Supplement Exercises 2 (Exam II)

1. Given the list <+,1,<*,x,<+,2,<*,x,<+,3,<*,x,4>>>>>>> write out the algebraic expression represented by the list, * is multiplication. Draw the tree that the list represents.
2. Draw the binary tree represented by the expression:
   <<<< >,a,< >>,b,< >>,c,<< >,d,<< >,e,<>>>>

Chapter 2: Facts about Functions

Definitions and examples

1. What are f, A, and B in the expression f: $A \rightarrow B$?
2. What is f(C), the image of C under f?
3. What is the range of a function
4. What is $f^{-1}(D)$, the pre-image of D under f?
5. What is the floor and the ceiling function
6. What is gcd(a,b)
7. What is a mod b
8. What is $\log_b x$.
9. Evaluate expressions
10. Evaluate floor and ceil expressions, gcd and mod expressions.
11. Evaluate gcd(235,110), gcd(15,63), 25 mod 4, -16 mod 3, $\log_2(1024)$, $\log_2(2^5 4^3)$.

Constructing functions

1. What does $f_o g$ mean?
2. What is the map function
3. Evaluate: dist(x,<a,b,c>), seq(4).
4. For each function find the image of the set {1,2,3}: f(x)=ceiling($\log_2$(x))
5. Express f as a composition of given functions.

Properties of functions

1. What is an injective function
2. What is a surjective function
3. What is a bijective function
4. What does f^{-1} mean
5. What is a hash function
6. What properties of functions are useful to ciphers
7. Given a property, define a function that satisfied the property. Choose domain and co-domain from the given sets.
8. For each property, find an example of a function $f: N \rightarrow N$ satisfying the given condition
9. Show that a function is injective, etc.
10. The function $f: N_7 \rightarrow N_7$ defined by f(x) =3x mod 7 is bijective. Find it's inverse.
11. Give a function f: $N_8 \rightarrow N_8$ of the form f(x) = (ax +b)mod 8 such that f is a bijection with no fixed points (ie. a good cipher function).

Additional Exercises:

I.TREES

Find the parsing tree for the following expressions

a)$x^2+2xy-5$
b) $(x-2)^2+2(x-2)y$
c) xy-(x-y)(x+y)

Find spanning trees for the following graphs

a) G={{1, 2, 3, 4, 5,6}, {{1,2}, {2,4}, {4,6}, {4,5}, {2,5}, {1,3}, {3,5}}) starting at 6, and starting at 1

b) G = {{A,B,C,D,E,F,G}, {{A,B}, {A,C}, {B,D}, {B,E}, {C,F}, {C,G}, {B,C}, {D,E}, {F,G}}) starting at C, and starting at F

Draw a picture for the tree whose list representation is

a) <<<<I>,E,<J>>,B,<<K>,F,<L>>>,A,<<<M>,H,<N>>,C,<>>>>>

b) <A,<B>,<C, <E>,<F>>,<D,<J,<K>,<L>>>>

Find an adequate list representation (binary style if the tree is binary) for each of the following trees

a) T=({1, 2, 3, 4, 5,6},{{3,5}, {3,4}, {5,6}, {4,2}, {4,1}})

b) T =({A,B,C,D,E,F,G,H,I}, {{A,B}, {A,C}, {A,D}, {C,E}, {C,F}, {D,G}, {D,H}, {D,I}})

FUNCTIONS

1. Design an algorithmic function to
a. Compute nth power of a given real number.
b. Compute factorial of a natural number
2. Write the type of the following functions:
    a) Sum(numbers)=sum of numbers
    b) Count(A)= number of elements in A
    c) Int(A,B)=intersection of A and B
    d) Uni(A,B)= union of A and B
    e) Dif(A,B)=difference of A and B
3. Compute the gcd of (a) (2700,-50), (b) (-297,351)
4. Write the composition of the functions (a) F(x) = x mod 20 G(x) = floor(x) x is real.
(b) F(x,n)=xn, G(n,x) = ceil (nx+12) , n is natural and x is real