Supplement Exercises 3 (Exam III)

Chapter 4.2: Equivalence relations

Definitions and examples

1. What is an equivalence class
2. What is a partition
3. What does [x] mean?
4. What is a kernel relation
5. Describe Kruskal's algorithm
6. Let R be defined on N by xRy iff |x-y| is odd. Show that R is not an equivalence relation on N
7. Given the relation over integers defined by a~b iff |a|=|b|, either prove that ~ it is an equivalence relation or not.
8. The binary relation R on N defined by xRy iff xmod 3=y mod 3 is an equivalence relation. Describe the partition of N induced by the relation.
9. Given the binary relation R on N defined by xRy iff |x-y| is even. Verify that R is an equivalence relation. Describe the partition of N induced by R.
10. A graph with vertex set {a,b,c,d,e} has its edges sorted by weight as <{a,b}, {b,d},{a,d},{c,e},{c,d}, {b,e}>. Trace Kruskal's algorithm to find a minimal spanning tree for the graph by showing the value of the spanning tree T and the corresponding equivalence classes produced at each stage of the algorithm, including initialization.

Chapter 4.2: Order relations

1. What are the two characteristics of a partial order relation
2. What do successor and predecessor mean for a poset
3. What does it mean to topologically sort a poset
4. What is the minimal element of a subset of a poset? Maximal element?
5. What does <A,≤ > mean
6. What does <A,< >mean
7. What does x<y mean
8. What does x≤y mean
9. Let S be the set of strings consisting of the days of the week. S={Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} A partial order can be defined on S by letting x<y iff |x|<|y|, where |s| is the length of string s. Draw a poset diagram of <S,< >.
10. Let D = {2,3,4,6,9,12,18,36} and for any x,y$\in D$ $let\ x < y\ iff\ x|y\ (ie.\ x\ divides\ y)$. Let S = {4,6,12}. Find the minimal element of S, maximal element of S.
11. Let D100 be the set of positive divisors of 100. Write down a topological sort of the elements in the poset <D100, | >. Draw the poset diagram for this poset.

Chapter 3.1: Inductively defined sets

1. What steps are needed to define a set inductively
2. Why is the closure case important
3. Describe the set S defined by $4 \in S$, and if $x \in S$, then $2x+1 \in S$
4. Write down an inductive definition for the set A={2,4,6,..}
5. Write an inductive definition for the set S of even integers
6. Write an inductive definition for $S = \{x \mid x \in Z \text{ and } x \bmod 5 = 0\}$
7. Write an inductive definition for A = {0,2,4,6,8,..} U {1,4,9,16,25,…}
8. Write an inductive definition for C ={L |L ∈lists({a,b}) where the a’s and b’s alternate}
9. Write an inductive definition for the set S of all binary trees over {a}
10. Write an inductive definition for the set B of binary trees over {a} where each node either has two identical children or no children
11. Write an inductive definition for S = {<a>, <a,b>, <a,b,b>,<a,b,b,b>,…}
12. Describe the set S defined by : $a \in S$, and if $x \in S$, then $bx \in S$.