**INEL 4102: CIRCUIT ANALYSIS – QUIZ II – NOV. 13, 2009**
**Prof. Domingo Rodríguez**

**Name_____ID. No._____**

## Problem One: Impulse Response of an Ideal Filter

**1a.- (30 points)** Obtain the impulse response of an ideal low-pass filter with cut-off frequency equal to $\omega = \omega_C$ by using the expression:

$$h_L(t) = \frac{1}{2\pi}\int_{t=-\infty}^{t=\infty} H(s)e^{+j\omega t}d\omega$$

Present your final result as a "*SinC*" $(\sin(\pi\theta)/\pi\theta)$ function of time.

**1b.- (20 points)** Obtain the output of the ideal filter if the input to the filter is the signal $x(t) = 2\delta(t-3)+4\delta(t-6)$.

**1c.- (20 points)** Obtain the output of the ideal filter if the input to the filter is the signal $x(t) = \cos(\omega_B t) = \frac{1}{2}e^{+j\omega_B t} + \frac{1}{2}e^{-j\omega_B t}$ and $\omega_B < \omega_C$. Remember that the Fourier transform of the input is $X(\omega) = \pi\delta(\omega+\omega_B)+\pi\delta(\omega-\omega_B)$.

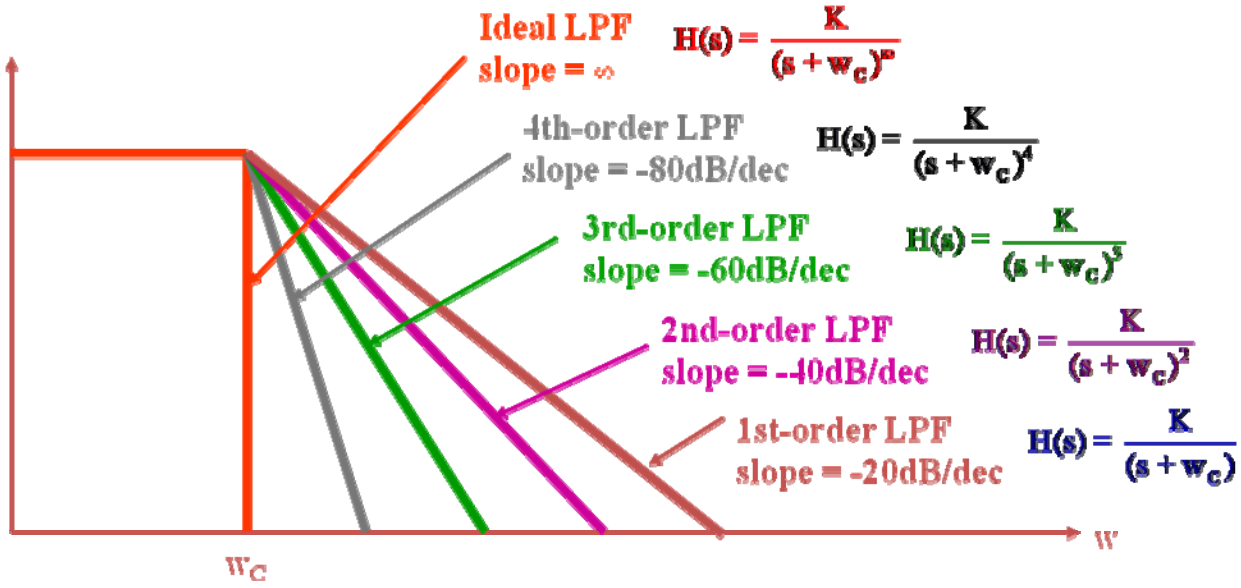


***Figure One***: *Ideal Low-pass Filter and Passive Filters*. Remember that the higher the order of a filter the closely it will approximate an ideal low-pass filter. The order of the filter is equal to the degree of the polynomial in the denominator of a rational transfer function. A rational transfer function is always the ratio of two polynomials. A transfer function is the Laplace transform of the impulse response of a filter. The frequency response is the Fourier transform of the impulse response of a filter. A transfer function is transformed into a frequency response through the substitution $s = j\omega$.

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# Problem Two: Sallen-Key Active Band-pass Filter

**Introduction:** The Sallen-Key filter is an active band-pass filter with center (resonant) frequency equal to $f = f_0$. At that frequency the maximum frequency response gain $H_0$. Remember that if $x(t)$ is the input and $y(t)$ is the output, then $H(f) = Y(f)/X(f) = V_o/V_i$. The gain will go down on either side of the resonant frequency as shown in ***Figure Two*** below. The lower ( $f_L$ ) and upper ( $f_H$ ) cut-off frequencies of the band-pass filter are defined at 3-dB less than the resonant or center frequency. The bandwidth of the filter is defined as $\beta = f_H - f_L$. The quality factor is defined as $Q = f_0/\beta$ and it is a figure of merit utilized to measure the degree of selectivity of a filter. The selectivity is expressed as the ratio of the resonant frequency over the bandwidth. The transfer function of the Sallen-Key band-pass filter is:

$$T(s) = \frac{V_o}{V_i} = \frac{\dfrac{Ks}{R_1C_1}}{s^2 + \left(\dfrac{1}{R_1C_1} + \dfrac{1}{R_3C_1} + \dfrac{1}{R_3C_2} + \dfrac{1-K}{R_2C_1}\right)s + \dfrac{R_1 + R_2}{R_1R_2R_3C_1C_2}} \quad ,$$

where $K = \dfrac{V_o}{V_i} = 1 + \dfrac{R_f}{R_i}$

The following simplification is used on the parameters of the transfer function which results in a simplification: $R_1 = R_2 = R_3/2 = R; \qquad C_1 = C_2 = C$

$$T(s) = \frac{\dfrac{Ks}{RC}}{s^2 + \left(\dfrac{3-K}{RC}\right)s + \left(\dfrac{1}{RC}\right)^2}$$

The following result is obtained as the poles of the transfer function. Remember that the poles of a transfer function are the zeros of the polynomial in the denominator of the transfer function expression above:

$$s_{1,2} = \frac{-\left(\dfrac{3-K}{RC}\right) \pm \sqrt{\left(\dfrac{3-K}{RC}\right)^2 - 4\left(\dfrac{1}{RC}\right)^2}}{2}$$

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**(Part 2a: 15 points)** Obtain the frequency response of Sallen-Key filter.

**(Part 2b: 15 points)** The stability of the Sallen-Key filter is ensured by the placement of the poles of the filter on the left-hand side of the complex s-plane. This implies that the real part of the roots of the denominator polynomial of the transfer function $T(s)$ given above must be less than zero:

$$\Re e(s_{1,2}) < 0$$

Determine what should be the numerical range of the gain $K$ of the amplifier in the Sallen Key band-pass filter (see ***Figure Three*** below) in order to ensure the stability of the filter.

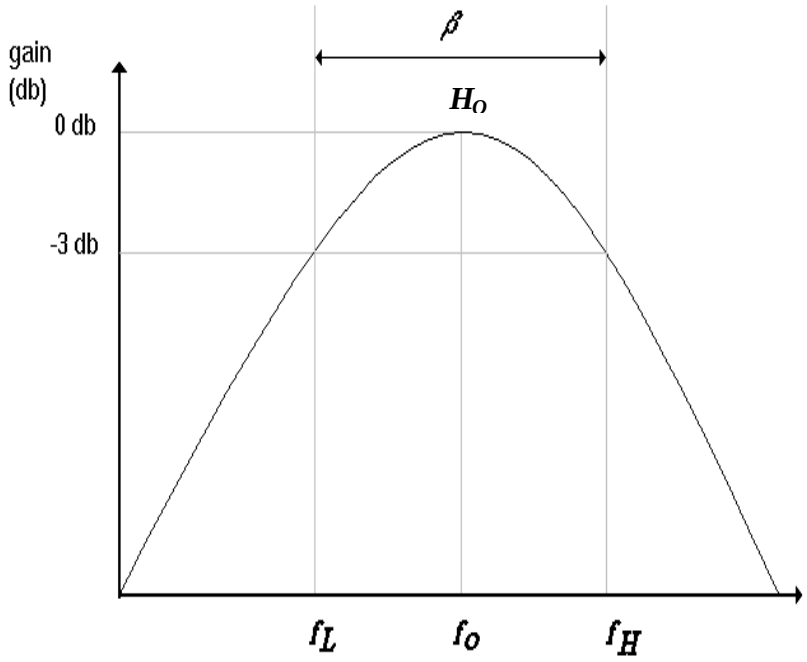


***Figure Two****: Frequency Response of Sallen Key Band-pass Filter*

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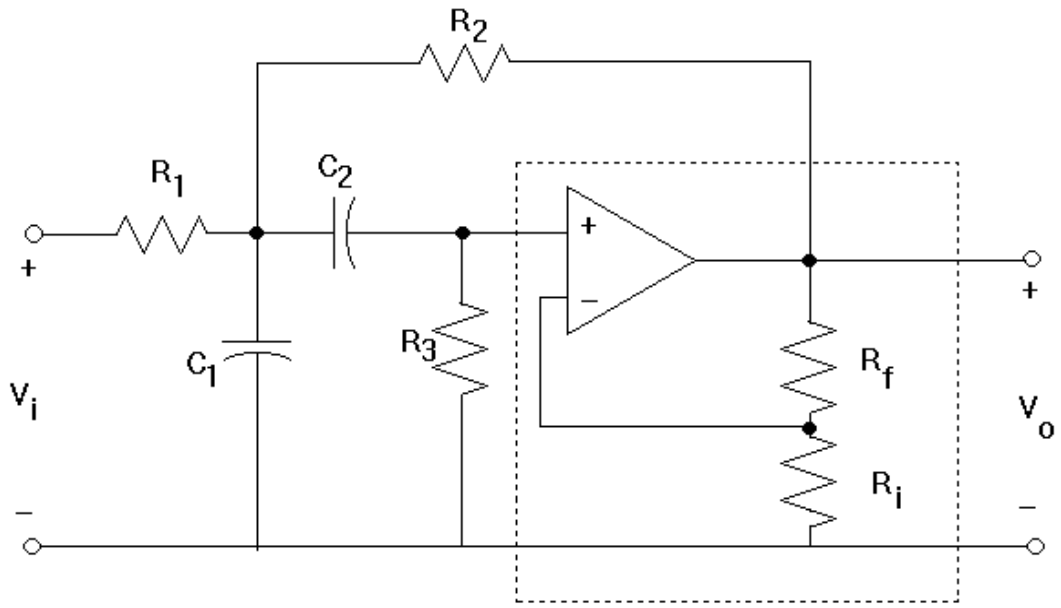


***Figure Three**: Active (Sallen-Key) Band-pass Filter*

**Solution:**