Supplement Exercises 4

Chapter 4.4: Inductive Proof

Definitions and examples

1. What is proof by the principle of mathematical induction?
2. What is proof by well-founded induction?
3. Give a carefully written induction proof of the following equation for all natural numbers $n \geq 0$. $1+3+5+\ldots+(2n+1)=(n+1)^2$
4. Given the following program: f(n)=if n=0 then 0 else 2*n+f(n-1). Use induction to prove that f(n)=n(n+1) for all $n \in N$.

Chapter 3.2: Recursive functions and procedures

Definitions and examples

1. What steps are needed to define a function recursively?
2. How is a sum of numbers recursively defined
3. How is a product of numbers recursively defined
4. What does n! mean
5. What do recursive definitions have to do with inductively defined sets?
6. Write a recursive definition for the function f: N→N defined by f(n)=02+12+..+n2.
7. Given the set S that is inductively defined by basis: $2 \in S$. Induction: If $x \in S$, then $x+7 \in S$.
   (a) Find a recursive definition for the function g: N-{0}→S such that g(n) is the nth element of S constructed from the basis element.
   (b) Find a recursive definition for the function f :S→{true,false} such that f(x) is true if and only if $x \in S$.
8. Write a recursive definition for the function f such that f(x,y) tests to see whether the length of string x is less than the length of string y.
9. The function f defined below takes two lists as input: f(A,B)=if A=<> then B else f(tail<A>, head(A)::B). Evalue the expression f(<a,b,c>,<>) by unfolding the definition of f.
10. Write a recursive definition for the function f defined by f(n)=<1,3,5,…,2n+1>, where n is a natural number.