ELECTRIC CURRENT An orderly motion of charges as defined by the (time) rate of flow of positive charges from the terminal a to the terminal b of a terminal pair a-b is called the current flowing from a to b and is shown by the symbol $i_{ab}(t)$. The order of the subscripts ab indicates that the function $i_{ab}(t)$ is the rate of flow of positive charges from a to b inside the terminal pair a-b. The definition of current implies that, in the terminal pair a-b, $i_{ab}(t) = -i_{ba}(t)$. Thus if, at one instant of time $t = t_1$, $i_{ab}(t_1)$ is a positive number, then at that instant positive charges are flowing from a to b through the terminal pair. If, at an instant $t = t_2$, $i_{ab}(t_2)$ is a negative number, then at that instant positive charges are flowing in the terminal pair, from b to a. We emphasize that the current i_{ab} is associated with the terminals a-b, although we may have no information about what is the exact motion of charges inside that particular part of the device which is represented by the terminal pair.

VOLTAGE FUNCTION With the assumption made in the foregoing paragraph, it can be shown that the rate of delivery of energy to a terminal pair can always be given as the product of the current $i_{ab}(t)$ and another function of time associated with the terminals a and b. This function is called the voltage between the two terminals of the terminal pair and is indicated by $v_{ab}(t)$. The order of the subscripts ab indicates that the function $v_{ab}(t)$ is the voltage of point a with respect to point b. Thus, if $p_{ab}(t)$ is the rate of delivery of the energy to the terminal pair a-b, we can always find a function $v_{ab}(t)$ such that $p_{ab}(t) \equiv v_{ab}(t)i_{ab}(t)$. This equation may be considered to be the defining equation of the voltage function $v_{ab}(t)$. Since $v_{ab}(t)$ and $i_{ab}(t)$ are both functions of time and at any given instant of time one may have a positive value and the other a negative value (resulting, at that instant, in a negative value of p_{ab}), we have to interpret the significance of the positive and negative values of p_{ab}. If, at a given instant of time, $p_{ab}(t)$ is a positive value, by this we understand that energy is being delivered to the terminal pair a-b at the rate given by the magnitude of p_{ab} at that instant. On the other hand, if $p_{ab}(t)$ is negative at another instant of time, by this we understand that the terminal pair is delivering energy to the rest of the circuit at a rate given by the magnitude of p_{ab} at that instant. Consider a terminal pair a-b. By definition, the rate of the delivery of energy to this terminal pair at time t is

$$p_{ab}(t) = v_{ab}(t)i_{ab}(t)$$

and also

$$p_{ba}(t) = v_{ba}(t)i_{ba}(t)$$

Since $p_{ba}(t)$ and $p_{ab}(t)$ refer to the rate of the delivery of energy to the same terminal pair, it follows that

$$p_{ab}(t) = p_{ba}(t)$$

and

$$v_{ab}(t)i_{ab}(t) = v_{ba}(t)i_{ba}(t)$$

Since from the definition of current

$$i_{ba}(t) = -i_{ab}(t)$$

it follows that

$$v_{ba}(t) = -v_{ab}(t)$$

It can be shown that the methods of circuit analysis may be applied to the study of other systems such as mechanical systems. In such cases $i_{ab}(t)$ may represent the velocity of a (point) mass a with respect to some reference (point) b, and $v_{ab}(t)$ may represent the force applied to a (point) mass a in a reference system designated by b. In this case also, the product of $v_{ab}i_{ab}$ will be power (work done on a per unit time), but the interpretations of the variables are different.

1-6 *Idealized lumped circuit elements*

In the field problem we discuss the energy density at a given point of space and compute it in terms of the $\mathcal{E}$ and $\mathcal{H}$ variables. In circuit theory we discuss the rate of delivery of energy to a terminal pair or combination of terminal pairs and compute it in terms of the voltage functions and the currents associated with the terminal pairs. In field theory energy stored in a unit volume of the magnetic field is $\mu H^2/2$, and $\epsilon E^2/2$ is the energy stored in the unit volume of the electric field, E^2/ρ being the rate of transformation of electromagnetic energy into heat per second per unit volume. The factors μ, ϵ, and ρ are the parameters of the medium in which the electric and magnetic fields exist. We should be able to find analogous parameters for terminal pairs which would correspond to the permeability, permittivity, and resistivity of the medium in which the fields exist. The latter parameters are called distributed parameters of the medium since their value may change continually from one point to another in the medium. In circuit theory we are not concerned with values which change from point to point, but deal with quantities which are defined in connection with two terminals. When energy is delivered to a terminal pair a-b at the rate $p_{ab} = v_{ab}i_{ab}$, the total energy delivered to the terminal pair between time t_1 and t_2 is $W(t_1,t_2) = \int_{t_1}^{t_2} v_{ab}i_{ab}\,dt$. Part of this energy will be stored in the electric field associated with the terminal pair; another part will be stored in its magnetic field; and the rest of the energy will be transformed into heat. This partition of energy within a terminal pair is shown symbolically in a diagram called a circuit diagram. Such a diagram consists of interconnected symbols called *circuit elements*.[1] The number of such elements and the manner in which they are interconnected are determined by the physical properties of the device or part of the device which the terminal pair represents. Five elements are necessary to represent the energy-storage and energy-conversion processes in an electrical device. These basic elements describe the

[1] Some authors use the term "parameter" instead of "element."

following processes:

1 Storage of energy in magnetic form; such energy storage is accounted for by the element *inductance.*
2 Storage of energy in electrical form; accounted for by the element *capacitance.*
3 Conversion of electromagnetic energy into heat; accounted for by the element *resistance.*
4 Transfer of energy from one part of a device to another part through a magnetic field. The circuit model for this phenomenon is *mutual inductance.*
5 Conversion of other forms of energy into electromagnetic energy for delivery to parts of a device, or reception of electromagnetic energy from parts of a device and its transformation into other forms of energy. *Ideal sources* are introduced in circuit analysis to account for these processes.

In contrast to the "distributed" parameters of the field theory, these elements are called *idealized lumped circuit elements.* The values attributed to these elements depend on the geometry of the path of the current between the terminals of the terminal pair, and are computed from the dimensions and field properties (ϵ, μ, ρ) of the path. The computation of the values of circuit elements corresponding to a given path of current is not studied in circuit analysis. Instead, we define circuit elements in terms of energy-storage and energy-conversion processes which they represent. It should be noted that this is not the only possible way of defining circuit elements. For example, the voltage-current relationship at the terminals of the elements can be used to define such elements. Although the computation of the values of circuit elements is based on field theory, it is possible to determine, by experiment, the values of the elements of a network representing a device.

1-7 Inductance

In circuit theory, the characteristic of a part of a device (circuit) which accounts for the storage of energy in a magnetic field associated with that part is termed the *inductance* of that part. The two-terminal element represented by the symbol shown in Fig. 1-3, whose value is designated by

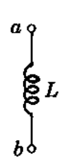

Fig. 1-3 Graphical symbol for the element inductance.

the letter L, represents an inductance. Quantitatively, inductance L may be defined by analogy with the field expression for energy density $\frac{1}{2}\mu H^2$ as

follows: In Fig. 1-3, if the current in the inductance is i_{ab}, the energy stored in the inductance (in the form of magnetic energy) is related to the current by the equation

$$w_M = \tfrac{1}{2}Li_{ab}^2 \tag{1-1}$$

From this definition one can deduce the *voltage-current* relationship at the terminals of the inductance as follows: The rate at which energy is delivered to a terminal pair a-b is given by the power $v_{ab}(t)i_{ab}(t)$. Therefore

$$v_{ab}i_{ab} = \frac{d}{dt}\left(\frac{1}{2}Li_{ab}^2\right)$$

Since L is defined as independent both of t and of i_{ab}, the result of the differentiation is

$$v_{ab}i_{ab} = \left(L\frac{di_{ab}}{dt}\right)i_{ab}$$

Thus, for an inductance L, connected between terminals a-b, the voltage v_{ab} is related to the current i_{ab} by the basic equation

$$v_{ab} = L\frac{di_{ab}}{dt} \tag{1-2}$$

If, from Eq. (1-2), we express the current i_{ab} as a function of the voltage v_{ab}, the result reads

$$i_{ab} = \frac{1}{L}\int v_{ab}\,dt + \text{const} \tag{1-3}$$

The constant can be made explicit if we recognize that the energy stored in the inductance at time t has been delivered over the period of time for which $v_{ab}(t)$ has existed. The current and energy are related by Eq. (1-1), and it is seen that the value of the current at any time t, like the value of the stored energy, will depend on the past history of the voltage across the inductance. This is taken care of by writing the integral expression (1-3) with the lower limit at minus infinity and allowing the integral to be a function of an upper limit t.

$$i_{ab}(t) = \frac{1}{L}\int_{-\infty}^{t} v_{ab}(\tau)\,d\tau \tag{1-4}$$

It frequently happens that interest is focused on the function i_{ab} beginning at some arbitrary instant of time, usually $t = 0$. In such cases it is convenient to write (1-4) in the form

$$i_{ab}(t) = \frac{1}{L}\int_{-\infty}^{0} v_{ab}\,dt + \frac{1}{L}\int_{0}^{t} v_{ab}(\tau)\,d\tau \tag{1-5}$$

Since the first of the integrals in (1-5) has numerical limits, it represents a number. This number is the value of the current at $t = 0$. Denoting this value by $i(0)$, we have, for an inductance,

$$i_{ab}(t) = i_{ab}(0) + \frac{1}{L}\int_0^t v_{ab}(\tau)\,d\tau \qquad (1\text{-}5a)$$

Equation (1-5*a*) reads, in words,

$$\begin{pmatrix}\text{Current in inductance}\\ \text{for all } t > 0\end{pmatrix} = \begin{pmatrix}\text{current at}\\ t = 0\end{pmatrix} + \begin{pmatrix}\text{current due to } v_{ab}\\ \text{from } t = 0 \text{ on}\end{pmatrix}$$

The value $i_{ab}(0)$ is usually called the initial value of the current i_{ab}. The reciprocal of inductance is called *inverse self-inductance* and is represented by the symbol $\Gamma = 1/L$.

From Eq. (1-2) we observe that the voltage-current relationship for an inductance is *linear;* that is, it is a *first-degree* differential relationship. We note that, if i_{ab} is a function of time represented, for example, by the graph shown in Fig. 1-4*a*, then v_{ab} will be a function of time proportional to the *slope*

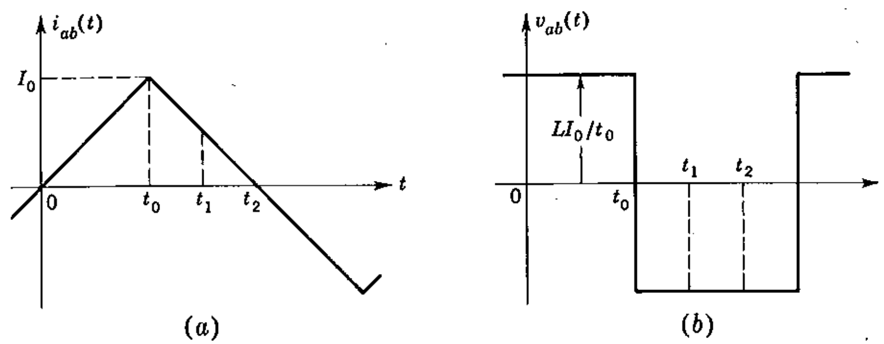

***Fig. 1-4** (a) Example of a current waveform i_{ab}. (b) Voltage v_{ab} across an inductance L when the current i_{ab} has the waveform of (a).*

of the i_{ab} graph as shown in Fig. 1-4*b*. We note also that the algebraic *sign* of v_{ab} depends on the *sign of the slope of* i_{ab}, *not on* i_{ab} *itself;* for example, i_{ab} is positive at t_1 whereas v_{ab} is negative. In connection with Fig. 1-4*a*, we observe that, at $t = t_0$, the slope of the graph i_{ab} changes abruptly, resulting in an abrupt change in v_{ab}.

Another interesting observation concerning inductance is the following: If we specify in addition to L the current and its slope at some instant, for example, $t = t_1$, we can calculate the stored energy in the inductance at that instant as $w_M(t_1) = \frac{1}{2}Li_{ab}^2(t_1)$ and the voltage v_{ab} at t_1 as

$$v_{ab}(t_1) = L(di_{ab}/dt)_{t=t_1}$$

In contrast, if we specify v or dv/dt at some instant, we cannot calculate i_{ab} at that moment because [from Eq. (1-4)] $i_{ab}(t_1)$ does not depend only on $v_{ab}(t_1)$, but also on how v_{ab} varied up to the time t_1. This dependence of one

circuit variable on the "history" of the other is characteristic of energy-storing devices.

1-8 Capacitance

In circuit theory, the characteristic of a part of a device which accounts for storage of energy in an electric field associated with that part is termed *capacitance* of that part. It is represented by the symbol shown in Fig. 1-5

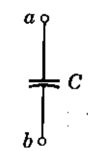

***Fig. 1-5** Graphical symbol for the element capacitance.*

and quantitatively denoted by the letter C. By analogy with the expression for electric field energy density $\frac{1}{2}\epsilon E^2$, the defining equation for energy stored in a capacitance owing to a voltage v_{ab} is given by the expression

$$w_E = \tfrac{1}{2}Cv_{ab}^2 \qquad (1\text{-}6)$$

The rate of delivery of energy to a capacitance is given by

$$v_{ab}i_{ab} = \frac{dw_E}{dt} = \frac{d}{dt}\left(\frac{1}{2}Cv_{ab}^2\right)$$

If C is independent of t and of v_{ab}, then

$$v_{ab}i_{ab} = \left(C\frac{dv_{ab}}{dt}\right)v_{ab}$$

Thus, for a capacitance connected between terminals *a-b*,

$$i_{ab} = C\frac{dv_{ab}}{dt} \qquad (1\text{-}7)$$

or in integral form,

$$v_{ab} = \frac{1}{C}\int_{-\infty}^{t} i_{ab}(\tau)\,d\tau = v_{ab}(0) + \frac{1}{C}\int_0^t i_{ab}(\tau)\,d\tau \qquad (1\text{-}8)$$

where $v_{ab}(0)$ is the value of the voltage across the terminals of the capacitance at $t = 0$. This value is called the initial value of the voltage across the capacitance. The reciprocal of capacitance is termed *elastance* and is denoted by $S = 1/C$.

We observe that the voltage-current equations for capacitance are analogous to those of inductance. Comparison of Eq. (1-7) with (1-2) shows that voltage and current have exchanged roles. Thus, in a capacitance, the current depends on the instantaneous rate of change (slope) of the voltage, whereas the voltage depends on the "history" of the current as indicated by Eq. (1-8).