KTR\Delta f$



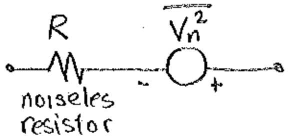
Assuming a 1Hz BW, $\overline{V_n^2} = 4KTR$ ①

Equation ① implies that thermal noise is independent of frequency (white)

At room temperature (T = 300°K) the thermal noise PSD on a 1KΩ resistor is $\overline{V_n^2} = 16 \times 10^{-18}\ V^2/Hz$ RMS: $4nV/\sqrt{Hz}$

Thermal Noise Models:

From ①, a suitable model for thermal noise is

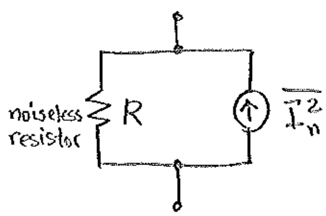


, where $\overline{V_n^2} = 4KTR$ (V^2/Hz)

Applying Norton on this model, a current equivalent can be derived

$\overline{I_n^2} = \frac{\overline{V_n^2}}{R^2}$, thus $\overline{I_n^2} = \frac{4KT}{R}$ (A^2/Hz) assumes $\Delta f = 1Hz$

Parallel model

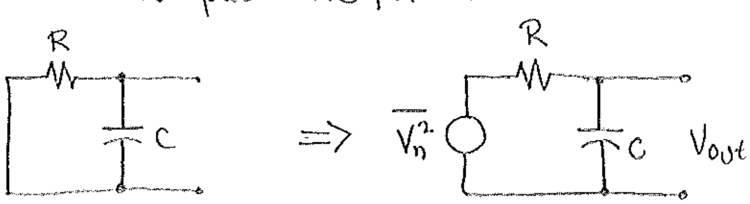


- Thermal noise is present in any linear passive resistor, i.e. it's found on wires, microphones, antennas, poly, etc.

  Polarity of $\overline{V_n^2}$ or $\overline{I_n^2}$ is uninportant (random quantities). Must be maintained once one is chosen.

- Thermal noise has a Gaussian PDF and white spectrum.

Example: Calculate the thermal noise PSD and the total noise power of a low-pass RC filter



Applying the theorem for PSD on LTI systems

$$S_{V_{out}}(f) = S_{V_n}(f) \cdot |H(f)|^2$$

$$H(s) = \frac{V_{out}}{V_R} \qquad V_{out} = V_R \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = V_R \frac{1}{1+RCs}$$

$$H(s) = \frac{1}{1+RCs} \qquad H(f) = \frac{1}{1+RC(j2\pi f)} \qquad |H(f)|^2 = H(f) \cdot H^*(f)$$

Thus $\boxed{S_{V_0}(f) = 4KTR \dfrac{1}{1+4\pi^2R^2C^2f^2}}$

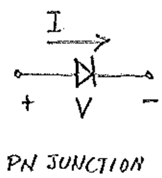
$$P_{nout} = \int_0^\infty S_{V_0}(f)\,df = \int_0^\infty \frac{4KTR}{1+4\pi^2R^2C^2f^2}\,df$$

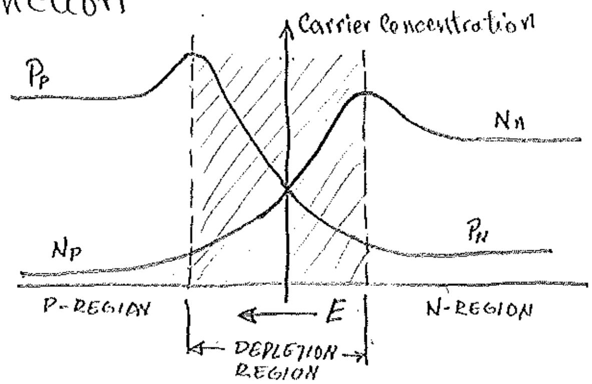
Making $x^2 = 4\pi^2R^2C^2f^2 \Rightarrow x = 2\pi RCf \quad dx = 2\pi RC\,df$

$$P_{n_{out}} = \frac{2KT}{\pi C}\int_0^\infty \frac{1}{1+x^2}\,dx = \frac{2KT}{\pi C} \tan^{-1} x \Big|_0^\infty = \frac{2KT}{\pi C}\left(\frac{\pi}{2}\right)$$

Thus $\boxed{P_{n_{out}} = \dfrac{KT}{C}}$

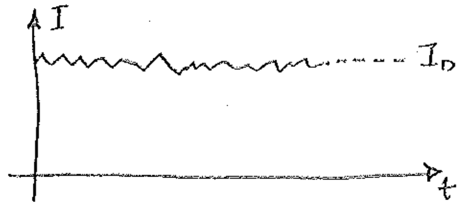
<u>Shot Noise</u>: Caused by the current flow through a forward-biased pn junction





- Electric-field developed in the depletion region
- Forward current I caused by those carriers with enough energy to overcome the potential barrier
- The passage of carriers across the junction is a random event

- The current fluctuations due to the randomness of carrier crossing is shot noise



- Shot noise is specified as the mean-square variation of I about the average value I_D

$$\overline{I_n^2} = \overline{(I-I_D)^2} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}(I-I_D)^2\,dt$$

For a current I composed of a series of random, independent impulses with average I_D, the noise current will have a PSD

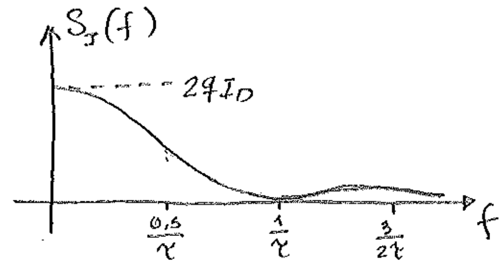
$$S_I(f) = \overline{I_n^2} = 2qI_D, \quad \text{in } (A^2/Hz) \qquad (2)$$

where

q = electron charge = 1.6×10^{-19} C

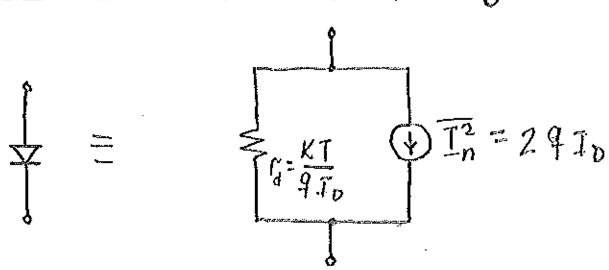
Note that shot noise has a white spectrum.

Equation (2) is valid for frequencies up to $1/\tau$, wher τ is the mean carrier transit time.



Typical values of τ are in the order of ps. Thus $1/\tau$ falls in the order of GHz.

<u>Shot Noise Model</u> (Low frequency, small signal)



Shot noise has a Gaussian distribution

- The probability of I_x lying between I and $I+\Delta I$

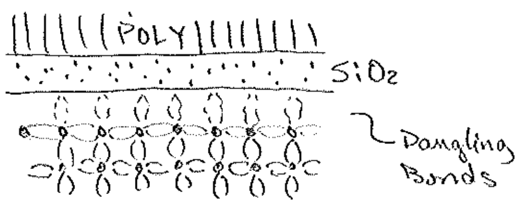
$$P(I < I_x < I+\Delta I) = f_{I_x}(I)\,dI$$

Thus, given that $f_{I_x}(I)$ is Gaussian then $|I_x|$ fall between $I_D \pm \sigma$ 68% of the time

By def. σ^2 is the mean-square value of $(I_x - I_D)$, thus

$$\sigma^2 = \overline{I_N^2} \quad , \text{ and } \sigma = \sqrt{2qI_D}$$

Flicker Noise: Associated to active devices, flicker noise is caused by "dangling" bonds created by defects in the crystaline structure of semiconductos. In some cases it also appears in carbon film resistors. A typical scenary is in the channel-oxide interface in MOS transistors



- Charge carriers are randomly trapped and released by dangling bonds
- Power concentrated at low frequencies
- Difficult to estimate. Depends on I

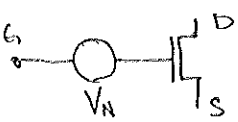
- The one-sided PSD of flicker noise

$$\overline{I_N^2} = K\frac{I^a}{f} \quad (A^2/Hz)$$

where

K = constant dependent on the device
I = Direct current flow
a = constant from 0.5 to 2

As an example, in a MOSFET the approximate expression of flicker noise is modeled as

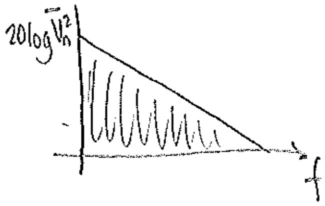


where $\overline{V_n^2} = \frac{K}{C_{ox}WL}\cdot\frac{1}{f}$, where K is process dependent.

$I_D = g_m V_{gs}$

Typical value $K \simeq 10^{-25}\ V^2F$

Flicker noise is inversely proportional to f, therefore it's PSD shows most strength at low frequencies. Also called $1/f$ noise

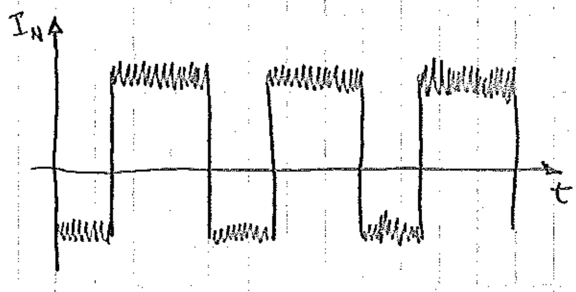


- In MOSFETs the dependence on $\frac{1}{WL}$ of Flicker indicates that low noise applications should have huge transistors.
- Often has non-Gaussian distribution

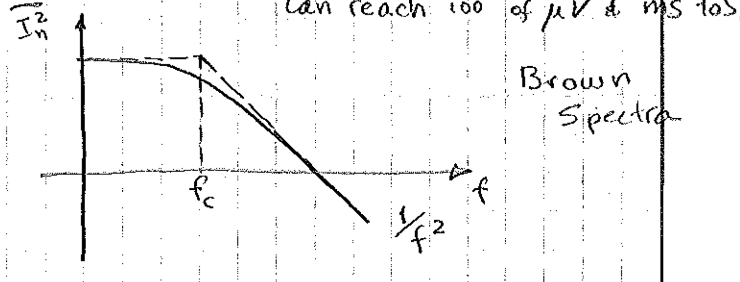
Other types of Noise include

Burst Noise: Low frequency noise associated to heavy-metal ion contamination in semiconductors. Also called RTS noise or popcorn noise.

- Appears as a burst on a number of discrete levels in audio freq. band
  Can reach 100' of μV & ms to s.



BURST NOISE WAVEFORM

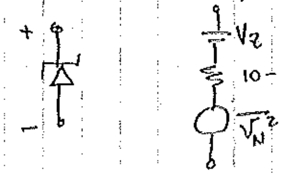


SPECTRAL DENSITY

- Origin not well understood [GrMe93]
- Spectral density $\overline{I_N^2} = K \dfrac{I^c}{1+(f/f_c)^2}$ where
  K = Device dependent constant
  I = Direct current
  c = Constant between 0.5 & 2
  f_c = Cut-off frequency
- Non-Gaussian distribution

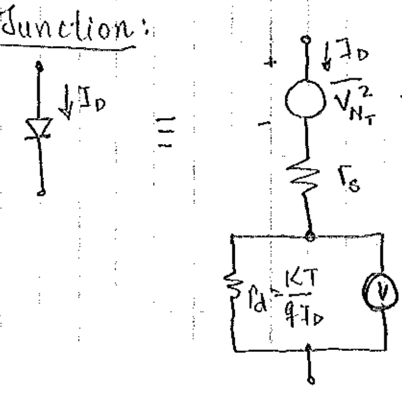
Avalanche Noise: Produced by Zener or avalanche breakdown in pn junctions

- In avalanche breakdown holes and electrons gain enough energy to create hole-electron pairs by collisions. This random event produces spurious noise spikes.
- Noise level much greater than shot noise.
- Associated to the use of zener diodes
- Not easily defined expression for PSD. Typical value: $\overline{V_n^2} \approx 10^{-14}\, V^2/Hz$ @ 1/2 mA. White spectra



Noise Models for IC components

- PN Junction:



$\overline{V_{N_T}^2} = 4KTr_s$ ← Thermal noise assoc. to silicon resistivity

Shot Noise, Flicker Noise

$\overline{I_N^2} = 2qI_D + K\dfrac{I_D^a}{f}$