Overview of Discrete-Time Filters

- First-order filters
- Ideal filters
- Practical filters
- Frequency-selective filter specifications
- Ripple versus filter order tradeoff
- Application example

Discrete-Time Filters Overview

$$
\begin{aligned}
\sum_{k=0}^{N} a_k \, y[n-k] &= \sum_{k=0}^{M} b_k \, x[n-k] \\
Y(\mathrm{e}^{j\omega}) &= \frac{\sum_{k=0}^{M} b_k \mathrm{e}^{-jwk}}{\sum_{k=0}^{N} a_k \mathrm{e}^{-jwk}} X(\mathrm{e}^{j\omega})
\end{aligned}
$$

- Discrete-time filters are divided into two categories
  - **Finite impulse response (FIR)**: $h[n] = 0$ for some a and b such that $-\infty < a < n < b < +\infty$
  - **Infinite impulse response (IIR)**: not FIR
- Filters that can be described with difference-equations
  - FIR: $N = 0$
  - IIR: $N > 0$
- A simple FIR filter is the moving average filter
- A simple IIR filter is the first-order lowpass filter

Example 1: First-Order Filters

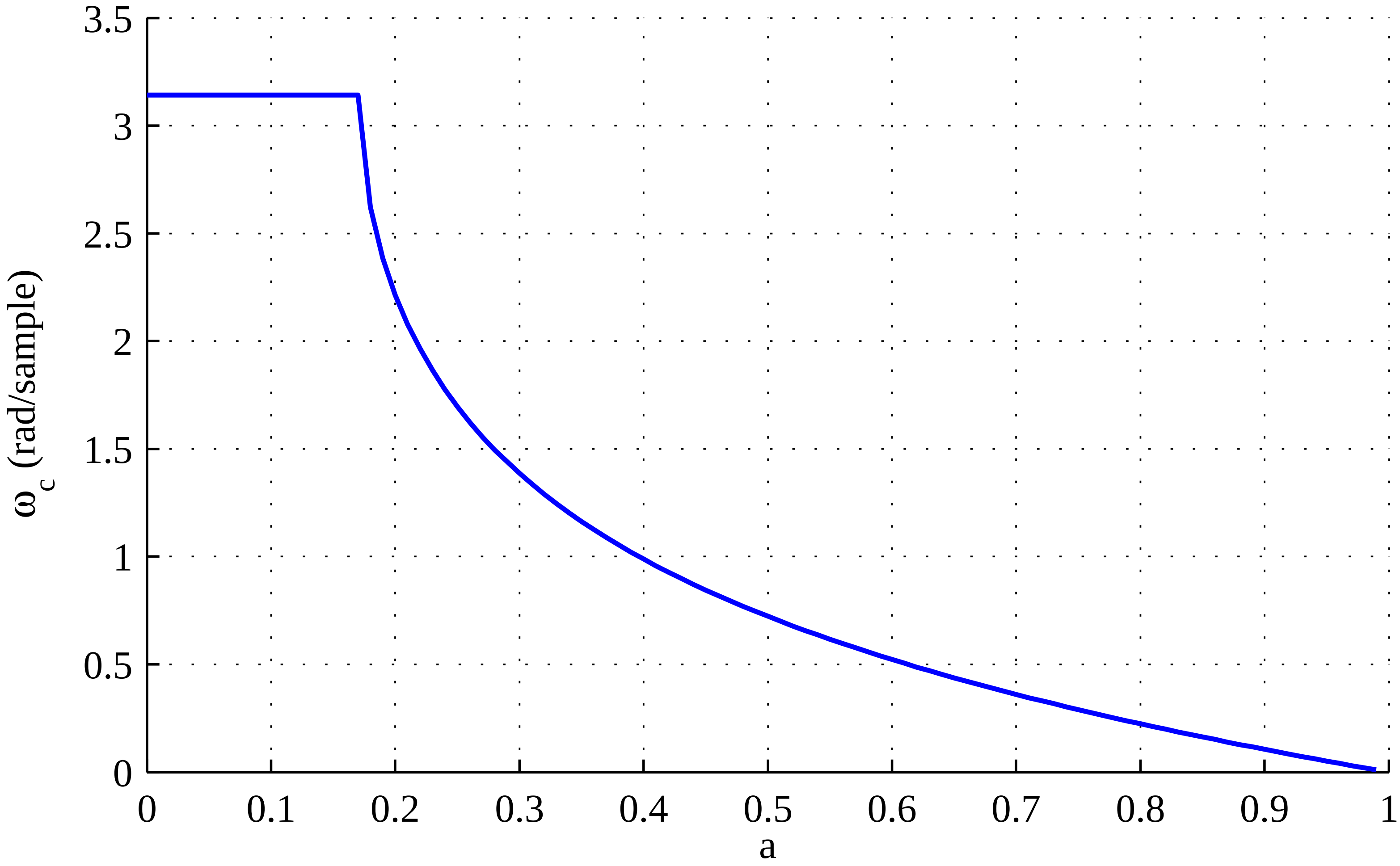
Consider the following filter:

$$y[n] - ay[n-1] = (1-a)x[n]$$

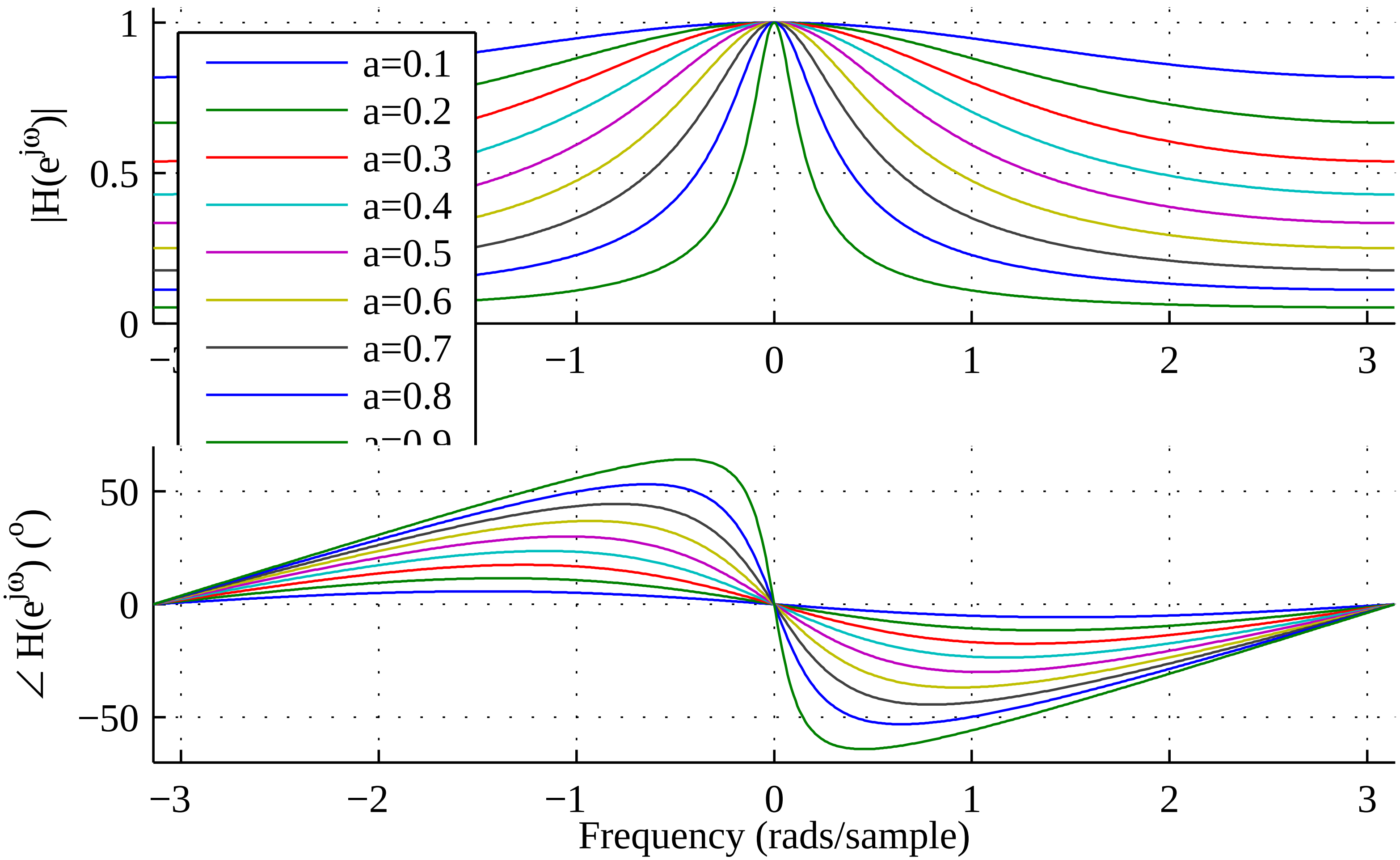
1. Solve for the filter's transfer function
2. Find the cutoff frequency as a function of a

Example 1: Workspace

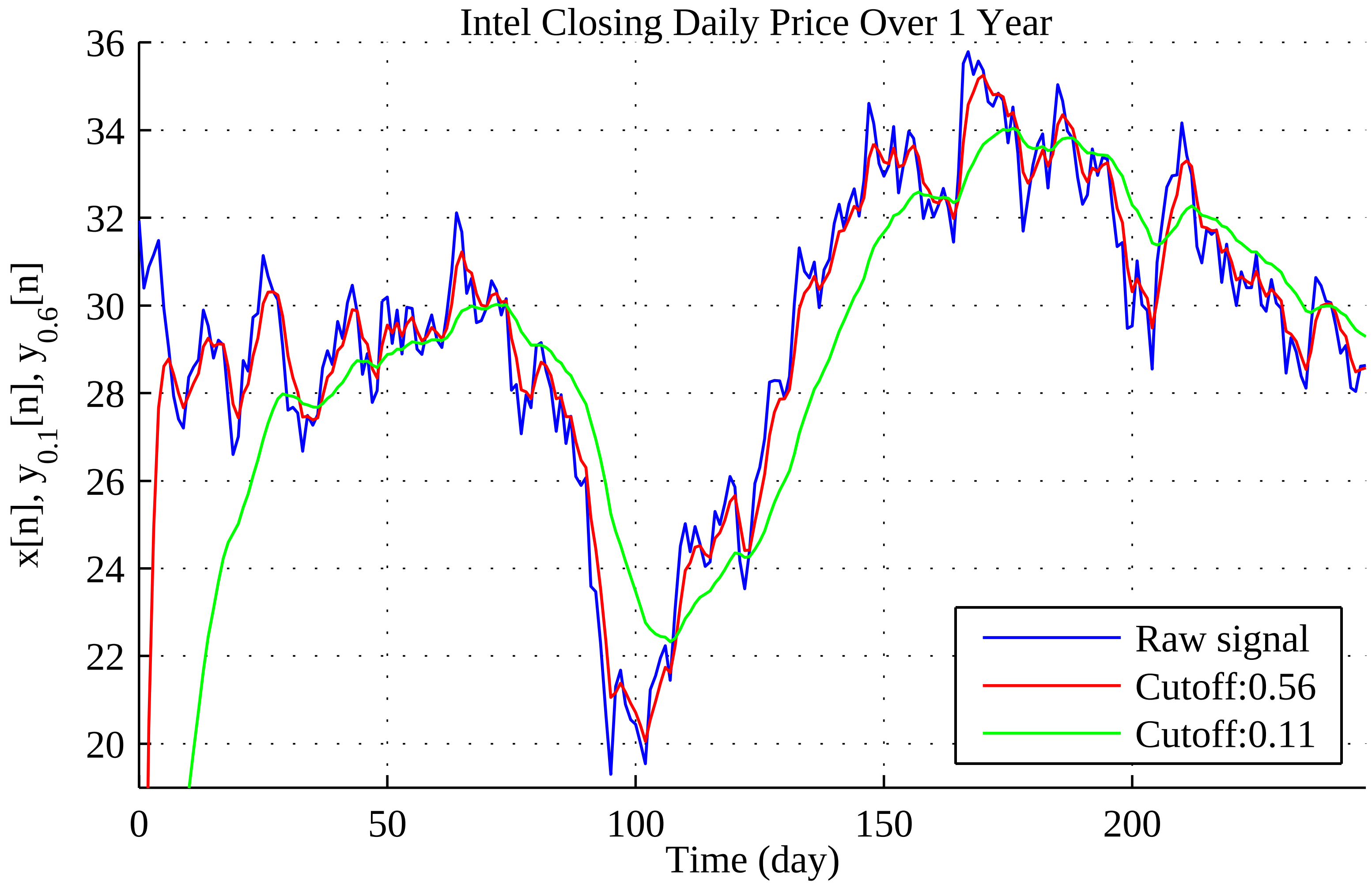
Example 1: Ω_c versus a



Example 1: $H(e^{j\omega})$ for various a



Example 1: Filtered Signals



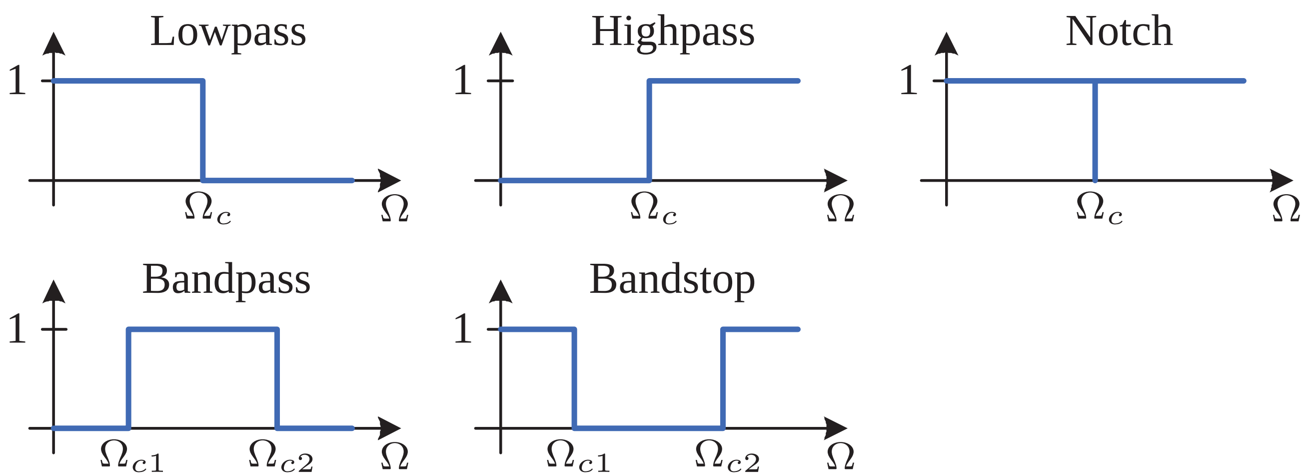
Example 1: MATLAB Code

```
%function [] = FirstOrderApplied();
close all;

d  = load('Intel.txt'); % Closing daily price
nd = length(d);
x  = d(nd:-1:1); % Reorder so first element is oldest data
n  = (0:nd-1)'; % Discrete time index

%==============================================================================
% Plot the relationship between cutoff frequency and a
%==============================================================================
a = 0.00001:0.01:1;
wc = acos((1-4*a+a.^2)./(-2*a));

figure
FigureSet(1,'LTX');
h = plot(a,wc,'LineWidth',1.0);
ylabel('\omega_c (rad/sample)');
xlabel('a');
grid on;
box off;
AxisSet(8);
print -depsc FOCutoff;

%==============================================================================
% Plot the relationship between cutoff frequency and a
%==============================================================================
a = 0.1:0.1:0.9;
w = -pi:0.01:pi;
H = zeros(length(a),length(w));

for cnt = 1:length(a)
    [h,w] = freqz(1-a(cnt),[1 -a(cnt)],w);
    H(cnt,:) = h;
```

```
    end;

figure
FigureSet(1,'LTX');
subplot(2,1,1);
    h = plot(w,abs(H));
    xlim([-pi pi]);
    ylim([0 1.05]);
    legend('a=0.1','a=0.2','a=0.3','a=0.4','a=0.5','a=0.6','a=0.7','a=0.8','a=0.9',2);
    ylabel('|H(e^{j\omega})|');
    grid on;
    box off;
subplot(2,1,2);
    h = plot(w,rem(angle(H)*180/pi,180));
    xlim([-pi pi]);
    ylim([-70 70]);
    ylabel('\angle H(e^{j\omega}) (^o)');
    xlabel('Frequency (rads/sample)');
    grid on;
    box off;
AxisSet(8);
print -depsc FOTransferFunctions;


%================================================================================
% Filter & Plot
%================================================================================
a   = 0.58; % cutoff frequency approximately 0.1
w1 = acos((1-4*a+a.^2)./(-2*a));
y1 = zeros(nd,1);
for cnt = 2:nd,
    y1(cnt) = a*y1(cnt-1) + (1-a)*x(cnt);
    end;

a  = 0.90; % cutoff frequency approximately 0.1
w2 = acos((1-4*a+a.^2)./(-2*a));
y2 = zeros(nd,1);
```

```
for cnt = 2:nd,
    y2(cnt) = a*y2(cnt-1) + (1-a)*x(cnt);
    end;

figure
FigureSet(1,'LTX');
h = plot(n,x,'b',n,y1,'r',n,y2,'g');
set(h,'LineWidth',0.6);
ylabel('x[n], y_{0.1}[n], y_{0.6}[n]');
xlabel('Time (day)');
title('Intel Closing Daily Price Over 1 Year');
xlim([0 nd-1]);
ylim([19 36]);
box off;
grid on;
AxisSet(8);
legend('Raw signal',sprintf('Cutoff:%4.2f',w1),sprintf('Cutoff:%4.2f',w2),4);
print -depsc FOSignalFiltered;
```

Ideal Filters



- MATLAB can be used to design standard frequency selective filters that meet user-specified requirements
- These filters include: lowpass, highpass, bandpass, and bandstop
- Unlike continuous-time filters, these must have cutoff frequencies that range between 0 and π

Practical Filters

- Practical filters are usually designed to meet a set of specifications
- Lowpass and highpass filters usually have the following requirements
  - Passband range
  - Stopband range
  - Maximum ripple in the passband
  - Minimum attenuation in the stopband
- If we know the specifications, we can ask MATLAB to generate the filter for us
- There are four popular types of standard filters
  - Butterworth
  - Chebyshev Type I
  - Chebyshev Type II
  - Elliptic

Ripple Tradeoff

Filter	Order	Passband	Stopband
Butterworth	Largest	Smooth	Smooth
Chebyshev Type I	Moderate	Ripple	Smooth
Chebyshev Type II	Moderate	Smooth	Ripple
Elliptic	Lowest	Ripple	Ripple

- The four popular filter types differ in how they satisfy the specifications
- In the passband and stopband, each filter is either *smooth* or contains *ripple*
- Elliptic filters are also called **equiripple filters** and **Cauer filters**

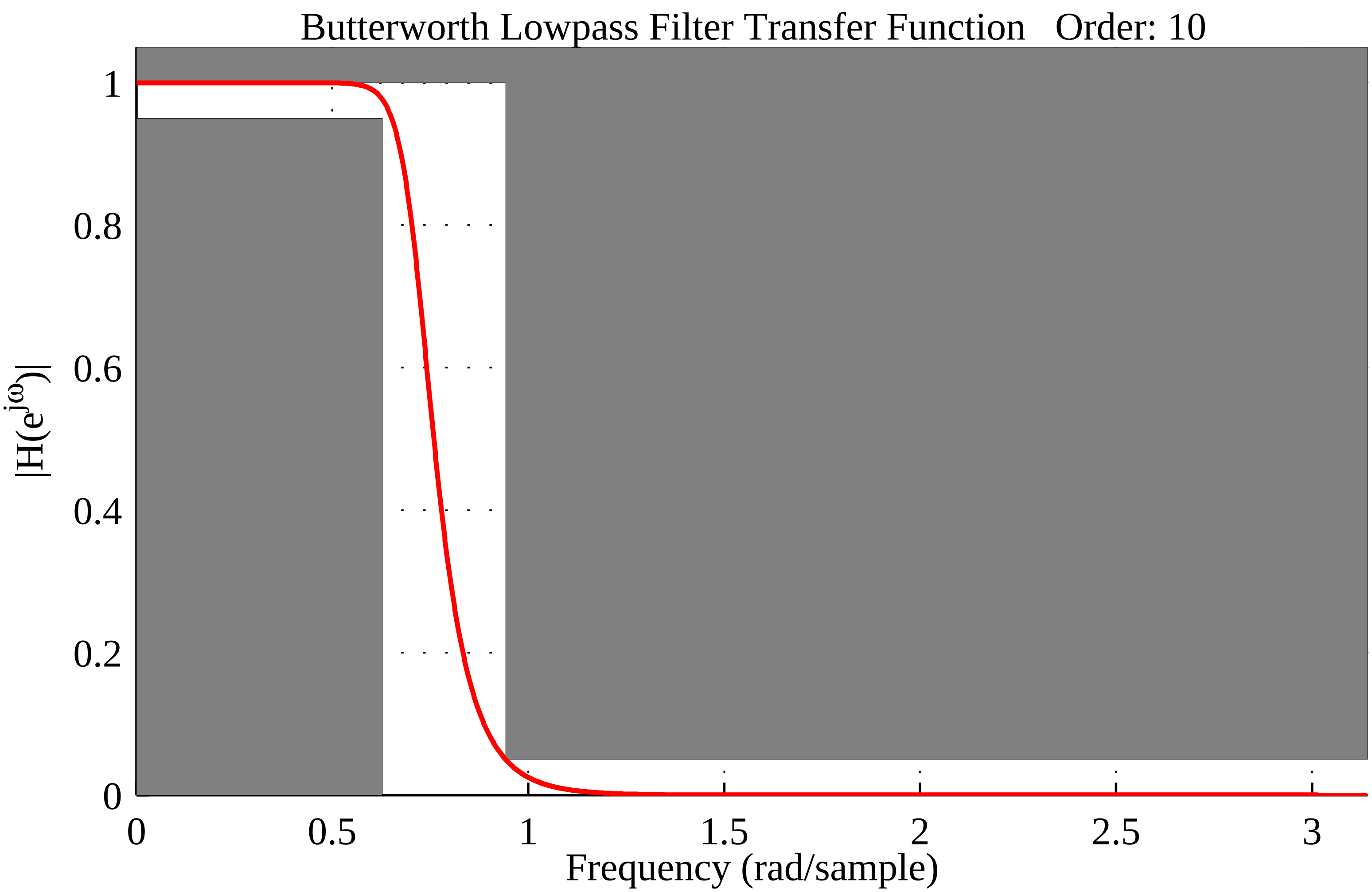
Example 2: Lowpass Filter Specifications

Design a lowpass filter that meets the following specifications:

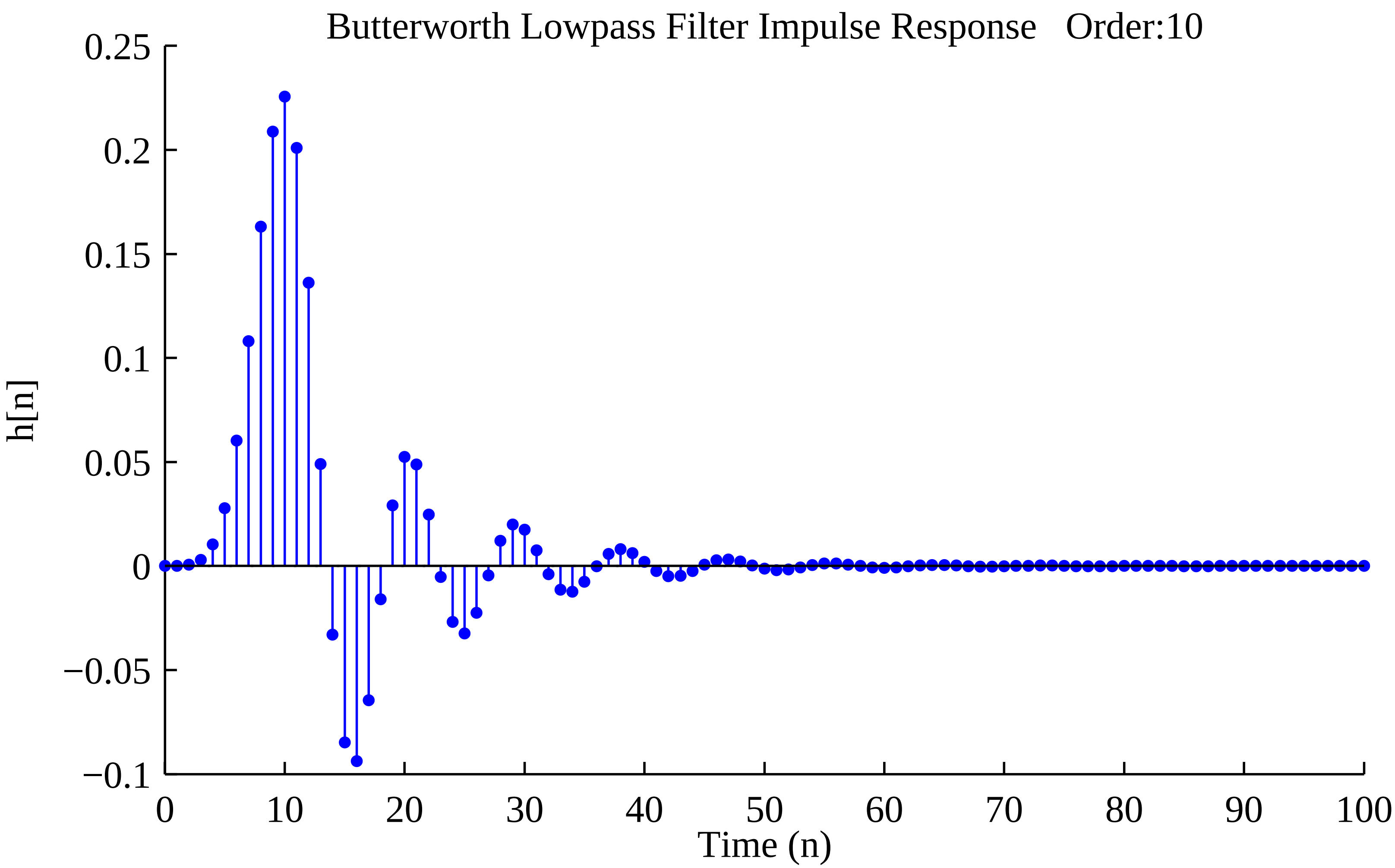
- The passband ripple is no more than 0.4455 dB ($0.95 \leq |H(\mathrm{e}^{j\omega})| \leq 1$)
- The stopband attenuation is at least 26.02 dB ($|H(\mathrm{e}^{j\omega})| \leq 0.05$)
- The passband ranges from 0–0.2π rad/sample
- The stopband ranges from 0.3π–π rad/sample

Plot the magnitude of the resulting transfer function on a linear-linear plot, the impulse response, and the step response. Try the Butterworth, Chebyshev I, Chebyshev II, and elliptic filters.

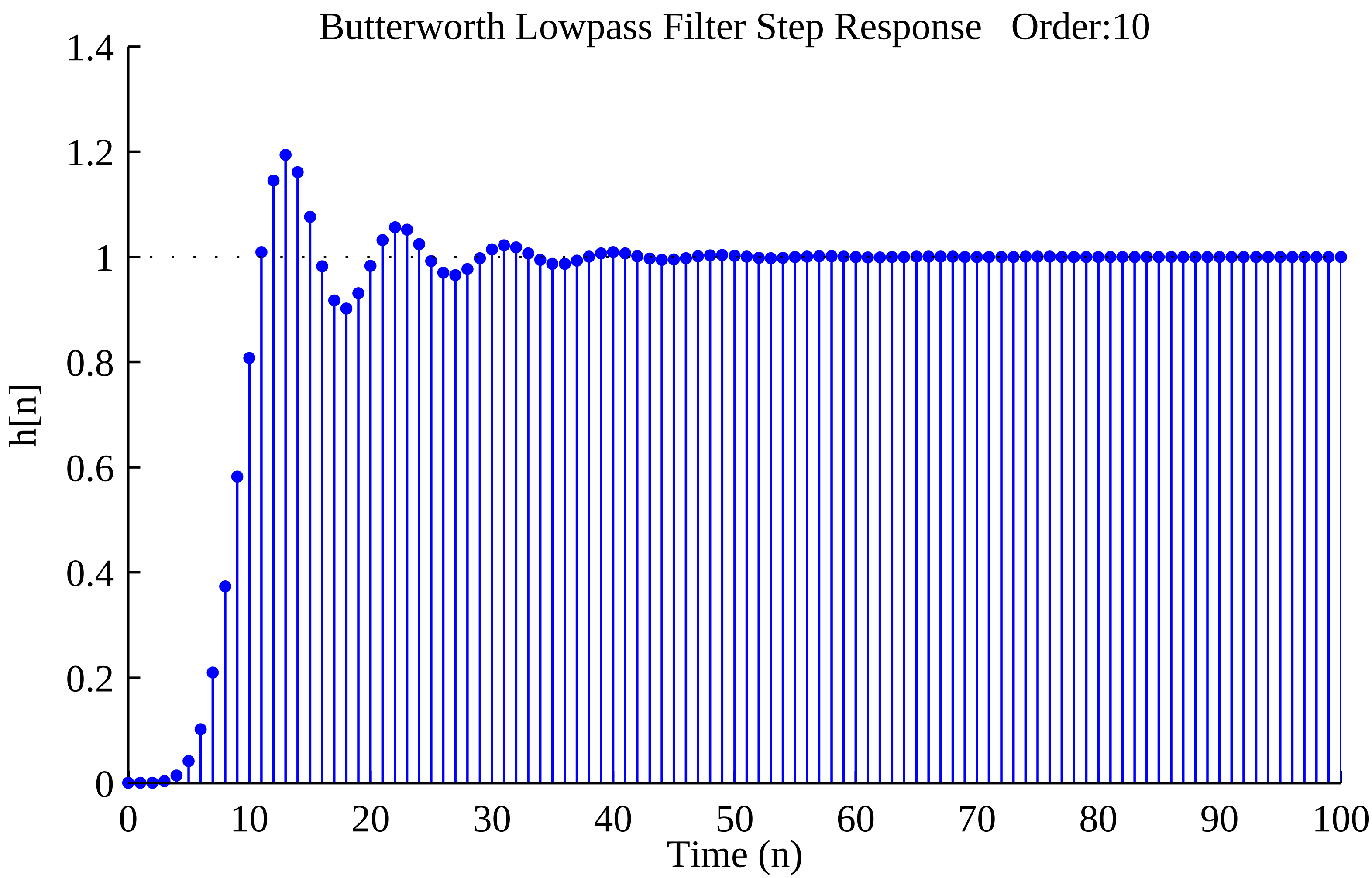
Example 3: Butterworth Lowpass



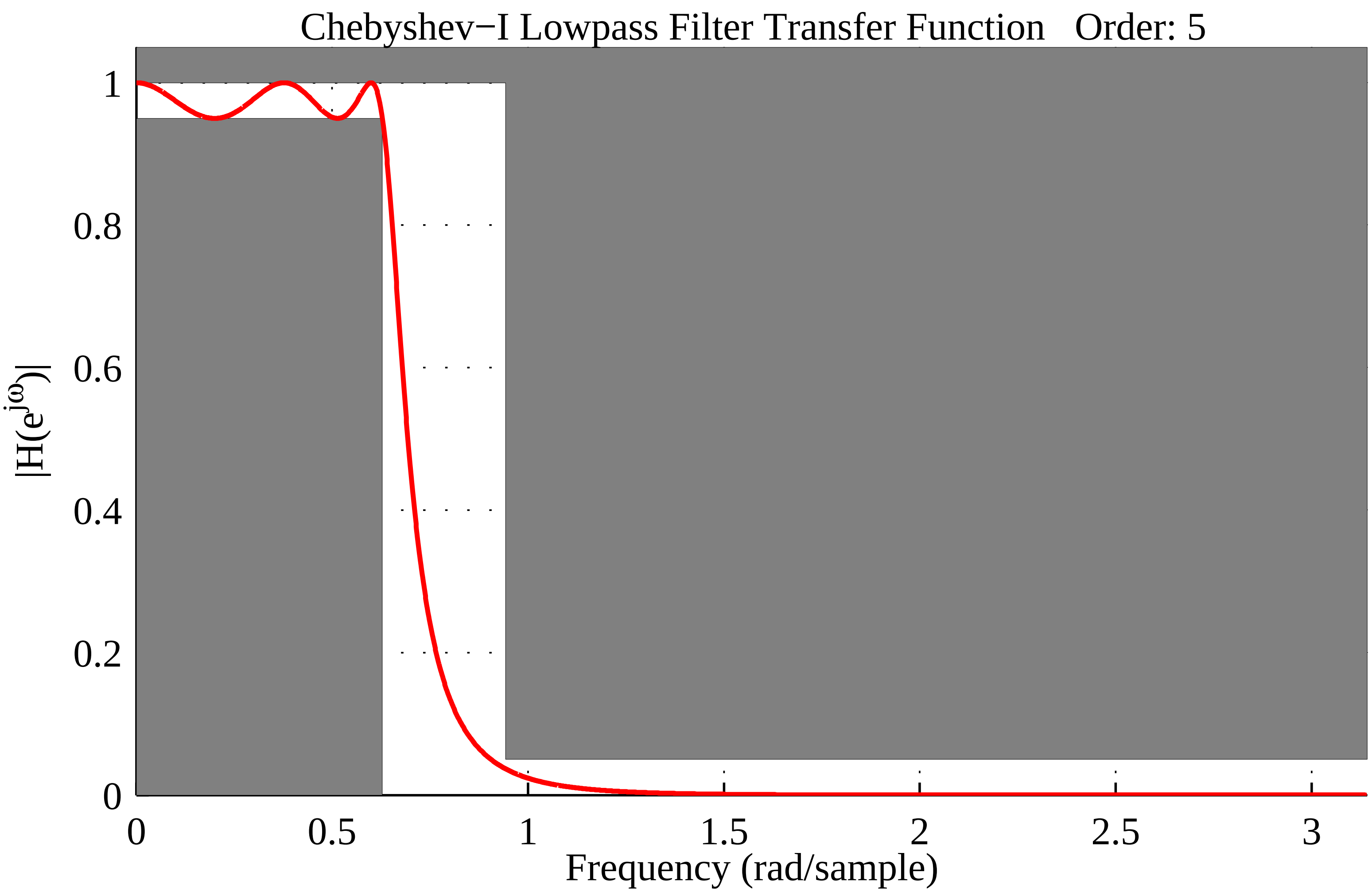
Example 3: Butterworth Lowpass



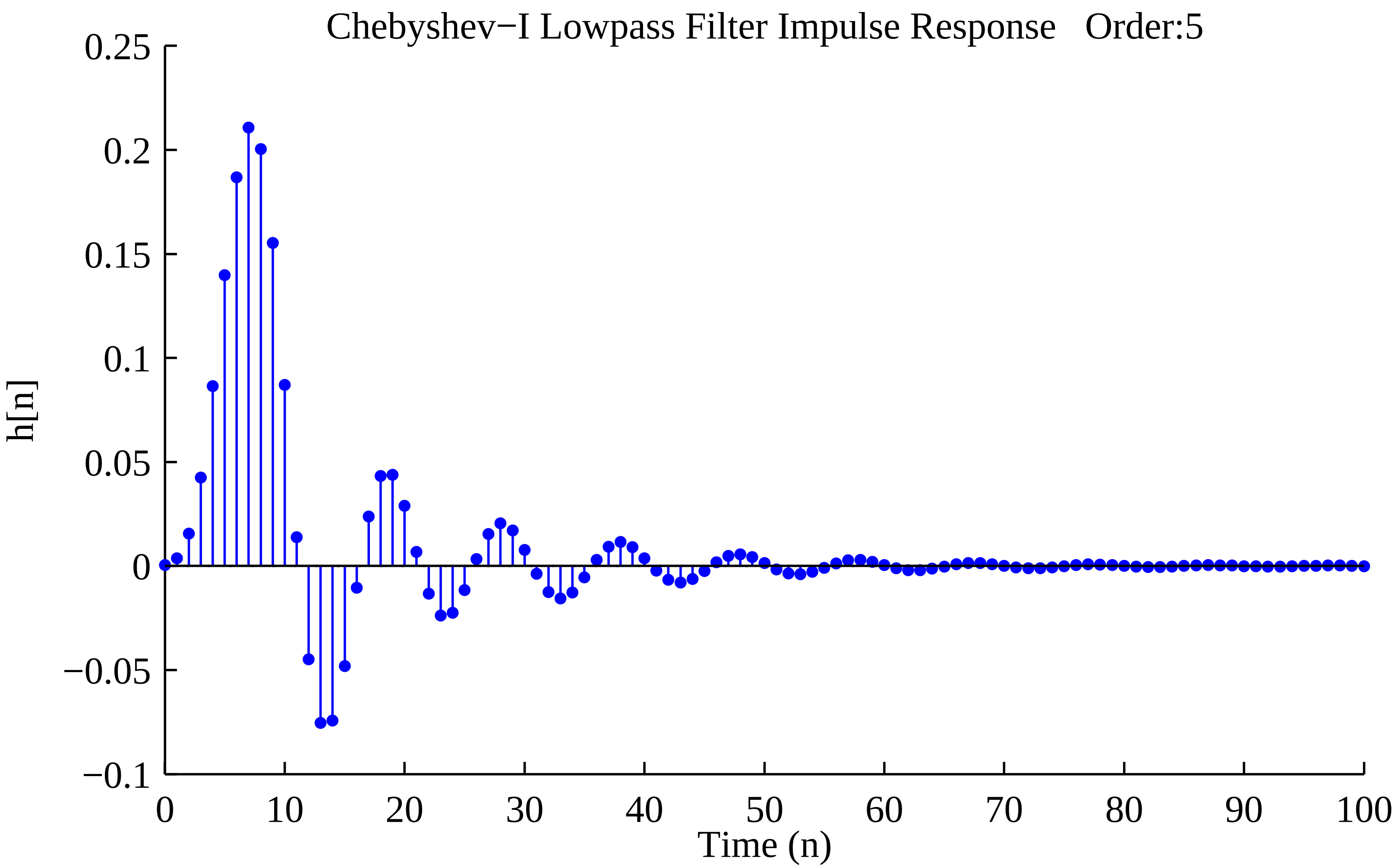
Example 3: Butterworth Lowpass



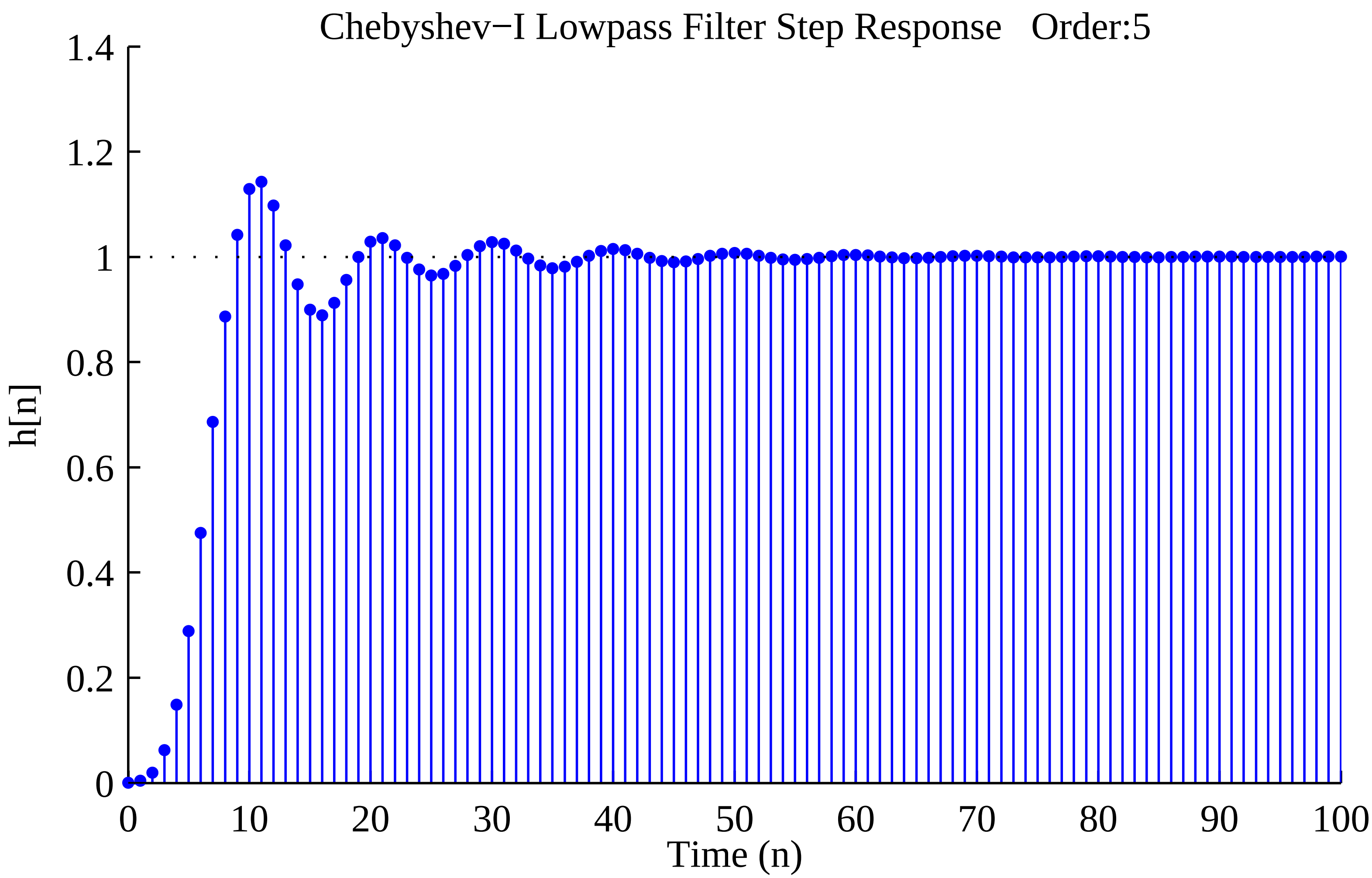
Example 3: Chebyshev-I Lowpass



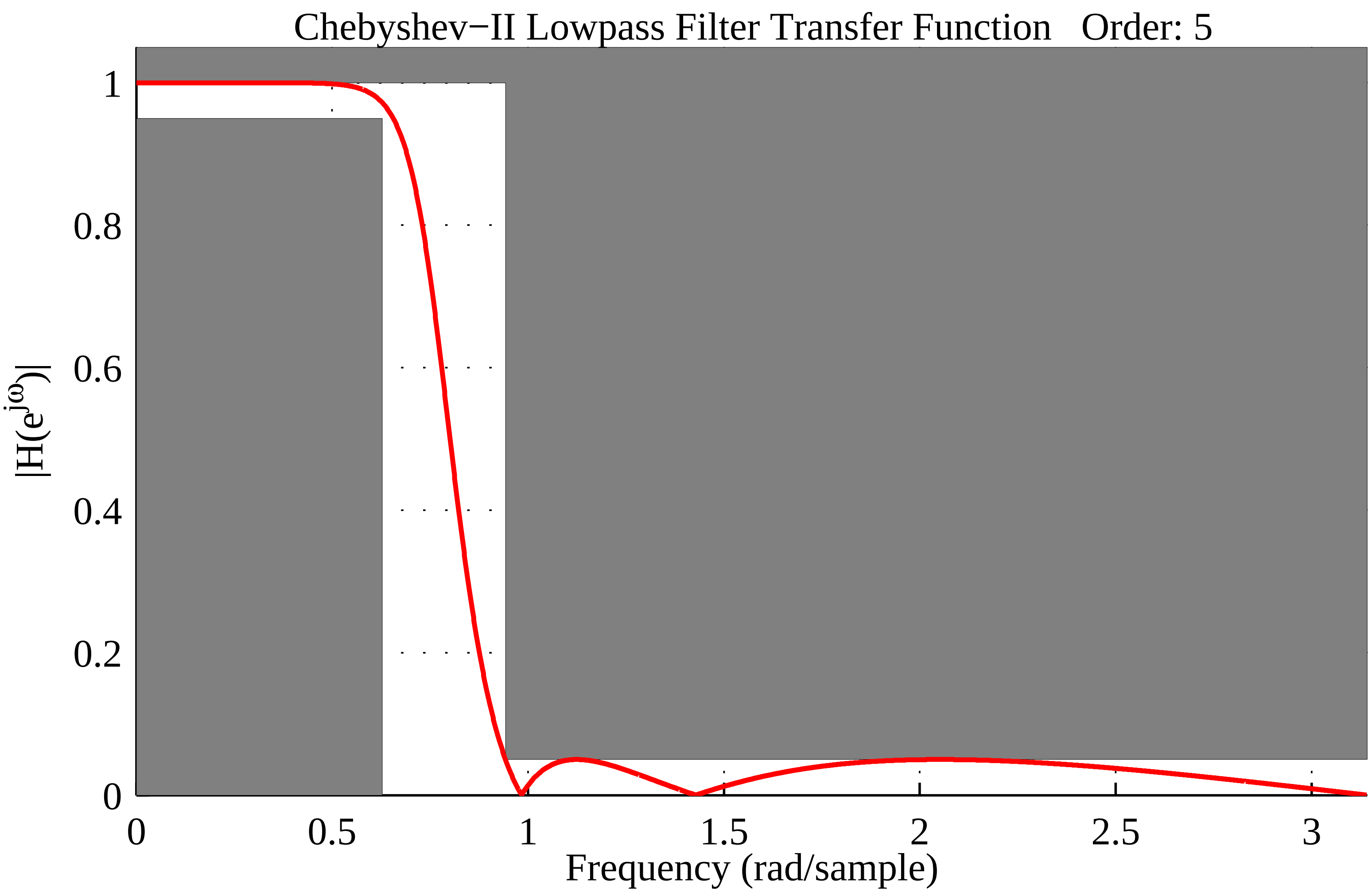
Example 3: Chebyshev-I Lowpass



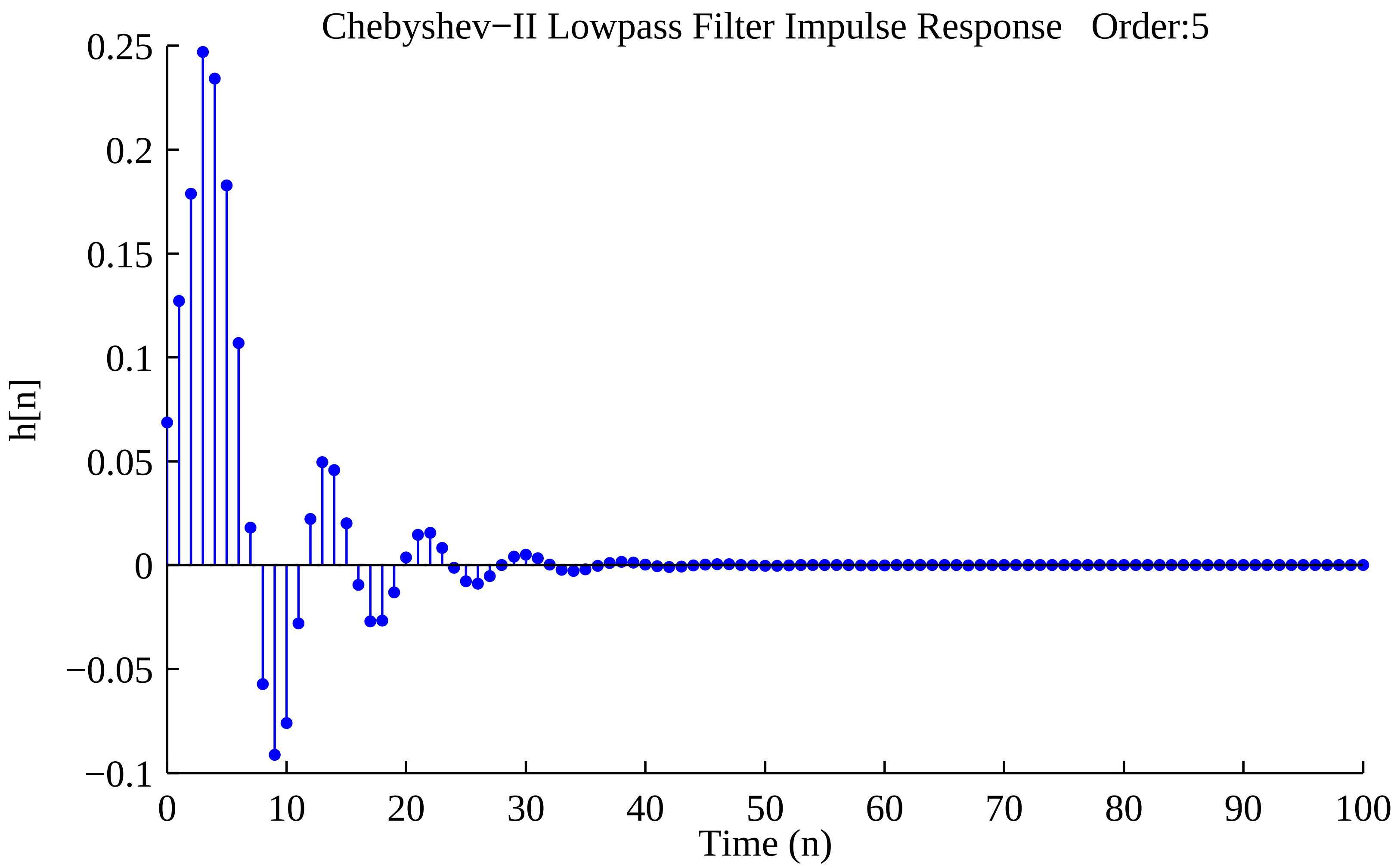
Example 3: Chebyshev-I Lowpass



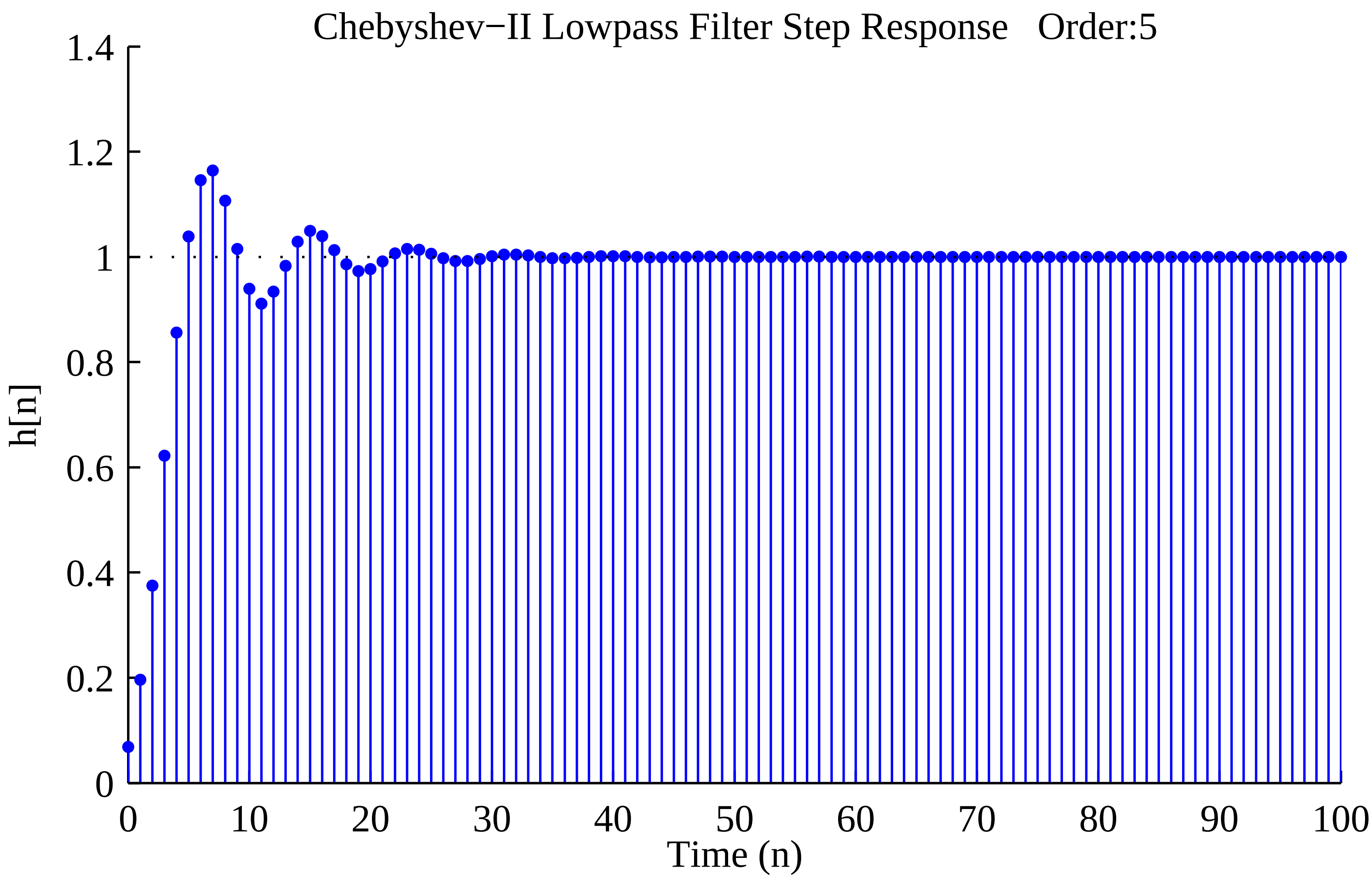
Example 3: Chebyshev-II Lowpass



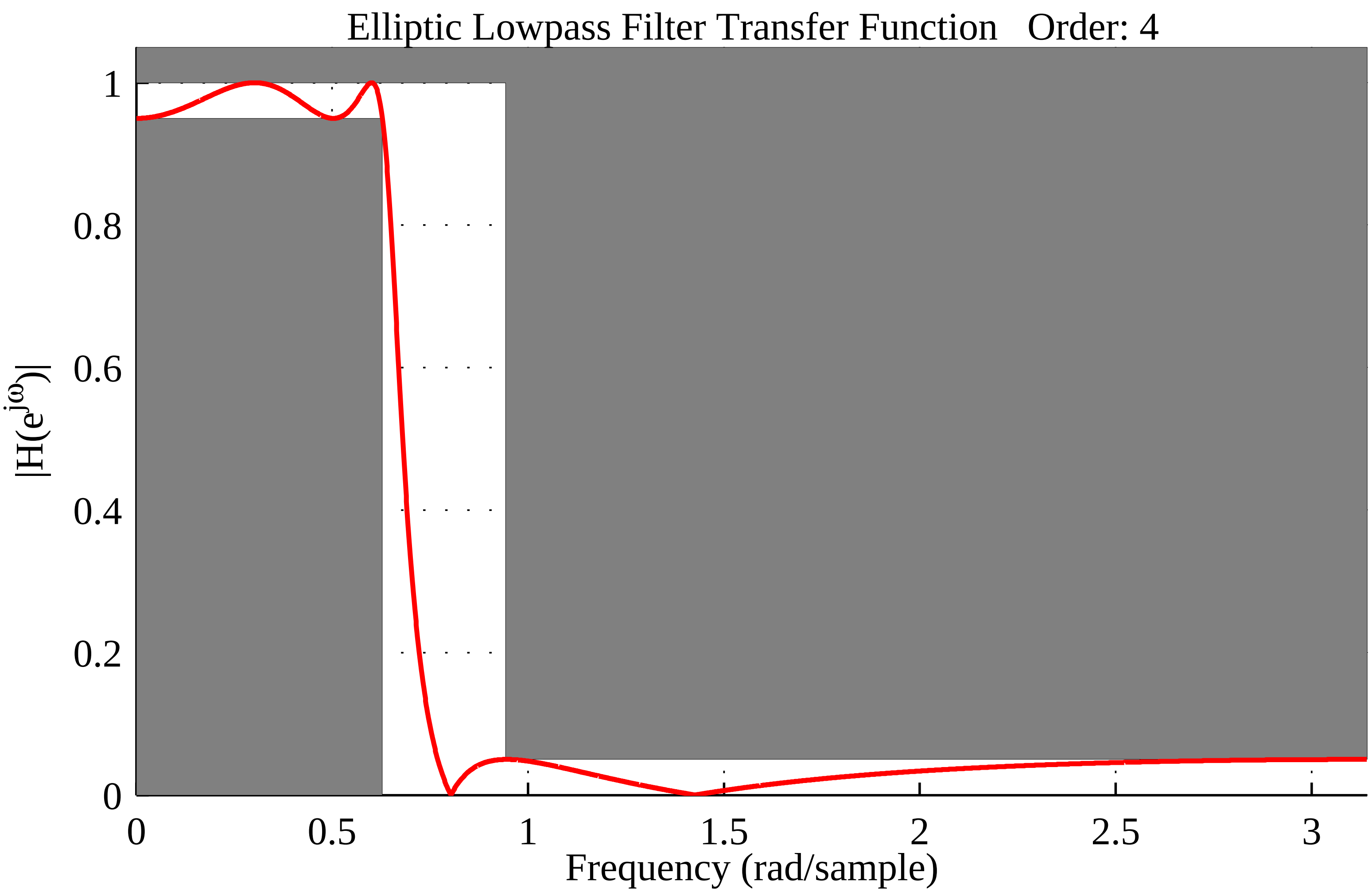
Example 3: Chebyshev-II Lowpass



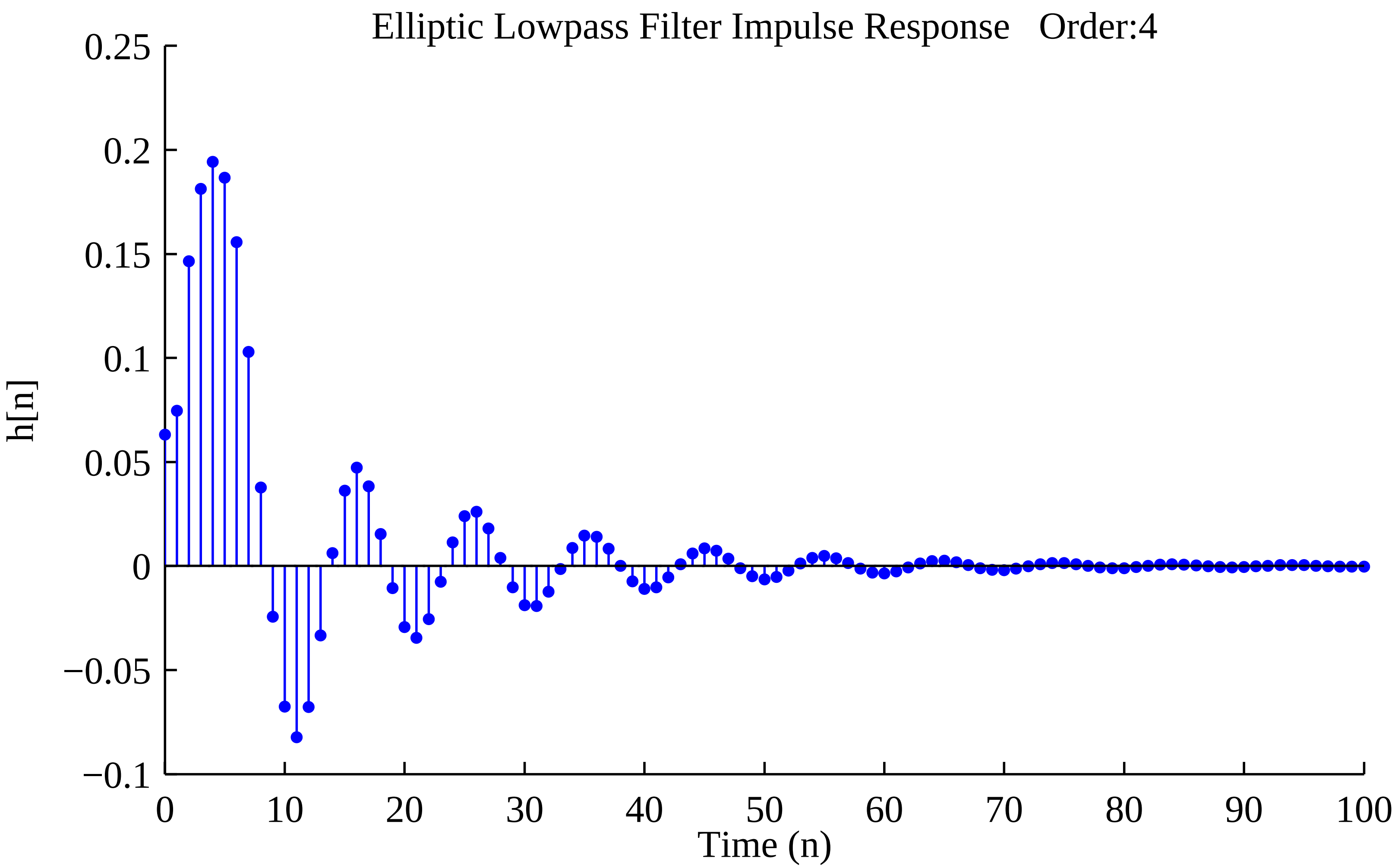
Example 3: Chebyshev-II Lowpass



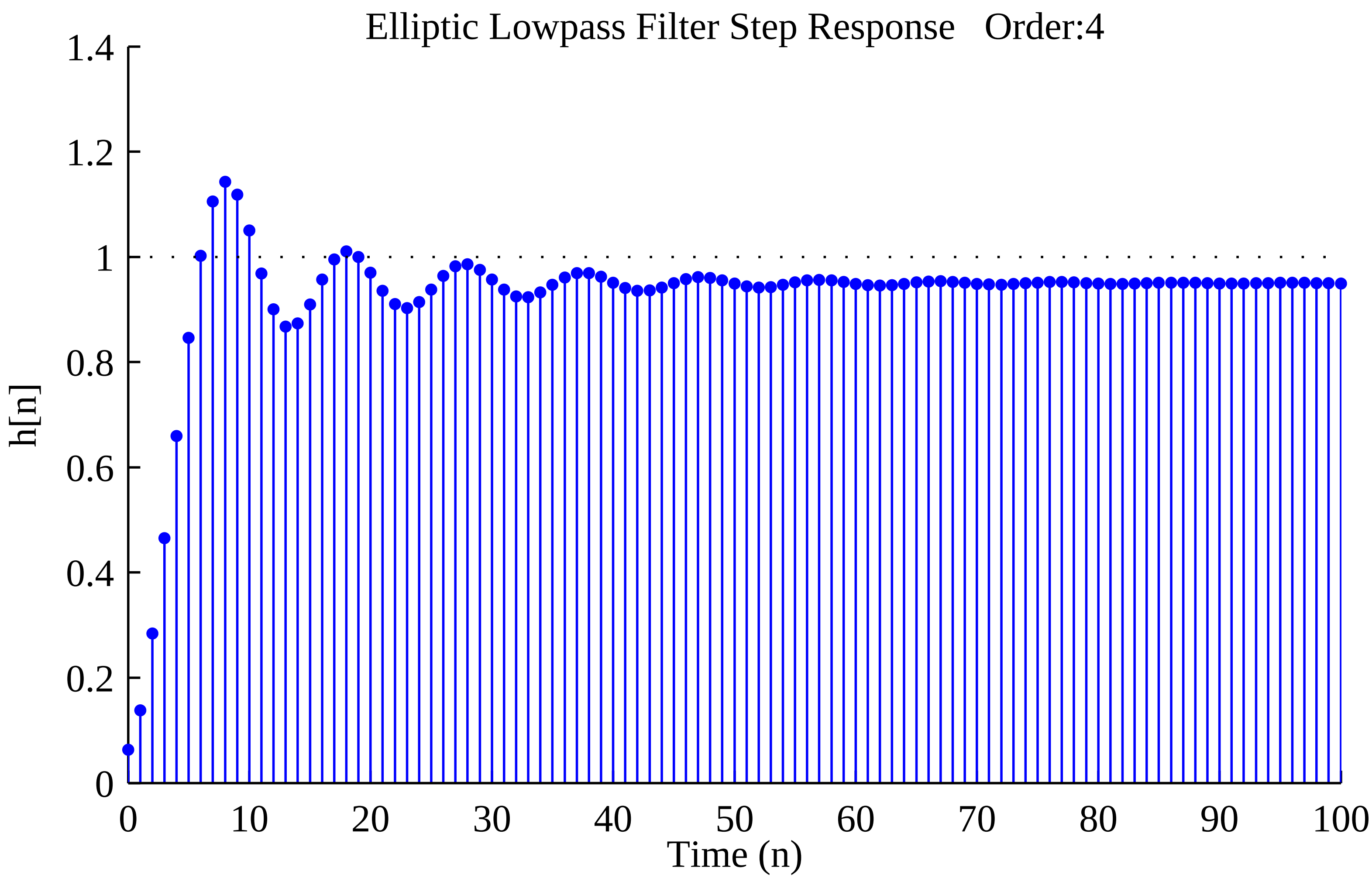
Example 3: Elliptic Lowpass



Example 3: Elliptic Lowpass



Example 3: Elliptic Lowpass



Example 3: MATLAB Code

```
%function [] = Lowpass();
clear all;
close all;

Wp = 0.20;            % Passband ends
Ws = 0.30;            % Stopband begins
Rp = -20*log10(0.95); % Maximum deviation from 1 in the passband (dB)
Rs = -20*log10(0.05); % Minimum attenuation in the stopband (dB)

for cnt = 1:4,
    if cnt==1,
        [od,wn] = buttord(Wp,Ws,Rp,Rs);
        [B,A]   = butter(od,wn);
        stFilter = 'Butterworth';
    elseif cnt==2,
        [od,wn] = ellipord(Wp,Ws,Rp,Rs);
        [B,A]   = ellip(od,Rp,Rs,wn);
        stFilter = 'Elliptic';
    elseif cnt==3,
        [od,wn] = cheb1ord(Wp,Ws,Rp,Rs);
        [B,A]   = cheby1(od,Rp,wn);
        stFilter = 'Chebyshev-I';
    elseif cnt==4,
        [od,wn] = cheb2ord(Wp,Ws,Rp,Rs);
        [B,A]   = cheby2(od,Rs,wn);
        stFilter = 'Chebyshev-II';
    else
        break;
        end;

    sys = tf(B,A,-1);
    wp  = Wp*pi;
    ws  = Ws*pi;
```

```
%==============================================================================
% Plot Magnitude on Linear Scale
%==============================================================================
ymax  = 1.05;
pbmax = 1;
pbmin = 10^(-Rp/20);
sbmax = 10^(-Rs/20);
wmax  = pi;
w     = 0:0.001:wmax;
[H,w] = freqz(B,A,w);
mag = abs(H);
phs = angle(H);
figure;
FigureSet(1,'LTX');
h = patch([0 wp wp 0],[0 0 pbmin pbmin],0.5*[1 1 1]);
set(h,'LineWidth',0.0001);
hold on;
    h = patch([0 ws ws wmax wmax 0],[pbmax pbmax sbmax sbmax ymax ymax],0.5*[1 1 1]);
    set(h,'LineWidth',0.0001);
    h = plot(w,mag,'r');
    set(h,'LineWidth',1.0);
    hold off;
ylim([0 ymax]);
xlim([0 wmax]);
grid on;
ylabel('|H(e^{j\omega})|');
title(sprintf('%s Lowpass Filter Transfer Function   Order: %d',stFilter,od));
xlabel('Frequency (rad/sample)');
box off;
AxisSet(8);
st = sprintf('print -depsc Lowpass%s',stFilter);
eval(st);

%==============================================================================
% Impulse Response
%==============================================================================
figure;
```

```
FigureSet(1,'LTX');
n = 0:100;
[x,t] = impulse(sys,n);
h = stem(t,x,'b');
set(h(1),'MarkerFaceColor','b');
set(h(1),'MarkerSize',2);
ylabel('h[n]');
xlabel('Time (n)');
title(sprintf('%s Lowpass Filter Impulse Response   Order:%d',stFilter,od));
box off;
hold on;
    h = plot(xlim,[0 0],'k:');
    hold off;
AxisSet(8);
st = sprintf('print -depsc Lowpass%sImpulse',stFilter);
eval(st);

%==============================================================================
% Step Response
%==============================================================================
figure;
FigureSet(1,'LTX');
n = 0:100;
[x,t] = step(sys,n);
h = stem(t,x,'b');
set(h(1),'MarkerFaceColor','b');
set(h(1),'MarkerSize',2);
ylabel('h[n]');
xlabel('Time (n)');
title(sprintf('%s Lowpass Filter Step Response   Order:%d',stFilter,od));
box off;
hold on;
    h = plot(xlim,[1 1],'k:');
    hold off;
AxisSet(8);
st = sprintf('print -depsc Lowpass%sStep',stFilter);
eval(st);
```

```
end;
```

Practical Filter Tradeoffs

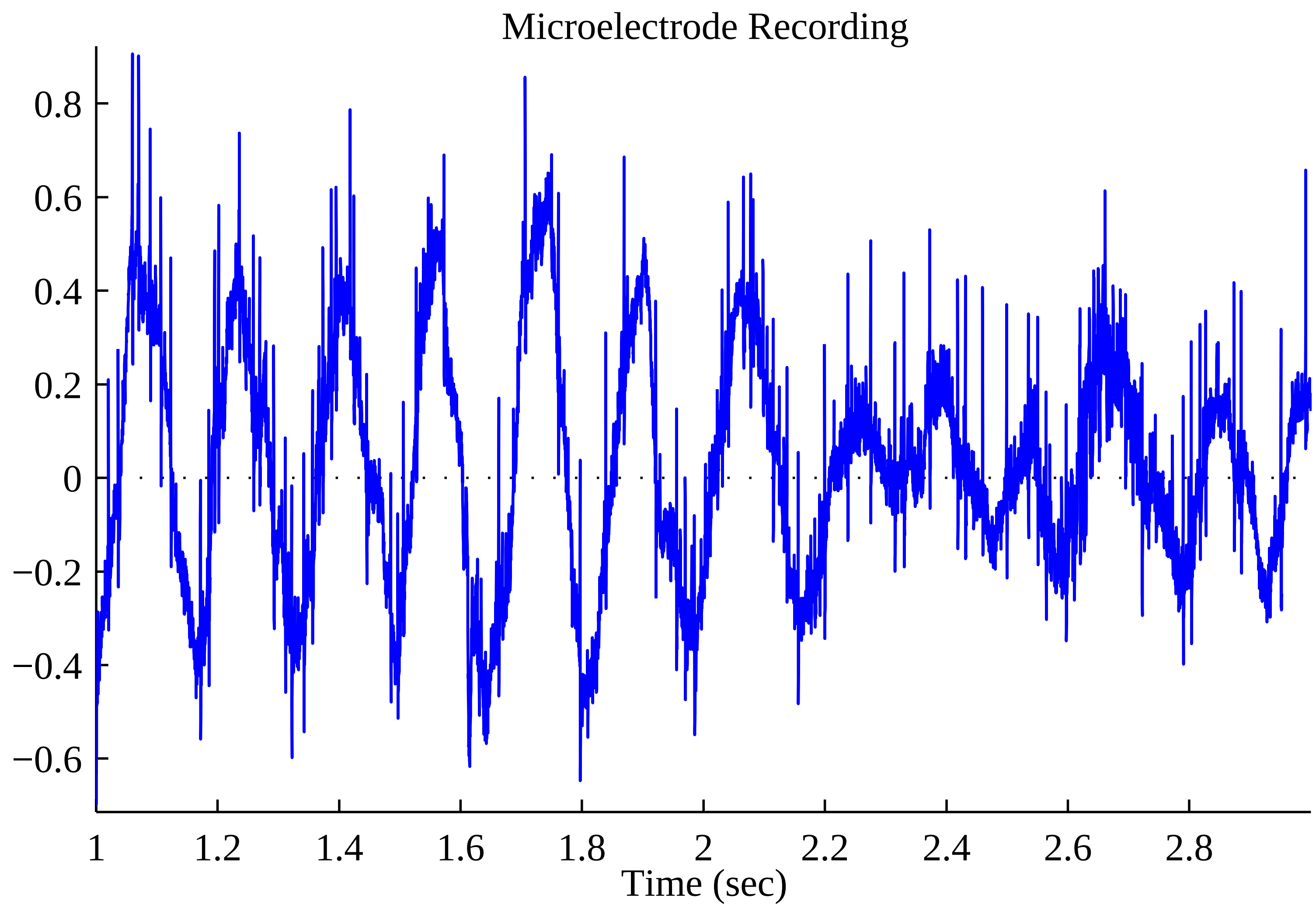
- Butterworth
  - Highest order $H(e^{j\omega})$
  + No passband or stopband ripple
- Chebyshev Type I
  + No stopband ripple
- Chebyshev Type II
  + No passband ripple
- Elliptic
  + Lowest order $H(e^{j\omega})$
  - Passband and stopband ripple

Application Example 1: Microelectrode Recording Filter

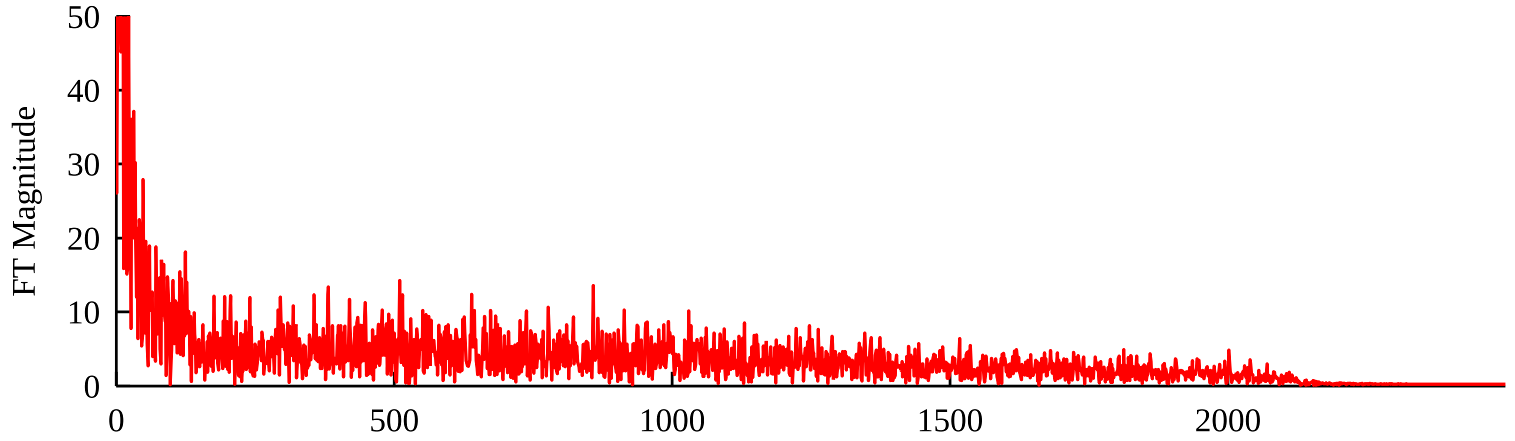
An engineer wishes to detect action potentials in a microelectrode recording with a simple threshold detector. The signal contains significant baseline drift. Action potentials typically last about 1 ms.

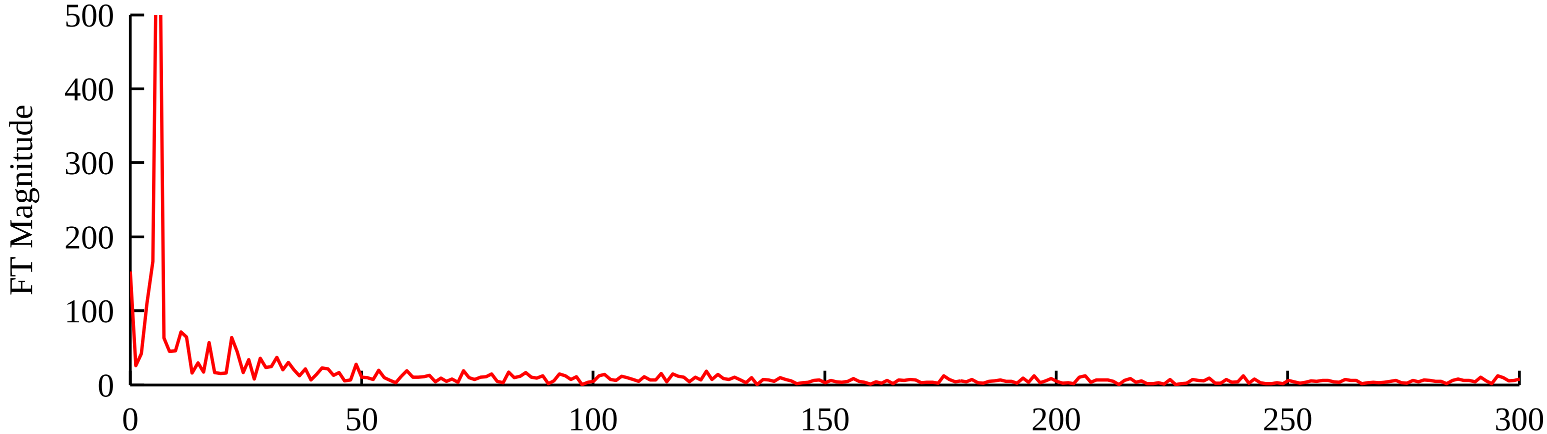
- What type of filter should the engineer use?
- What should the filter specifications be?
- What should the cutoff frequency(ies) be?

Application Example 1: Frequency-Selective Filters

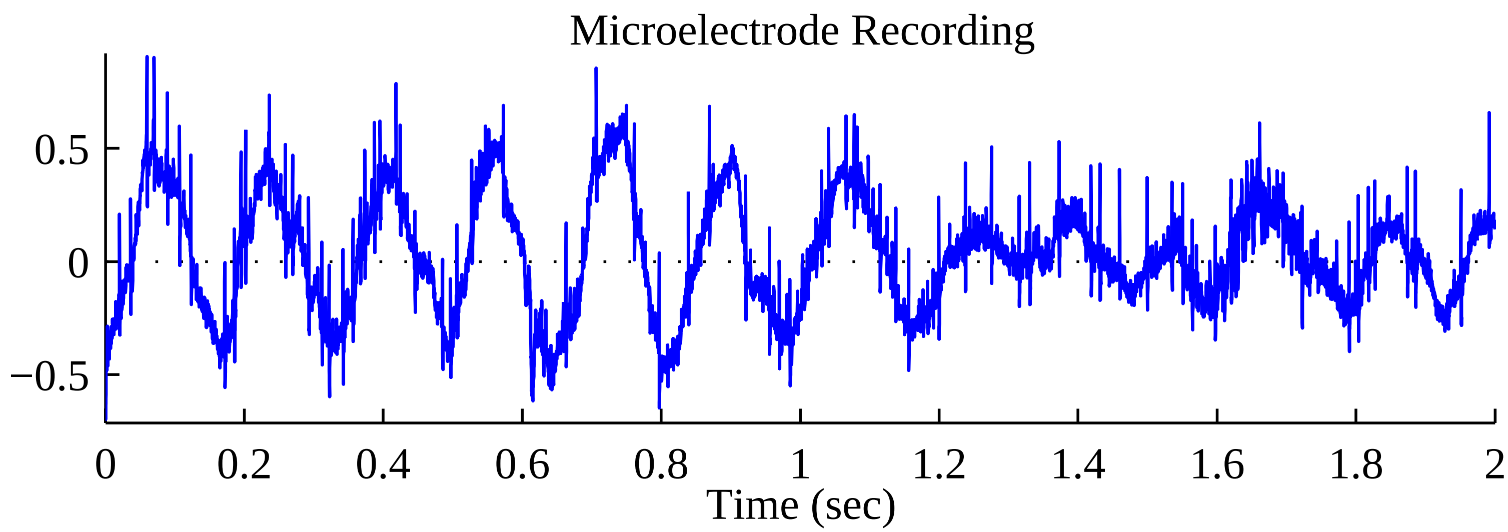


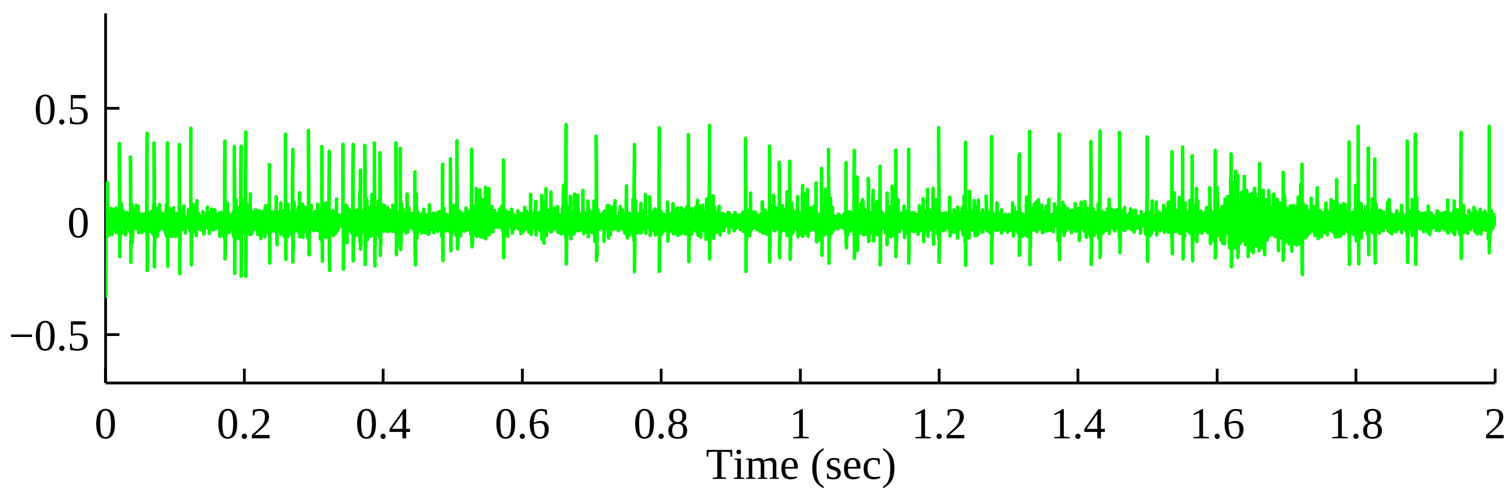
Application Example 1: Frequency-Selective Filters



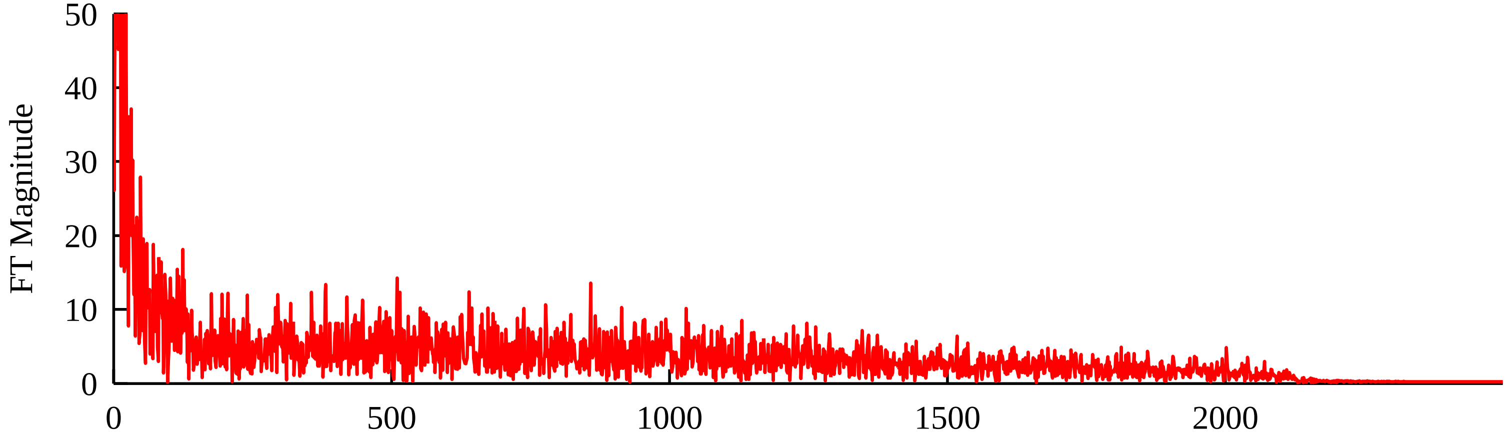


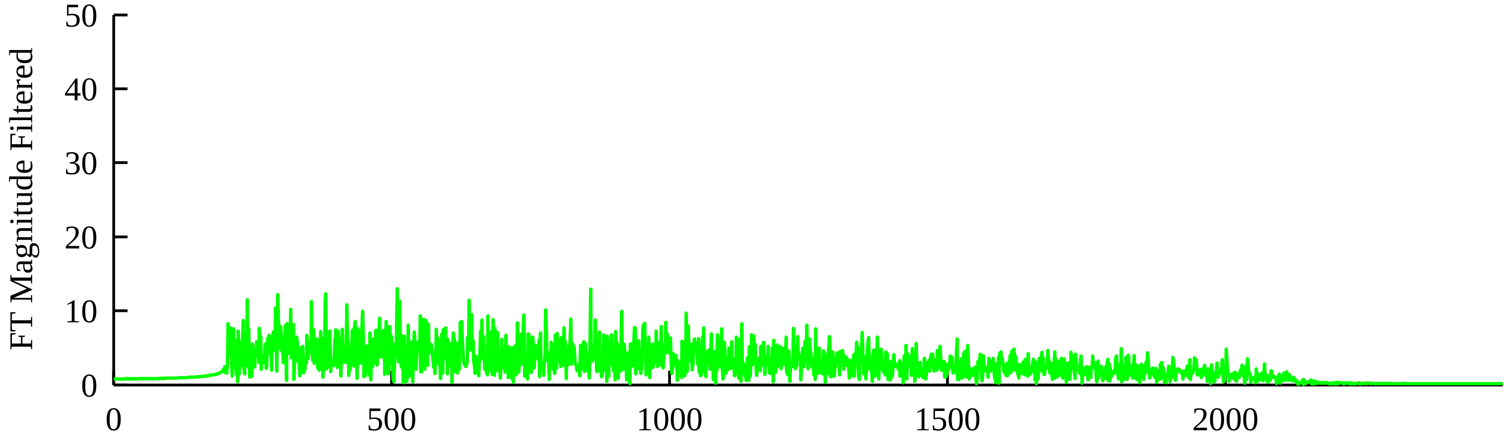
Application Example 1: Frequency-Selective Filters





Application Example 1: Frequency-Selective Filters





Application Example 1: MATLAB Code

```
%function [] = MER();
close all;

[x,fs,nbits] = wavread('Henderson2.wav');
x = decimate(x,2);
fs = fs/2;

k  = round(fs*1):round(fs*3); % Look at only 5 s
x  = x(k);
nx = length(x);

figure;
FigureSet(1,'LTX');
t = (k-1)/fs;
h = plot(t,x,'b');
set(h,'LineWidth',0.6);
xrng = max(x)-min(x);
xlim([min(t) max(t)]);
ylim([min(x)-0.01*xrng max(x)+0.01*xrng]);
AxisLines;
xlabel('Time (sec)');
ylabel('');
title('Microelectrode Recording');
box off;
AxisSet(8);
print -depsc MERSignal;

X  = fft(x,2^12);
nX = length(X);
k  = 1:floor((length(X)+1)/2);
f  = (k-1)*(fs)./(nX+1);

figure;
FigureSet(1,'LTX');
```

```
subplot(2,1,1);
h = plot(f,abs(X(k)),'r');
set(h,'LineWidth',0.6);
xlim([min(f) max(f)]);
ylim([0 50]);
set(gca,'XTick',[0:500:max(f)]);
box off;
ylabel('FT Magnitude');
subplot(2,1,2);
h = plot(f,abs(X(k)),'r');
set(h,'LineWidth',0.6);
xlim([0 300]);
ylim([0 500]);
%set(gca,'XTick',[0:500:max(f)]);    S
box off;
ylabel('FT Magnitude');
AxisSet(6);
print -depsc MERSpectralDensity;


Wp = 210/(fs/2);      % Passband ends
Ws = 190/(fs/2);      % Stopband begins
Rp = -20*log10(0.95); % Maximum deviation from 1 in the passband (dB)
Rs = -20*log10(0.05); % Minimum attenuation in the stopband (dB)
[od,wn] = ellipord(Wp,Ws,Rp,Rs);
[B,A]   = ellip(od,Rp,Rs,wn,'high');
stFilter = 'Elliptic';

y = filtfilt(B,A,x);

figure;
FigureSet(1,'LTX');
k  = 1:length(x);
t = (k-1)/fs;
subplot(2,1,1);
t = (k-1)/fs;
h = plot(t,x,'b');
```

```
set(h,'LineWidth',0.6);
xrng = max(x)-min(x);
xlim([min(t) max(t)]);
ylim([min(x)-0.01*xrng max(x)+0.01*xrng]);
AxisLines;
xlabel('Time (sec)');
ylabel('');
title('Microelectrode Recording');
box off;
subplot(2,1,2);
h = plot(t,y,'g');
set(h,'LineWidth',0.6);
xlim([min(t) max(t)]);
ylim([min(x)-0.01*xrng max(x)+0.01*xrng]);
AxisLines;
xlabel('Time (sec)');
ylabel('');
box off;
AxisSet(8);
print -depsc MERSignalFiltered;

Y  = fft(y,2^12);
nY = length(Y);
k  = 1:floor((length(Y)+1)/2);
f  = (k-1)*(fs)./(nY+1);

figure;
FigureSet(1,'LTX');
subplot(2,1,1);
h = plot(f,abs(X(k)),'r');
set(h,'LineWidth',0.6);
xlim([min(f) max(f)]);
ylim([0 50]);
set(gca,'XTick',[0:500:max(f)]);
box off;
ylabel('FT Magnitude');
subplot(2,1,2);
```

```
h = plot(f,abs(Y(k)),'g');
set(h,'LineWidth',0.6);
xlim([min(f) max(f)]);
ylim([0 50]);
set(gca,'XTick',[0:500:max(f)]);
box off;
ylabel('FT Magnitude Filtered');
AxisSet(6);
print -depsc MERSpectralDensityFiltered;
```