Texture Analysis and Synthesis

Prof. Vidya Manian
Dept. of Electrical and Computer Engineering

INEL 6209 (Spring 2010) ECE, UPRM Wavelets-1
Overview

• Definition
• Background
• Statistical methods
• Structural methods
• Filtering based methods
• Model methods
• Applications
Definitions

• A macroscopic pattern with placement of texture elements ‘texels’ using a placement rule.

• It is used in early processing. There is no particular definition.

• There is a notion of local order, with a nonrandom arrangement of elementary parts.
Psychophysical background

• Julesz conjecture: Textures with same second order statistics cannot be discriminated
• He proposed the theory of textons to explain preattentive discrimination of texture pairs
• Psychophysics studies suggest that retinal cells perform multi-channel, orientation and frequency analysis
• De Valois et al studied the brain of macaque monkey and recorded response of cells in visual cortex to sinusoidal gratings
Textures discriminable?

FIGURE 4. Texture pairs with identical second-order statistics. The bottom halves of the images consist of texture tokens that are different from the ones in the top half. (a) Humans cannot perceive the two regions without careful scrutiny. (b) The two different regions are immediately discriminable by humans.
Applications

- Automated inspection
- Remote sensing
- Medical image analysis
- Document processing
- Agriculture
- Defense and security
- Biometric identification
Texture

- Defined as the structural pattern of surfaces *which is homogeneous in spite of fluctuations in brightness and color*
- Most important visual cue in identifying regions
- Main categories: regularity, randomness and directionality
Texture features

• Statistical approach
  – From first order statistics (histogram)
  – From second order statistics (grey-level cooccurrence matrix)
• Spectral approach - Fourier power spectrum
• Gabor filter
• Multi-channel approaches (multiresolution wavelet)
The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)
Feature extraction methods

• Grey level co-occurrence
• Grey level run length
• Grey level difference method
• Autocorrelation function
• Autoregressive models
• Fourier power spectrum
• Voronoi tessellation features
• Structural methods
• Model based – Markov Random Field methods
• fractals
Signal processing methods

- Spatial domain filters
- Fourier domain filtering
- Gabor and wavelet
- Level set methods
- PDE diffusion filtering
Statistics

<table>
<thead>
<tr>
<th>Texture</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>$R$ (normalized)</th>
<th>Third Moment</th>
<th>Uniformity</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>82.64</td>
<td>11.79</td>
<td>0.002</td>
<td>-0.105</td>
<td>0.026</td>
<td>5.434</td>
</tr>
<tr>
<td>Coarse</td>
<td>143.56</td>
<td>74.63</td>
<td>0.079</td>
<td>-0.151</td>
<td>0.005</td>
<td>7.783</td>
</tr>
<tr>
<td>Regular</td>
<td>99.72</td>
<td>33.73</td>
<td>0.017</td>
<td>0.750</td>
<td>0.013</td>
<td>6.674</td>
</tr>
</tbody>
</table>

Texture measures for the subimages.
Generating co-occurrence matrix

Image $f$

Co-occurrence matrix $G$
Gray-Level Co-occurrence Matrix

The gray-level co-occurrence matrix $P[i,j]$ is defined by specifying a displacement vector $d=(dx,dy)$ and counting all pairs of pixels separated by $d$ having gray levels $i$ and $j$

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

$5 \times 5$ image with 3 gray levels 0, 1, and 2

The co-occurrence matrix for $d=(1,1)$
Co-occurrence Features

- The co-occurrence matrices can be computed for 0, 45, 90 and 135 degrees and at distances 1, 2, 3, ...
- Statistical features are computed from the co-occurrence matrices
- Entropy is a feature which measures the randomness of gray-level distribution
- The angular second-moment is a measure of homogeneity of the image
- The contrast is a measure of the amount of local variations present in the image
- The correlation is a measure of linearity in the image
<table>
<thead>
<tr>
<th>Descriptor</th>
<th>Explanation</th>
<th>Formula</th>
</tr>
</thead>
</table>
| Maximum probability   | Measures the strongest response of \( G \). The range of values is \([0, 1]\).                                                                                                                           | \[
\max_{ij}(p_{ij})
\]                                                                                                                                                                                 |
| Correlation           | A measure of how correlated a pixel is to its neighbor over the entire image. Range of values is 1 to \(-1\), corresponding to perfect positive and perfect negative correlations. This measure is not defined if either standard deviation is zero. | \[
\sum_{i=1}^{K} \sum_{j=1}^{K} \frac{(i - m_r)(j - m_c)p_{ij}}{\sigma_r \sigma_c}
\] \( \sigma_r \neq 0; \sigma_c \neq 0 \)                                                                                                  |
| Contrast              | A measure of intensity contrast between a pixel and its neighbor over the entire image. The range of values is 0 (when \( G \) is constant) to \((K - 1)^2\).                                                  | \[
\sum_{i=1}^{K} \sum_{j=1}^{K} (i - j)^2 p_{ij}
\]                                                                                                                                                     |
| Uniformity (also called Energy) | A measure of uniformity in the range \([0, 1]\). Uniformity is 1 for a constant image.                                                                                     | \[
\sum_{i=1}^{K} \sum_{j=1}^{K} p_{ij}^2
\]                                                                                                                                                           |
| Homogeneity           | Measures the spatial closeness of the distribution of elements in \( G \) to the diagonal. The range of values is \([0, 1]\), with the maximum being achieved when \( G \) is a diagonal matrix. | \[
\sum_{i=1}^{K} \sum_{j=1}^{K} \frac{p_{ij}}{1 + |i - j|}
\]                                                                                                                                               |
| Entropy               | Measures the randomness of the elements of \( G \). The entropy is 0 when all \( p_{ij} \)'s are 0 and is maximum when all \( p_{ij} \)'s are equal. The maximum value is \( 2 \log_2 K \). (See Eq. (11.3-9) regarding entropy). | \[
- \sum_{i=1}^{K} \sum_{j=1}^{K} p_{ij} \log_2 p_{ij}
\]                                                                                                                                                  |

**Table 11.3**

Descriptors used for characterizing co-occurrence matrices of size \( K \times K \). The term \( p_{ij} \) is the \( ij \)th term of \( G \) divided by the sum of the elements of \( G \).
FIGURE 11.30
Images whose pixels have (a) random, (b) periodic, and (c) mixed texture patterns. Each image is of size $263 \times 800$ pixels.
FIGURE 11.32 Values of the correlation descriptor as a function of offset (distance between “adjacent” pixels) corresponding to the (a) noisy, (b) sinusoidal, and (c) circuit board images in Fig. 11.30.
(a) Image showing periodic texture. (b) Spectrum. (c) Plot of $S(r)$. (d) Plot of $S(\theta)$. (e) Another image with a different type of periodic texture. (f) Plot of $S(\theta)$.

(Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)
FIGURE 11.31
256 × 256 co-occurrence matrices, $G_1$, $G_2$, and $G_3$, corresponding from left to right to the images in Fig. 11.30.
<table>
<thead>
<tr>
<th>Normalized Co-occurrence Matrix</th>
<th>Max Probability</th>
<th>Correlation</th>
<th>Contrast</th>
<th>Uniformity</th>
<th>Homogeneity</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1/n_1$</td>
<td>0.00006</td>
<td>-0.0005</td>
<td>10838</td>
<td>0.00002</td>
<td>0.0366</td>
<td>15.75</td>
</tr>
<tr>
<td>$G_2/n_2$</td>
<td>0.01500</td>
<td>0.9650</td>
<td>570</td>
<td>0.01230</td>
<td>0.0824</td>
<td>6.43</td>
</tr>
<tr>
<td>$G_3/n_3$</td>
<td>0.06860</td>
<td>0.8798</td>
<td>1356</td>
<td>0.00480</td>
<td>0.2048</td>
<td>13.58</td>
</tr>
</tbody>
</table>

**TABLE 11.4**

Descriptors evaluated using the co-occurrence matrices displayed in Fig. 11.31.
FIGURE 11.33
A zoomed section of the circuit board image showing periodicity of components.
Structural approaches

• String rule $S \rightarrow aS$, results in string $aaaS$, $a$ represents a circle.

• Add rules $S \rightarrow bA$, $A \rightarrow cA$, $A \rightarrow c$, $A \rightarrow bS$, $S \rightarrow a$

• String ‘aaabccbaa” corresponds to a 3x3 matrix of circles

• Generate texture patterns using rules. Texture primitives are used to form more complex patterns by rules that limit number of possible arrangements of primitive(s)
Figure 11.34
(a) Texture primitive.
(b) Pattern generated by the rule $S \rightarrow aS$.
(c) 2-D texture pattern generated by this and other rules.
Spectral approaches

• Fourier spectrum for describing the directionality of periodic or almost periodic 2D patterns in an image
• (1) prominent peaks in the spectrum give direction
• (2) location of peaks give fundamental spatial period of patterns
• (3) eliminating any periodic components via filtering leaves nonperiodic image elements, which can be described by statistical techniques
• Express spectrum in polar coordinates $S(r, \theta)$, for each frequency $r$, $S_r(\theta)$ is a 1-D function.
• For fixed value of $\theta$, yields behavior of spectrum in radial direction from original
• Analyzing for fixed value of $r$ yields behavior along a circle centered on the origin
• Integrating above two gives a global description.
FIGURE 11.35
(a) and (b) Images of random and ordered objects. (c) and (d) Corresponding Fourier spectra. All images are of size 600 × 600 pixels.
Figure 11.36
Plots of (a) $S(r)$ and (b) $S(\theta)$ for Fig. 11.35(a). (c) and (d) are plots of $S(r)$ and $S(\theta)$ for Fig. 11.35(b). All vertical axes are $\times 10^5$. 
Gabor filter feature extraction

• Bank of Gabor filters in the spatial domain

\[ f_{a,b}(x, y) = \frac{1}{2\pi\sigma_a^2} \exp\left\{ -\frac{x^2 + y^2}{2\sigma_a^2} \right\} \cos(2\pi(\omega_a x \cos \theta_b + \omega_a y \sin \theta_b)) \]

• Response from convolving Image sample with filter

\[ G_{dab}(x, y) = X_d(x, y) \ast f_{ab}(x, y) \]
Daubechies wavelet

- Scaling function (h)
- Wavelet function (g)
Wavelet decomposition of an image

- Approximate subimages are further decomposed
- Detail images constitute the wavelet coefficients ($C_{ij}$)
Multi-resolution wavelet features

• A 2-level wavelet transform of the image is constructed using the orthonormal Daubechies filter

• Features

1. Wavelet energy

\[ f_7 = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} |C_{ij}| \]

2. Variance

\[ f_8 = \left[ \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} |C_{ij} - M|^2 \right]^{1/2} \]

3. Residual energy

\[ f_9 = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} |C_{ij} - M| \]
Classifier distance metric equation

\[ D_{ij} = \sum_{r=1}^{q} \left( \frac{f_r^i - f_r^j}{\alpha(f_r)} \right)^2 \]

where

- \((f_1^i, f_2^i, ..., f_q^i)\) is the feature vector for texture class i
- \((f_1^j, f_2^j, ..., f_q^j)\) is the feature vector for texture class j
- \(q\) is the number of features
- \(\alpha(fr)\) is the std. deviation of the feature fr over the C texture classes.
Classifications of IKONOS image (La Parguera)

- 4 bands
  - R, Near IR, G, B

Total features = 4
(Band R spectral value and 3 Statistical features)
8x8 regions
Accuracy (testing) = 99%

Maximum likelihood classification using 4 bands of spectral values
Benthic habitat classification
IKONOS image

Coral/algae
Seagrass
Sand
Water
Mangroves

4 bands – R, NIR, G and B
Resolution - 1mt.

Class map – using spectral and gray level statistical features - 3x3 regions
Total features = 5 (NIR band and 4 statistical features)
Accuracy (testing samples) = 95.0745%