Oversampling A/D Converter

- For digital processing of an analog continuous-time signal, the signal is first passed through a S/H circuit whose output is then converted into a digital form by means of an A/D converter
- According to the sampling theorem, a band-limited continuous-time signal woth a lowpass spectrum can be fully recovered from its uniformly sampled version if it is sampled at a sampling frequency that is at least twice the highest frequency contained in the signal

Oversampling A/D Converter

- If the condition of the sampling theorem is not satisfied, the original continuous-time signal cannot be recovered from is sampled version because of aliasing
- To prevent aliasing, the analog signal is thus passed through an analog anti-aliasing lowpass filter prior to sampling to enforce the condition of the sampling theorem

Oversampling A/D Converter

- The passband cutoff frequency of the lowpass filter is chosen equal to the frequency of the highest signal frequency component that needs to be preserved at the output
- The anti-aliasing filter also cuts off all out-of-band signal components and any high-frequency noise that may be present in the original analog signal, which otherwise would alias into the baseband after sampling

Oversampling A/D Converter

- The filtered signal is then sampled at a rate that is at least twice that of the cutoff frequency
- Let the signal band of interest be the frequency range $0 \le f \le F_m$
- Then, the Nyquist rate is given by $2F_m$
- Now, if the sampling rate F_T is the same as the Nyquist rate, we need to use before the sampler an anti-aliasing filter with a very sharp cutoff in its frequency response

Oversampling A/D Converter

- This requires the design of a very high-order anti-aliasing filter structure built with high-precision analog components, and it is usually very difficult to implement such a filter in VLSI technology
- Moreover, such a filter also introduces undesirable phase distortion in its output

Oversampling A/D Converter

- An alternate approach is to sample the analog signal at a rate much higher than the Nyquist rate, use a fast low-resolution A/D converter, and then decimate the digital output of the A/D converter to the Nyquist rate
- This approach relaxes the sharp cutoff requirements of the analog anti-aliasing filter, resulting in a simpler filter structure that can be built using low-precision analog components

Oversampling A/D Converter

- But, requiring fast, more complex digital signal processing hardware at a later stage
- The overall structure is not only amenable to VLSI fabrication but also can be designed to provide linear-phase response in the signal band of interest
- Consider a b-bit A/D converter operating at F_T Hz

Oversampling A/D Converter

- Now, for a full-scale peak-to-peak input analog voltage of R_{FS}, the smallest volatge step represented by b bits is

$$\Delta V = \frac{R_{FS}}{2^b - 1} \cong \frac{R_{FS}}{2^b}$$

- The rms quantization noise power σ_e^2 of the error voltage, assuming a uniform distribution of the error between $-\Delta V/2$ and $\Delta V/2$, is given by $$\sigma_e^2 = \frac{(\Delta V)^2}{12}$$

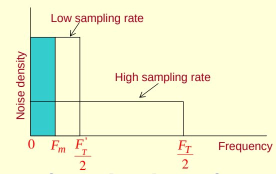
Oversampling A/D Converter

- The rms noise voltage, given by σ_e, therefore has a flat spectrum in the frequency range from 0 to $F_T/2$
- The noise power per unit bandwidth, called the noise density, is then given by

$$P_{e,n} = \frac{(\Delta V)^2/12}{F_T/2} = \frac{(\Delta V)^2}{6F_T}$$

- A plot of the noise densities for two different sampling rates is shown in the next slide

Oversampling A/D Converter



- As can be seen from the above figure, the total amount of noise in the signal band of interest for the high sampling rate case is smaller than that for the low sampling rate case

Oversampling A/D Converter

- The total noise in the signal band of interest, called the in-band noise power, is given by

$$P_{total} = \frac{(R_{FS}/2^b)^2}{12} \cdot \frac{F_m}{F_T/2}$$

- Substituting $F_T = 2F_m$ and replacing b with β in the above equation, we get

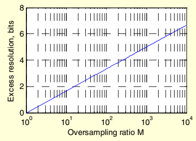
$$P_{total} = \frac{(R_{FS}/2^\beta)^2}{12} = \frac{(R_{FS}/2^b)^2}{12} \cdot \frac{F_m}{F_T/2}$$

Oversampling A/D Converter

- From the last equation we arrive at $$\beta = b + \tfrac{1}{2}\log_2 M$$ where $M = F_T/2F_m$ is the oversampling ratio (OSR)
- Thus, $\beta - b$ denotes the increase in the resolution of a b-bit converter whose oversampled output is filtered by an ideal brick-wall lowpass filter

Oversampling A/D Converter

- A plot of the increase in resolution as a function of the oversampling ratio is shown below



- For example, for an OSR of $M = 1000$, an 8-bit oversampling A/D converter has an effective resolution equal to that of an 13-bit A/D converter operating at the Nyquist rate

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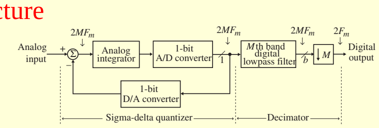
Oversampling A/D Converter

- Further improvement in the noise performance is obtained by employing a sigma-delta ($\Sigma\Delta$) quantization scheme
- The figure below shows the block diagram of an oversampling sigma-delta A/D converter structure



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Oversampling A/D Converter

- Note: The 1-bit output samples of the quantizer after decimation become b-bit samples at the output of the sigma-delta A/D converter due to the filtering operations involving b-bit multiplier coefficients of the M-th band digital lowpass filter
- To analyze the operation of the sigma-delta quantizer, it is convenient to use the discrete-time equivalent circuit shown in the next slide

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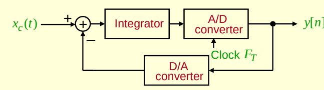
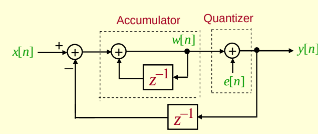
Oversampling A/D Converter

- Note: In the equivalent circuit, the integrator has been replaced with an accumulator



Sigma-delta quantizer



Discrete-time equivalent

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Oversampling A/D Converter

- In the discrete-time equivalent circuit, the input $x[n]$ is a discrete-time sequence of analog samples developing an output sequence of binary-valued samples $y[n]$
- Note: At each discrete instant of time, the circuit forms the difference (Δ) between the input and the delayed output, which is accumulated by a summer (Σ) whose output is then quantized by a one-bit A/D converter

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Oversampling A/D Converter

- Even though the input-output relation of the sigma-delta converter is basically nonlinear, the low-frequency content of the input $x_c(t)$ can be recovered from the output $y[n]$ by passing it through a digital lowpass filter
- This property can be easily shown for a constant input analog signal $x_a(t)$ with a magnitude less than +1

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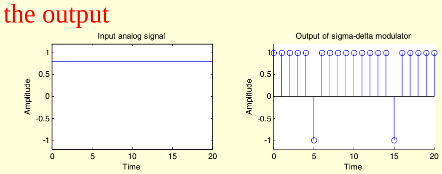
Oversampling A/D Converter

- In this case, the output $w[n]$ of the accumulator is a bounded sequence with sample values equal to either -1 or $+1$
- This can happen only if the input to the accumulator has an average value of zero
- Or in other words, the average value of $w[n]$ must be equal to the average value of the input $x[n]$

Oversampling A/D Converter

- Program 15_11 illustrates the operation of a sigma-delta quantizer
- It develops the binary equivalent of a constant input signal and computes the average value of the output

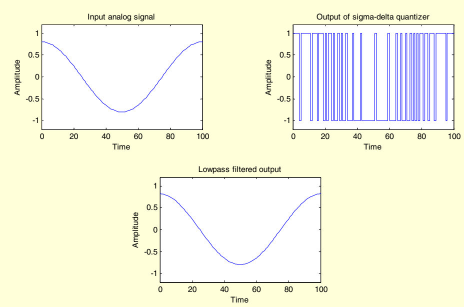


- Average value of output is 0.8095

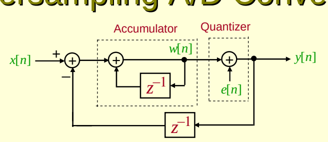
Oversampling A/D Converter

- Note: The average value of the output gets closer to the amplitude 0.8 of the input as the length of the input increases
- Program 15_12 is next used to verify the operation of the sigma-delta quantizer for a sinusoidal input of frequency 0.01 Hz
- Figures on next page show the input and the output of the sigma-delta quantizer, and the lowpass filtered version of the quantizer output

Oversampling A/D Converter



Oversampling A/D Converter



- Analysis of the discrete-time equivalent of the sigma-delta quantizer yields

$$y[n] = w[n] + e[n]$$

where

$$w[n] = x[n] - y[n-1] + w[n-1]$$

Oversampling A/D Converter

- From the last two equations we arrive at

$$y[n] = x[n] + (e[n] - e[n-1])$$

where the quantity inside the parentheses represents the noise due to sigma-delta modulation

- The noise transfer function is simply

$$G(z) = (1 - z^{-1})$$

Oversampling A/D Converter

- The power spectral density of the modulation noise is therefore given by

$$P_y(f) = \left|G(e^{j2\pi fT})\right|^2 P_e(f) = 4\sin^2\left(\tfrac{2\pi fT}{2}\right) P_e(f)$$

- In the above expression, we have assumed the power spectral density $P_e(\omega)$ of the quantization noise to be the one-sided power spectral density defined for positive frequencies only

Oversampling A/D Converter

- For a random signal input $x[n]$, $P_e(f)$ is constant for all frequencies and is given by

$$P_e(f) = \frac{(\Delta V)^2/12}{F_T/2}$$

- Substituting the above equation in the equation in the previous slide, we arrive at the power spectral density of the output noise, given by

$$P_y(f) = \frac{2}{3}\cdot\frac{(\Delta V)^2}{F_T}\sin^2(\pi/T)$$

Oversampling A/D Converter

- For a very large OSR, as is usually the case, the frequencies in the signal band of interest are much smaller than F_T
- We can thus approximate $P_y(f)$ as

$$P_y(f) \cong \frac{2}{3}\frac{(\Delta V)^2}{F_T}(\pi fT)^2 = \frac{2}{3}\pi^2(\Delta V)^2 T^3 f^2, \qquad f \ll F_T$$

Oversampling A/D Converter

- From the above equation, the in-band noise of the sigma-delta A/D converter is thus given by

$$P_{total,sd} = \int_0^{F_m} P_y(f)df$$
$$= \frac{2}{3}\pi^2(\Delta V)^2 T^3 \int_0^{F_m} f^2 df$$
$$= \frac{2}{9}\pi^2(\Delta V)^2 T^3 (F_m)^3$$

Oversampling A/D Converter

- Now, the in-band noise power of an oversampling A/D converter is given by

$$P_{total,os} = \frac{1}{6}(\Delta V)^2 T F_m$$

- The improvement in the noise performance resulting from the use of sigma-delta quantizer in the oversampling A/D converter is thus given by

$$10\log_{10}\left(\frac{P_{total,os}}{P_{total,sd}}\right) = -5.1718 + 20\log_{10}(M) \ \ dB$$

Oversampling A/D Converter

- In deriving the last equation, we have used $M = F_T/2F_m$
- For example, for an OSR of $M = 1000$, the improvement in the noise performance using the sigma-delta modulation scheme is about 55 dB
- In this case, the increase in the resolution is about 1.5 bits per doubling of the OSR

Oversampling A/D Converter

- The improved noise performance of the sigma-delta A/D converter results from the shape of $|G(e^{j2\pi fT})|$, which decreases the noise power spectral density in-band $0 \le f \le F_m$, while increasing it outside the band of interest ($f > F_m$)
- Since this type of converter also employs oversampling, it requires less stringent analog anti-aliasing filter

Oversampling A/D Converter

- As indicated in the figure in Slide 14, the quantizer output is passed through an M-th band lowpass digital filter whose output is then down-sampled by a factor-of-M to reduce the sampling rate to the desired Nyquist rate
- The function of the lowpass filter is to eliminate the out-of-band quantization noise and the out-of-band signals that would be aliased into the passband by the down-sampling operation

Oversampling A/D Converter

- As a result, the filter must exhibit a very sharp cutoff frequency response with a passband edge at F_m
- This necessitates the use of a very high-order digital filter
- In practice, it is preferable to use a filter with a transfer function having simple integer-valued coefficients to reduce the cost of hardware implementation and to permit all multiplication operation to be carried out at the doen-sampled rate

Oversampling A/D Converter

- In addition, most application requires the use of linear-phase digital filters, which can be easily implemented using FIR filters
- The simplest lowpass FIR filter is the N-point moving average filter given by the transfer function

$$H(z) = 1 + z^{-1} + z^{-2} + \cdots + z^{-(N-1)}$$

where we have ignored the scale factor $1/N$, which is needed to provide a dc gain of 0 dB

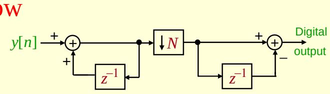
Oversampling A/D Converter

- A more convenient form of $H(z)$ is given by

$$H(z) = (1 - z^{-N})/(1 - z^{-1})$$

also known as the recursive running-sum filter or a boxcar filter

- A realization of a factor-of-N decimator based on the above running-sum filter is shown below

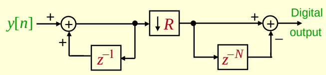


Oversampling A/D Converter

- Since the decimator based on a running-sum filter does not provide sufficient out-of-band attenuation, often a multistage decimator formed by a cascade of the running-sum decimators, more commonly known as cascaded integrator comb (CIC) filters, is used in practice

Oversampling A/D Converter

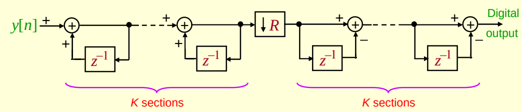
- The structure of a 2-stage CIC decimator is shown below



- It can be easily shown that the above structure is equivalent to a factor-of-R decimator with a length-RN running sum decimation filter

Oversampling A/D Converter

- Further flexibility in the design of a CIC decimator is obtained by including K feedback paths before and K feedforward paths after the down-sampler as indicated below



Oversampling A/D Converter

- The corresponding transfer function is given by

$$H(z) = \left(\frac{1 - z^{-RN}}{1 - z^{-1}} \right)^K$$

- The parameters N and K can be adjusted for a given down-sampling factor R to yield the desired out-of-band attenuation

Oversampling D/A Converter

- The digital-to-analog conversion process consists of two steps:
- (1) The conversion of input digital signal into a staircase continuous-time waveform by means of a D/A converter with a zero-order hold at its output
- (2) Filtering of the staircase waveform by an analog lowpass reconstruction filter

Oversampling D/A Converter

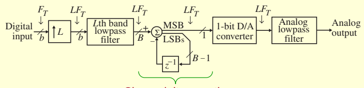
- If the sampling rate F_T of the input digital signal is the same as the Nyquist rate, the analog lowpass reconstruction filter must have a very sharp cutoff in its frequency response
- This involves the design of a very high-order analog filter with high precision analog circuit components
- To get around the above problem, often an oversampling approach is used

Oversampling D/A Converter

- The oversampling approach leads to the design of the reconstruction filter with a wide transition band resulting in its implementation with low-precision circuit components
- However, the approach requires a more complex digital interpolation filter at the front end
- Further improvement in the performance of an oversampling D/A converter is obtained by employing a digital sigma-delta 1-bit quantizer at the output

Oversampling D/A Converter

- The block diagram representation of an oversampling sigma-delta D/A converter is shown below



Sigma-delta quantizer

- The quantizer extracts the MSB from its input and subtracts the remaining LSBs, the quantization noise, from its input

Oversampling D/A Converter

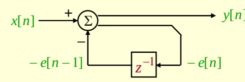
- The MSB output is then fed into a 1-bit D/A converter and then passed through an analog lowpass reconstruction filter to remove all frequency components beyond the signal band of interest
- Since the signal band occupies a very small portion of the baseband of the high-sample-rate signal, the reconstruction filter can have a very wide transition band

Oversampling D/A Converter

- As a result, a very low-order analog reconstruction filter can be used which can also be implemented using a Bessel filter to provide an approximately linear phase in the signal band
- The spectrum of the quantized 1-bit output of the digital sigma-delta quantizer is nearly the same as that of its input

Oversampling D/A Converter

- The quantizer also shapes the quantization noise spectrum by moving the noise power out of the signal band of interest



- The input-output relation of the sigma-delta quantizer shown above is given by

$$y[n]-e[n]=x[n]-e[n-1]$$

Oversampling D/A Converter

- Or, equivalently, by

$$y[n]=x[n]+e[n]-e[n-1]$$

where $y[n]$ is the MSB of the n-th sample of the adder output, and $e[n]$ is the n-th sample of the quantization noise composed all bits except the MSB

- From the above equation, it can be seen that with no quantization noise, the noise transfer function is given by $G(z)=1$

Oversampling D/A Converter

- The noise transfer function then is given by

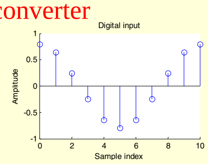
$$G(z)=1-z^{-1}$$

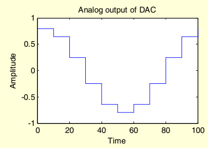
which is the same as that for the first-order sigma-delta modulator employed in the oversampling A/D converter discussed earlier

- We illustrate next the oversampling D/A conversion

Oversampling D/A Converter

- Figure below on the left show a digital sinusoidal sequence of frequency 100 Hz operating at a sampling rate F_T of 1 kHz
- The figure below on the right shows the analog staircase output of a conventional D/A converter





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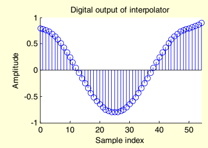
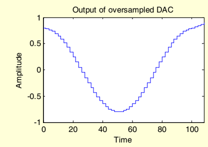
Oversampling D/A Converter

- Figures below show the output obtained by passing the the low-sampling-rate digital signal through a factor-of-5 interpolator and the corresponding analog output generated by a D/A converter operating at $5F_T$





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Oversampling D/A Converter

- Note: The staircase output of the oversampling D/A converter is much smoother with smaller jumps than that of the lower-rate D/A converter
- ➡ The oversampling D/A converter output has considerably smaller high-frequency components in contrast to the lower-rate D/A converter

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Oversampling D/A Converter

- Program 15_13.m can be used to illustrate using MATLAB the operation of the sigma-delta D/A converter
- Figures in the next slide show a sinusoidal sequence of frequency 100 Hz operating at a sampling rate of 5 kHz, the corresponding digital output of the sigma-delta quantizer, and the lowpass filtered analog output generated using Program 15_13.m

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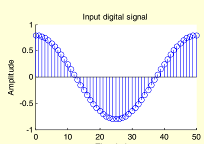
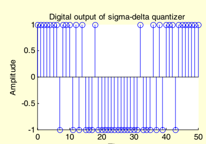
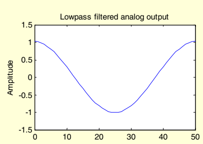
Oversampling D/A Converter







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Oversampling D/A Converter

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