20 The Laplace Transform

Recommended Problems

P20.1

Consider the signal $x(t) = 3e^{2t}u(t) + 4e^{3t}u(t)$.

(a) Does the Fourier transform of this signal converge?

(b) For which of the following values of σ does the Fourier transform of $x(t)e^{-\sigma t}$ converge?

(i) $\sigma = 1$

(ii) $\sigma = 2.5$

(iii) $\sigma = 3.5$

(c) Determine the Laplace transform $X(s)$ of $x(t)$. Sketch the location of the poles and zeros of $X(s)$ and the ROC.

P20.2

Determine the Laplace transform, pole and zero locations, and associated ROC for each of the following time functions.

(a) $e^{-at}u(t), \quad a > 0$

(b) $e^{-at}u(t), \quad a < 0$

(c) $-e^{-at}u(-t), \quad a < 0$

P20.3

Shown in Figures P20.3-1 to P20.3-4 are four pole-zero plots. For each statement in Table P20.3 about the associated time function $x(t)$, fill in the table with the corresponding constraint on the ROC.

(a)

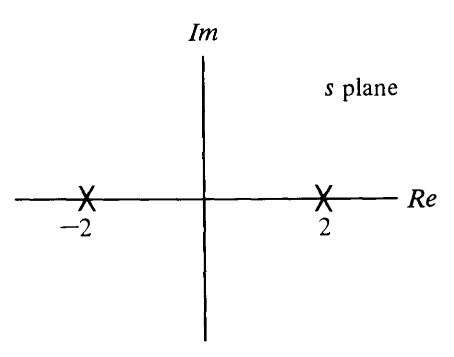

Figure P20.3-1

(b)

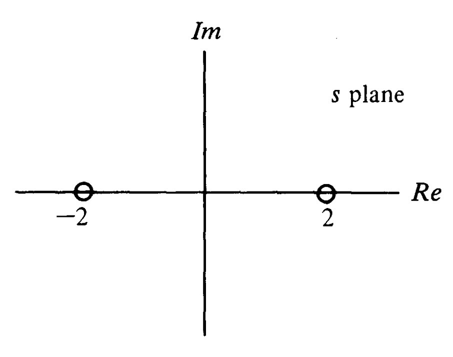

Figure P20.3-2

(c)

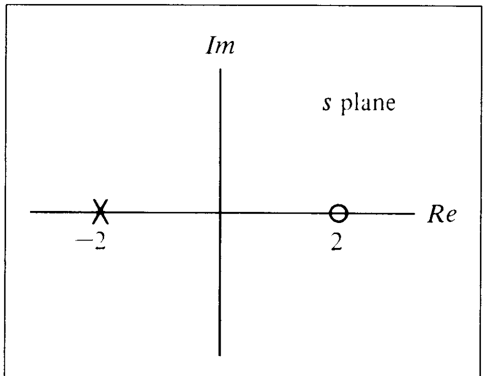

Figure P20.3-3

(d)

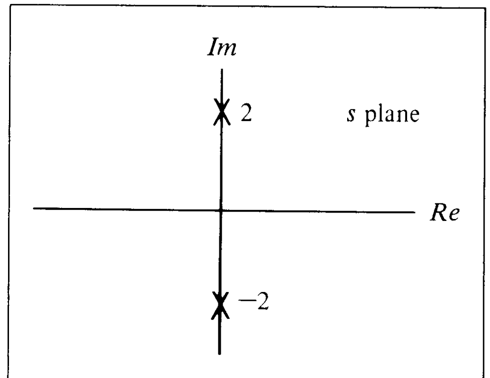

Figure P20.3-4

$x(t)$	Constraint on ROC for Pole-Zero Pattern (a)	(b)	(c)	(d)
(i) Fourier transform of $x(t)e^{-t}$ converges				
(ii) $x(t) = 0$, $t > 10$				
(iii) $x(t) = 0$, $t < 0$				

Table P20.3

P20.4

Determine $x(t)$ for the following conditions if $X(s)$ is given by

$$X(s) = \frac{1}{(s+1)(s+2)}$$

(a) $x(t)$ is right-sided

(b) $x(t)$ is left-sided

(c) $x(t)$ is two-sided

P20.5

An LTI system has an impulse response $h(t)$ for which the Laplace transform $H(s)$ is

$$H(s) = \int_{-\infty}^{+\infty} h(t)e^{-st}dt = \frac{1}{s+1}, \qquad Re\{s\} > -1$$

Determine the system output $y(t)$ for all t if the input $x(t)$ is given by

$$x(t) = e^{-t/2} + 2e^{-t/3} \qquad \text{for all } t.$$

P20.6

(a) From the expression for the Laplace transform of $x(t)$, derive the fact that the Laplace transform of $x(t)$ is the Fourier transform of $x(t)$ weighted by an exponential.

(b) Derive the expression for the inverse Laplace transform using the Fourier transform synthesis equation.

Optional Problems

P20.7

Determine the time function $x(t)$ for each Laplace transform $X(s)$.

(a) $\dfrac{1}{s+1}, \quad Re\{s\} > -1$

(b) $\dfrac{1}{s+1}, \quad Re\{s\} < -1$

(c) $\dfrac{s}{s^2+4}, \quad Re\{s\} > 0$

(d) $\dfrac{s+1}{s^2+5s+6}, \quad Re\{s\} > -2$

(e) $\dfrac{s+1}{s^2+5s+6}, \quad Re\{s\} < -3$

(f) $\dfrac{s^2-s+1}{s^2(s-1)}, \quad 0 < Re\{s\} < 1$

(g) $\dfrac{s^2-s+1}{(s+1)^2}, \quad -1 < Re\{s\}$

(h) $\dfrac{s+1}{(s+1)^2+4}, \quad Re\{s\} > -1$

Hint: Use the result from part (c).

P20.8

The Laplace transform $X(s)$ of a signal $x(t)$ has four poles and an unknown number of zeros. $x(t)$ is known to have an impulse at $t = 0$. Determine what information, if any, this provides about the number of zeros.

P20.9

Determine the Laplace transform, pole-zero location, and associated ROC for each of the following time functions.

(a) $e^{-at}u(t), \quad a < 0$

(b) $-e^{at}u(-t), \quad a > 0$

(c) $e^{at}u(t), \quad a > 0$

(d) $e^{-a|t|}, \quad a > 0$

(e) $u(t)$

(f) $\delta(t - t_0)$

(g) $\sum_{k=0}^{\infty} a^k \delta(t - kT), \qquad a > 0$

(h) $\cos(\omega_0 t + b)u(t)$

(i) $\sin(\omega_0 t + b)e^{-at}u(t), \qquad a > 0$

P20.10

(a) If $x(t)$ is an even time function such that $x(t) = x(-t)$, show that this requires that $X(s) = X(-s)$.

(b) If $x(t)$ is an odd time function such that $x(t) = -x(-t)$, show that $X(s) = -X(-s)$.

(c) Determine which, if any, of the pole-zero plots in Figures P20.10-1 to P20.10-4 could correspond to an even time function. For those that could, indicate the required ROC.

(i)

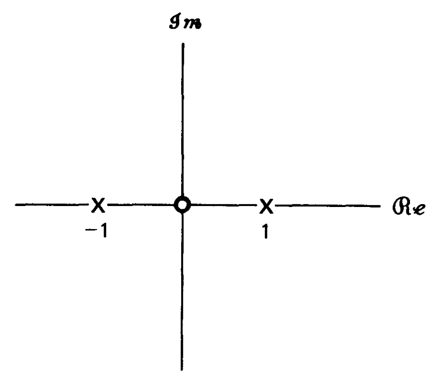

Figure P20.10-1

(ii)

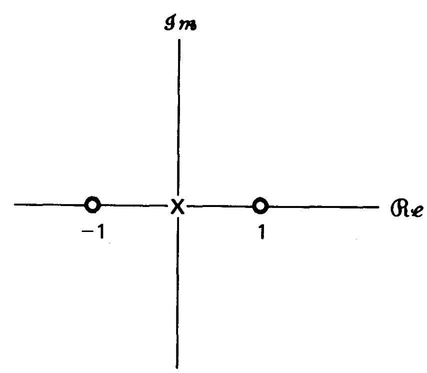

Figure P20.10-2

(iii)

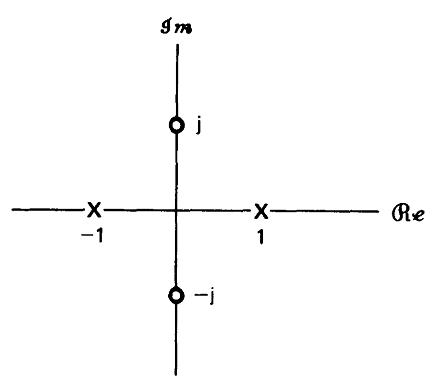

Figure P20.10-3

(iv)

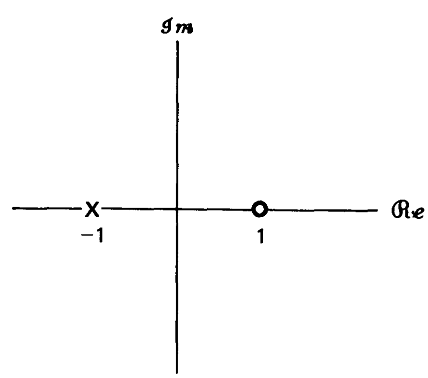

Figure P20.10-4

(d) Determine which, if any, of the pole-zero plots in part (c) could correspond to an odd time function. For those that could, indicate the required ROC.