21 Continuous-Time Second-Order Systems

Recommended Problems

P21.1

Consider the following four impulse responses:

(i) $h_1(t) = e^{-at}u(t), \quad a > 0,$

(ii) $h_2(t) = -e^{-at}u(-t), \quad a > 0,$

(iii) $h_3(t) = e^{-at}u(t), \quad a < 0,$

(iv) $h_4(t) = -e^{-at}u(-t), \quad a < 0$

(a) Verify that

$$H_2(s) = \frac{1}{s + a},$$

with an ROC corresponding to $Re\{s\} < -a$.

(b) It is easily verified that in all four cases the system functions (i.e., the Laplace transforms of the impulse responses) have the same algebraic expression. However, the ROC is different. For each of the four systems, identify which of the s plane diagrams in Figures P21.1-1 to P21.1-6 correctly represents the associated Laplace transform, including the ROC. Also indicate which diagrams correspond to stable systems.

(A)

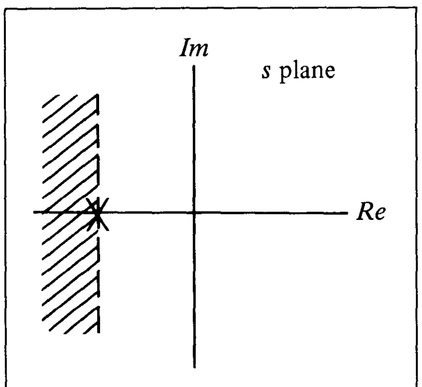

Figure P21.1-1

(B)

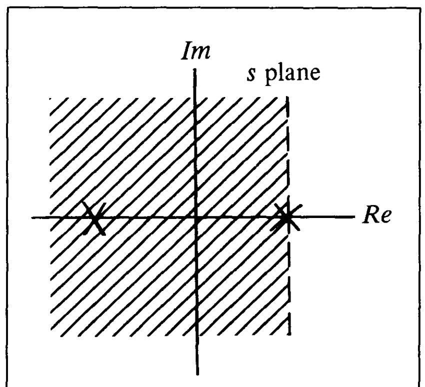

Figure P21.1-2

(C)

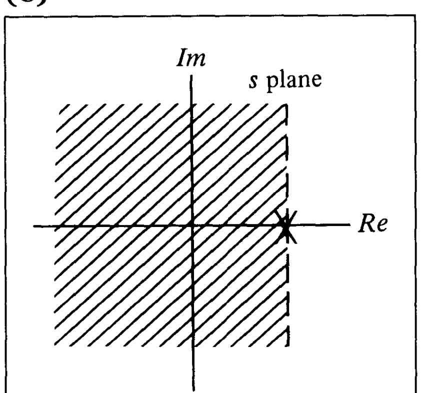

Figure P21.1-3

(D)

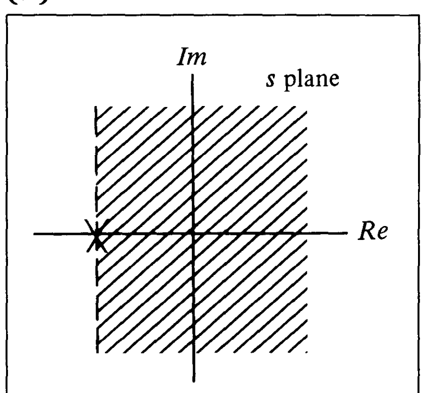

Figure P21.1-4

(E)

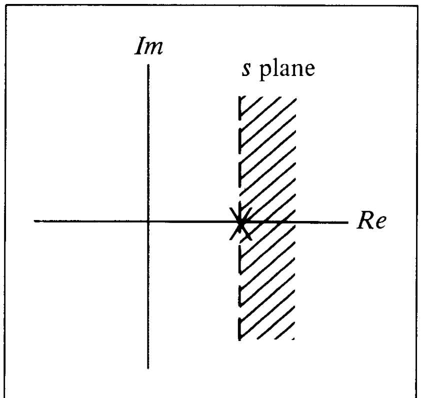

Figure P21.1-5

(F)

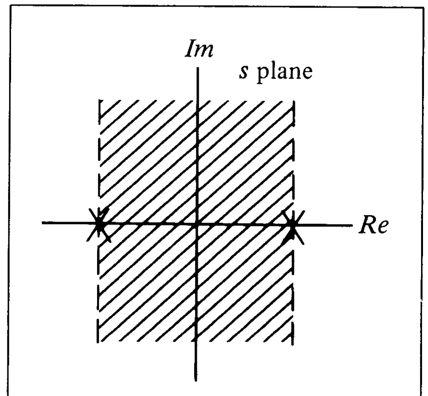

Figure P21.1-6

P21.2

Consider the LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

(a) Determine $X(s)$ and $H(s)$.

(b) Using the convolution property of the Laplace transform, determine $Y(s)$, the Laplace transform of the output, $y(t)$.

(c) From your answer to part (b), find $y(t)$.

P21.3

As indicated in Section 9.5 of the text, many of the properties of the Laplace transform and their derivation are analogous to corresponding properties of the Fourier transform developed in Chapter 4 of the text. In this problem you are asked to outline the derivation for some of the Laplace transform properties in Section 9.5 of the text.

By paralleling the derivation for the corresponding property for the Fourier transform in Chapter 4 of the text, derive each of the following Laplace transform properties. Your derivation must include a consideration of the ROC.

(a) Time-shifting property (Section 9.5.2)

(b) Convolution property (Section 9.5.5)

P21.4

Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

Let $X(s)$ and $Y(s)$ denote the Laplace transforms of $x(t)$ and $y(t)$, and let $H(s)$ denote the Laplace transform of the impulse response $h(t)$ of the preceding system.

(a) Determine $H(s)$. Sketch the pole-zero plot.

(b) Sketch the ROC for each of the following cases:

(i) The system is stable.

(ii) The system is causal.

(iii) The system is neither stable nor causal.

(c) Determine $h(t)$ when the system is causal.

P21.5

Consider the following system function $H(s)$ and its corresponding pole-zero plot in Figure P21.5.

$$H(s) = \frac{s}{s+1}$$

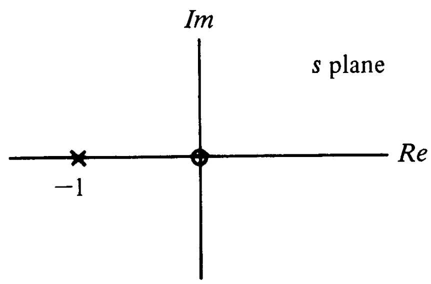

Figure P21.5

Using the graphical method discussed in the lecture, find $|H(0)|$, $\sphericalangle H(0)$, $|H(j1)|$, $\sphericalangle H(j1)$, $|H(j\infty)|$, and $\sphericalangle H(j\infty)$. Sketch the functions $|H(j\omega)|$ and $\sphericalangle H(j\omega)$.

P21.6

For the following system function, plot the pole-zero diagram and graphically determine the approximate locations of the peaks and valleys of $|H(j\omega)|$:

$$H(s) = \frac{(s+5)s}{[s-(-0.1+j2)][s-(-0.1-j2)]}$$

Optional Problems

P21.7

(a) Draw the block diagram for the following second-order system in terms of integrators, coefficient multipliers, and adders.

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(b) Sketch the pole-zero plot of $H(s)$ and plot $|H(j\omega)|$ under the following conditions.

(i) ω_n is kept constant, but ζ is varied from close to 0 to close to 1.

(ii) ζ is kept constant, but ω_n is varied from about 0 to infinity.

You don't have to be precise but show how the bandwidth and location of the peak changes for the two cases above.

P21.8

(a) Consider the following system function $H(s)$.

$$H(s) = \frac{s}{s^2 + s + 1} + \frac{1}{s^2 + 2s + 2}$$

Draw the block diagram for $H(s)$ implemented as

(i) a parallel combination of second-order systems,

(ii) a cascade combination of second-order systems.

(b) Is implementation (ii) unique?

P21.9

Complete the Laplace transform pairs given in Table P21.9. You may use the following transform of $e^{-\alpha t}u(t)$ as well as general properties of Laplace transforms to derive your answer. You may also use earlier entries in the table to help you derive later results.

$$e^{-\alpha t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+\alpha}, \qquad \text{ROC: } Re\{s\} > Re\{\alpha\}$$

$x(t)$	$X(s)$	ROC
(a) $\sin(\omega_0 t)u(t)$		
(b) $e^{-2t}\sin(\omega_0 t)u(t)$		
(c) $te^{-2t}u(t)$		
(d)	$\frac{s+1}{(s+2)(s+3)}$	$Re\{s\} > -2$
(e)	$\frac{1-e^{-2s}}{s+3}$	$Re\{s\} > -3$

Table P21.9

P.21.10

Figures P21.10-1, P21.10-2, and P21.10-3 contain impulse responses $h(t)$, frequency responses $H(j\omega)$, and pole-zero plots, respectively.

(a) For each $h(t)$, find the best matching $|H(j\omega)|$.

(b) For each $|H(j\omega)|$, find the best matching pole-zero plot of $H(s)$.

Consider entries with references to other plots, such as (3) and (5) or (b) and (h), as a pair.

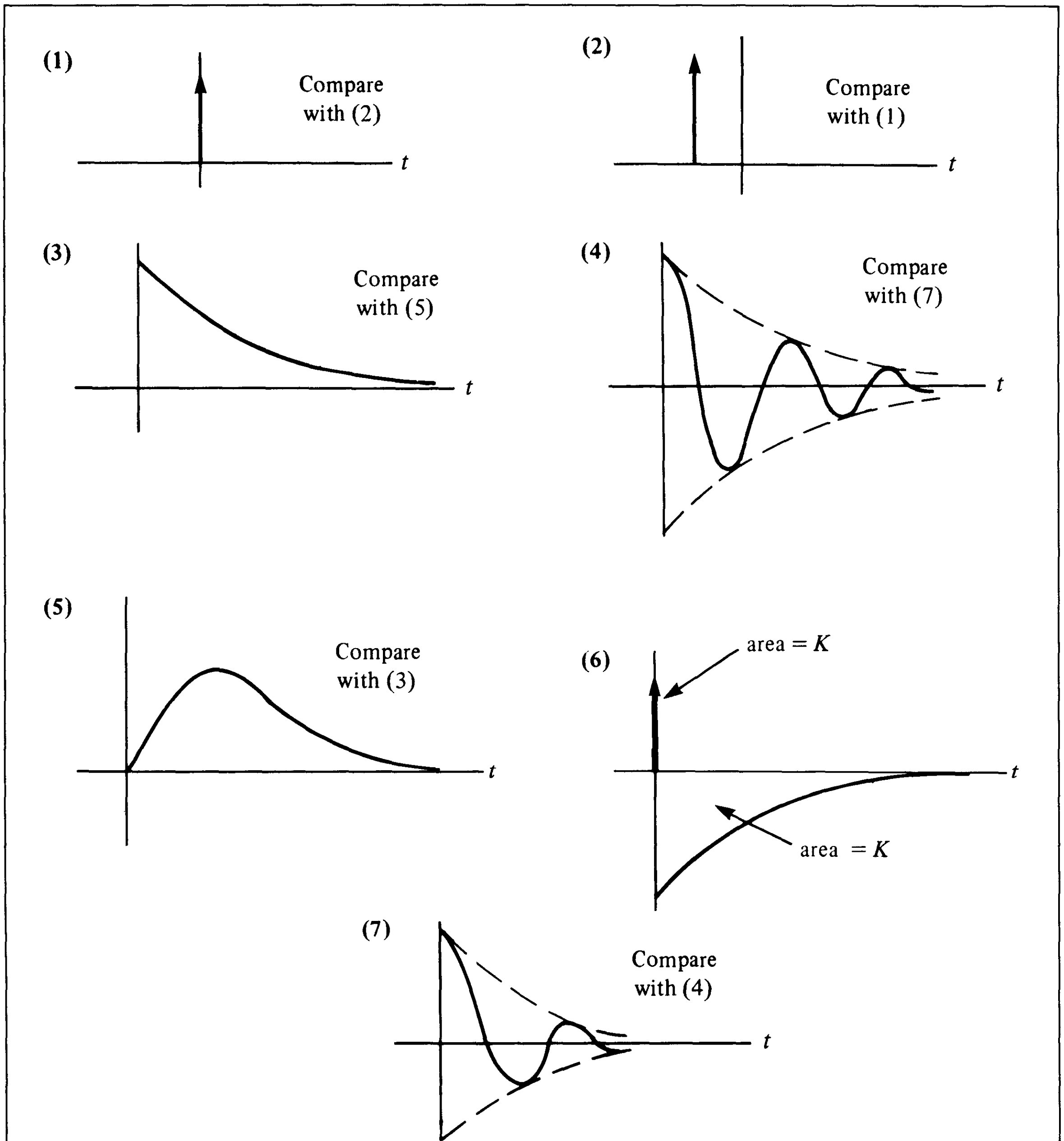

Figure P21.10-1

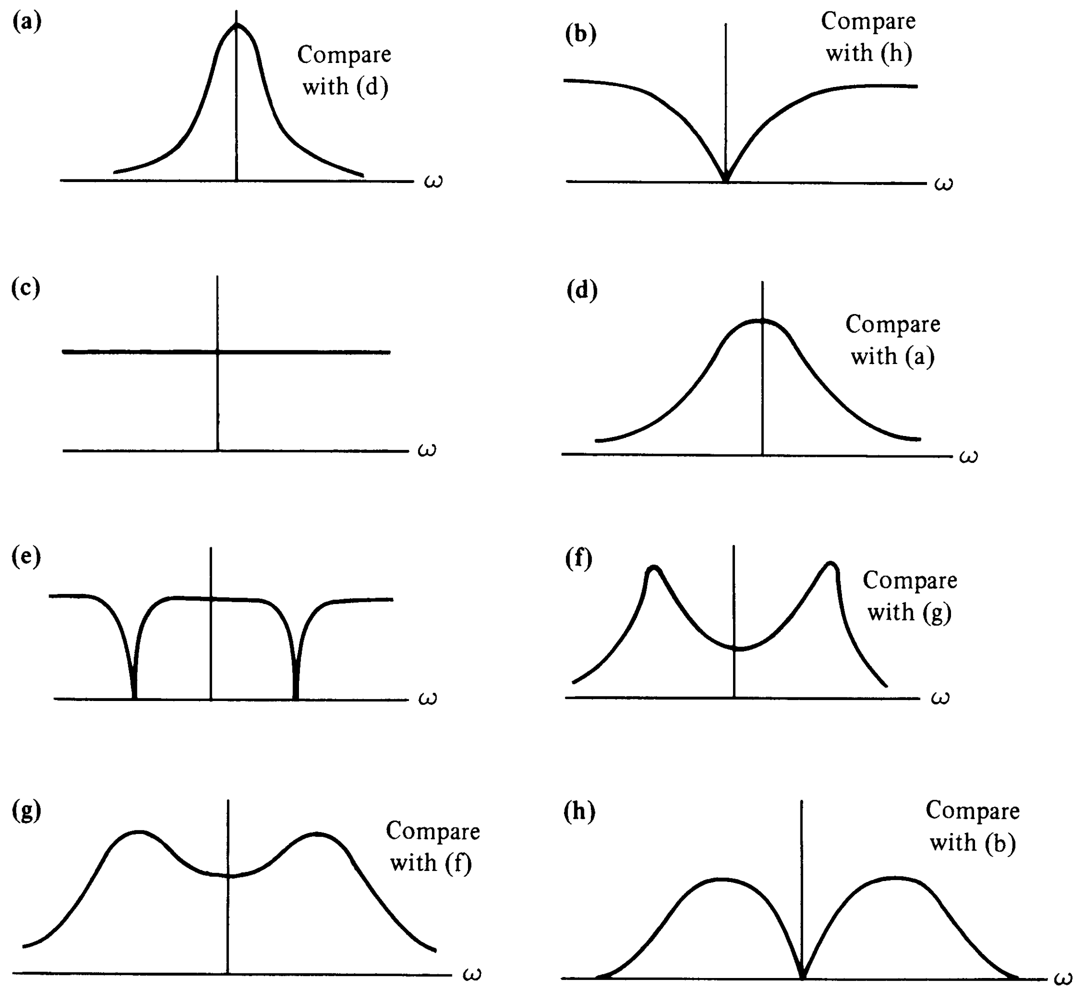

Figure P21.10-2

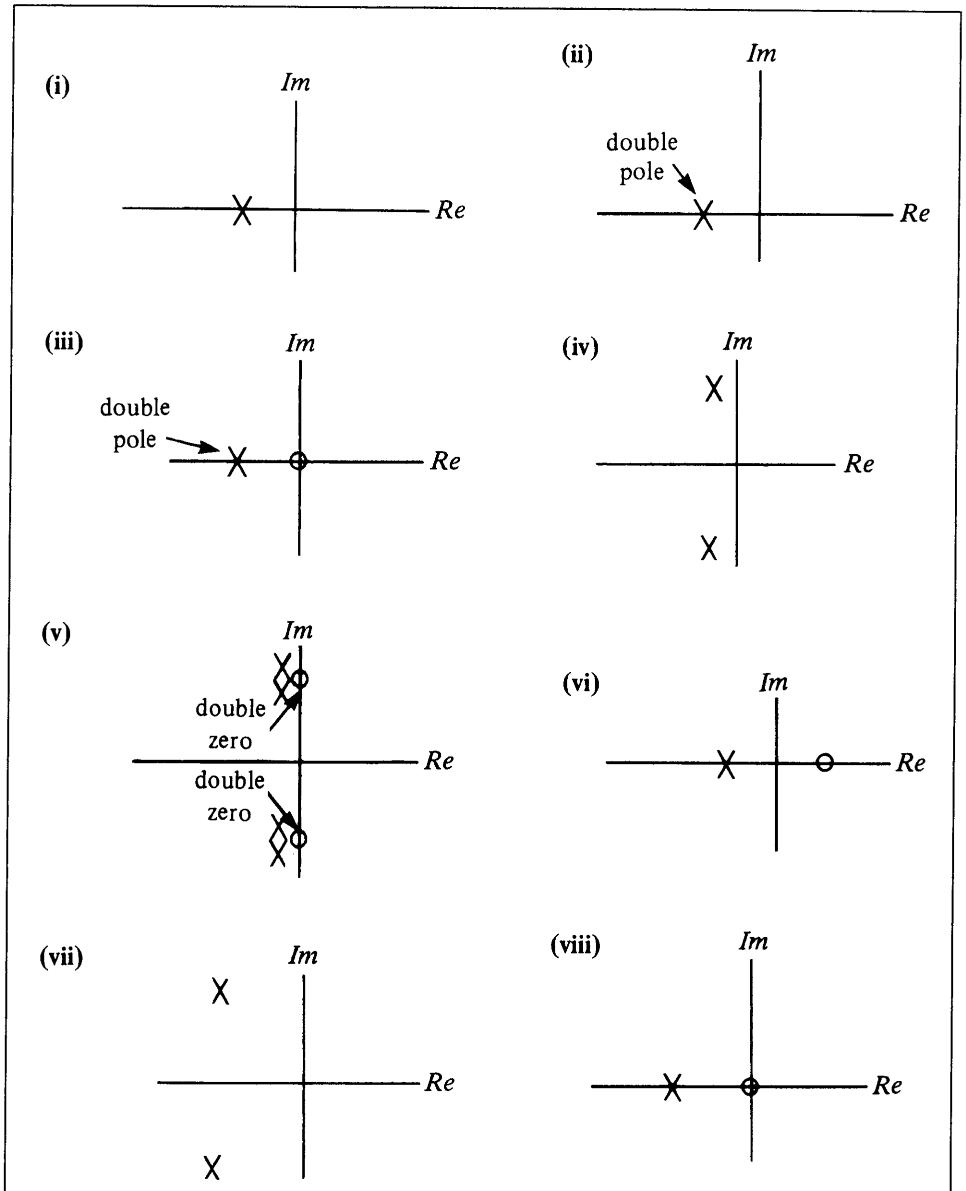

Figure P21.10-3