# 18 Discrete-Time Processing of Continuous-Time Signals

## Recommended Problems

### P18.1

Consider the system in Figure P18.1-1 for discrete-time processing of a continuous-time signal using sampling period $T$, where the C/D operation is as shown in Figure P18.1-2 and the D/C operation is as shown in Figure P18.1-3.

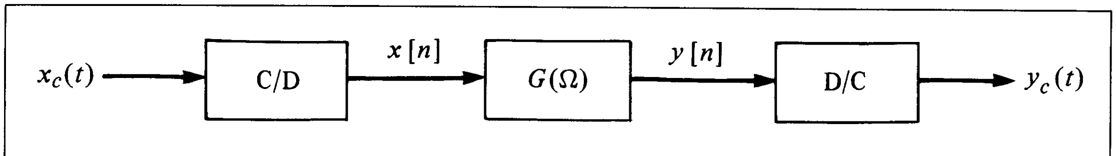


**Figure P18.1-1**

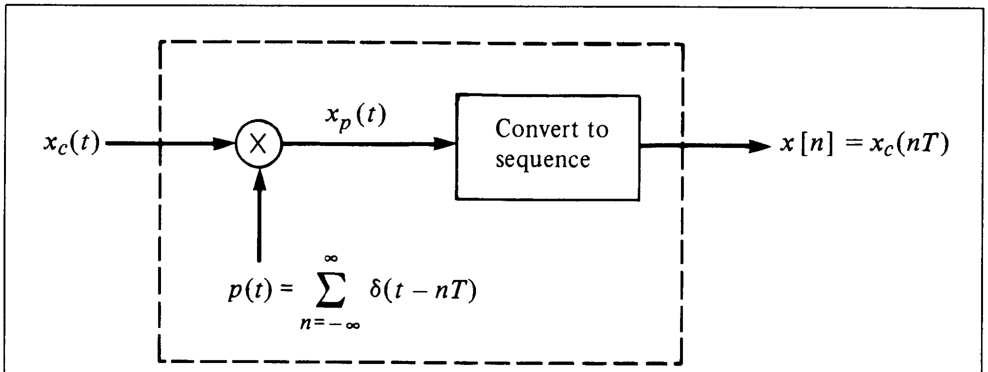


**Figure P18.1-2**

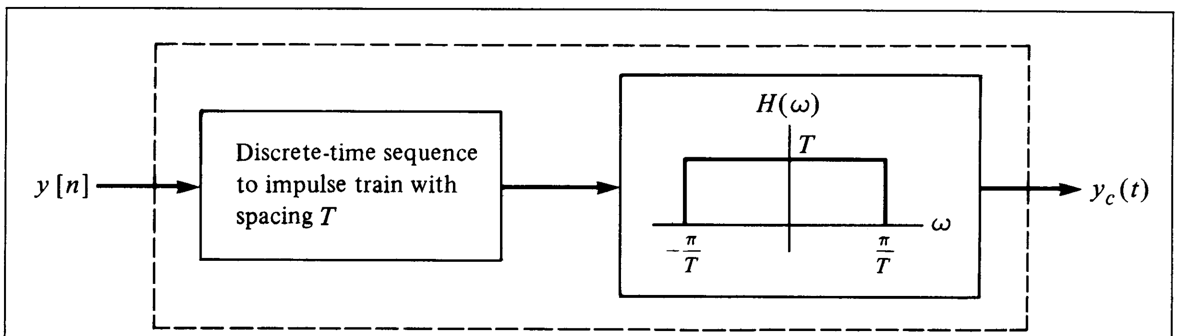


**Figure P18.1-3**

The filter $G(\Omega)$ is the lowpass filter shown in Figure P18.1-4.

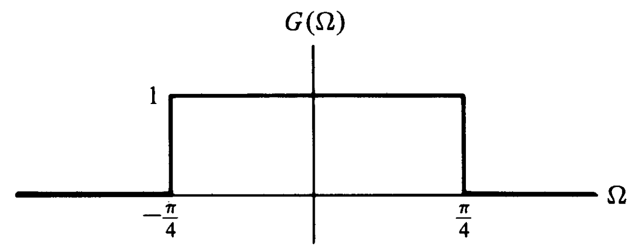


**Figure P18.1-4**

The Fourier transform of $x_c(t)$, $X_c(\omega)$ is given in Figure P18.1-5.

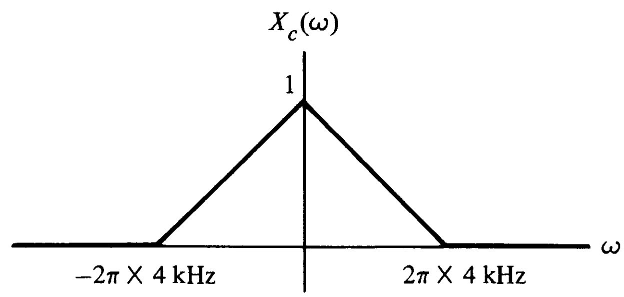


**Figure P18.1-5**

The sampling frequency is 8 kHz. Sketch accurately the following transforms.

**(a)** $X_p(\omega)$

**(b)** $X(\Omega)$

**(c)** $Y(\Omega)$

**(d)** $Y_c(\omega)$

## P18.2

Consider the continuous-time frequency response in Figure P18.2.

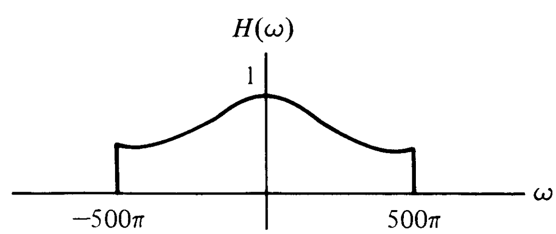


**Figure P18.2**

We want to implement this continuous-time filter using discrete-time processing.

**(a)** What is the maximum value of the sampling period $T$ required?

**(b)** What is the required discrete-time filter $G(\Omega)$ for $T$ found in part (a)?

**(c)** Sketch the total system.

## P18.3

The system in Figure P18.3 is similar to that demonstrated in the lecture. Note that, as in the lecture, there is no anti-aliasing filter.

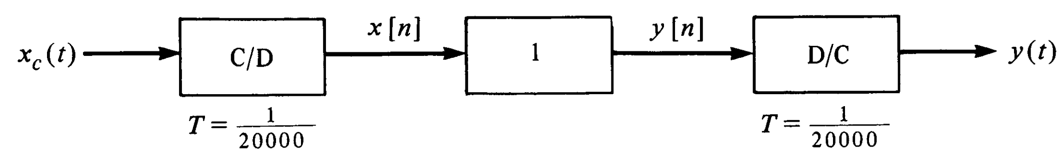


**Figure P18.3**

For the following signals, draw $X_c(\omega)$, $X(\Omega)$, and $Y_c(\omega)$.

**(a)** $x_c(t) = \cos(2\pi \cdot 5000t)$

**(b)** $x_c(t) = \cos(2\pi \cdot 27000t)$

**(c)** $x_c(t) = \cos(2\pi \cdot 17000t)$

## P18.4

Suppose we want to design a variable-bandwidth, continuous-time filter using the structure in Figure P18.4-1.

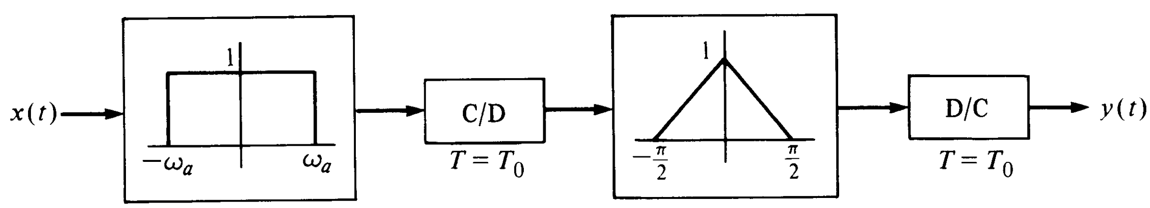


**Figure P18.4-1**

Find, in terms of $\omega_c$, the value of the sampling period $T_0$ and the corresponding value $\omega_a$ such that the total continuous-time filter has the frequency response shown in Figure P18.4-2.

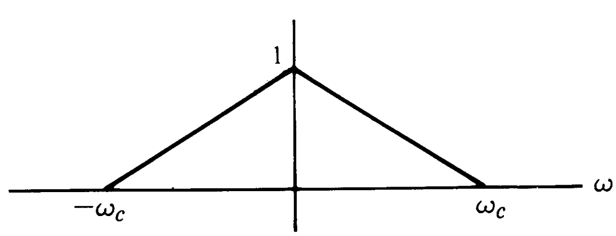


**Figure P18.4-2**

## P18.5

Consider the system in Figure P18.5-1.

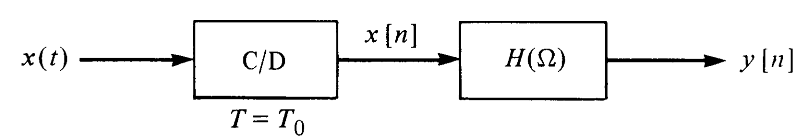


**Figure P18.5-1**

Let $H(\Omega)$ be as given in Figure P18.5-2 and $X(\omega)$ as given in Figure P18.5-3.

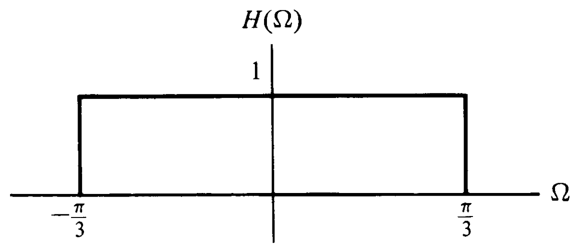


**Figure P18.5-2**

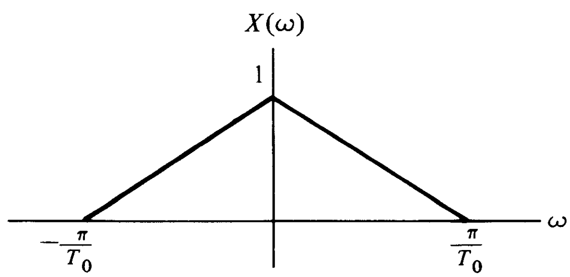


**Figure P18.5-3**

**(a)** Sketch $X(\Omega)$ and $Y(\Omega)$.

**(b)** Suppose we replace the system in Figure P18.5-1 by the system in Figure P18.5-4. Find $G(\omega)$ such that $y[n] = z[n]$.

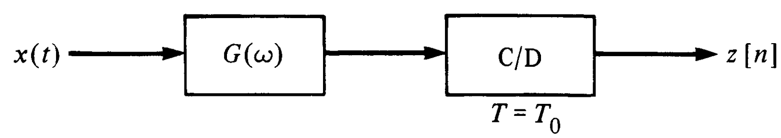


**Figure P18.5-4**

# Optional Problems

## P18.6

Suppose we are given the system in Figure P18.6-1.

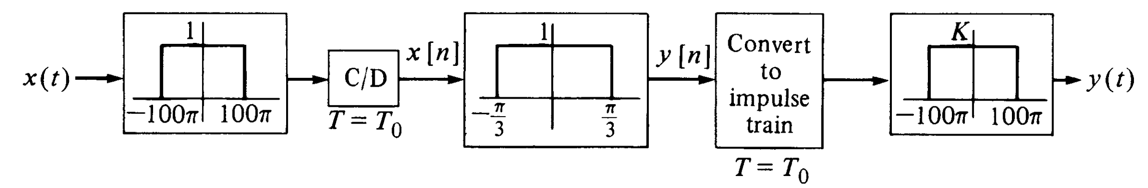


**Figure P18.6-1**

**(a)** Find the appropriate values of the sampling period $T_0$ to avoid aliasing. Also find the proper value for $K$ so that the overall system has a gain of unity at $\omega = 0$ (i.e., no overall dc gain).

**(b)** Suppose $T_0$ is halved, but the anti-aliasing and reconstruction filters are *not* modified.

(i) If $X(\omega)$ is as given in Figure P18.6-2, find $Y(\Omega)$.

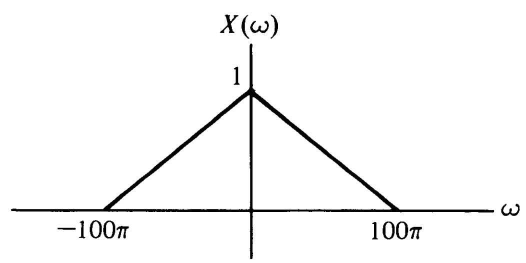


**Figure P18.6-2**

(ii) If $Y(\Omega)$ is as given in Figure P18.6-3, find $Y(\omega)$.

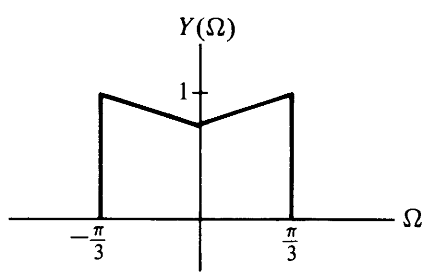


**Figure P18.6-3**

## P18.7

Figure P18.7 shows a system that processes continuous-time signals using a digital filter. The digital filter $h[n]$ is linear and causal with difference equation

$$y[n] = \tfrac{1}{2}y[n - 1] + x[n]$$

For input signals that are bandlimited so that $X_c(\omega) = 0$ for $|\omega| > \pi/T$, the system is equivalent to a continuous-time LTI system. Determine the frequency response $H_c(\omega)$ of the equivalent overall system with input $x_c(t)$ and output $y_c(t)$.

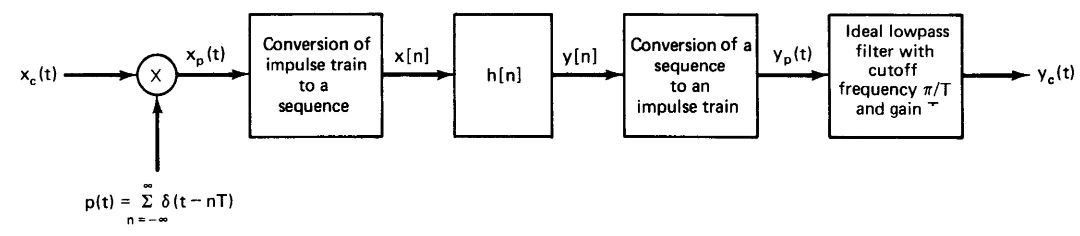


**Figure P18.7**

## P18.8

Figure P18.8-1 depicts a system for which the input and output are discrete-time signals. The discrete-time input $x[n]$ is converted to a continuous-time impulse train $x_p(t)$. The continuous-time signal $x_p(t)$ is then filtered by an LTI system to produce the output $y_c(t)$, which is then converted to the discrete-time signal $y[n]$. The LTI system with input $x_c(t)$ and output $y_c(t)$ is causal and is characterized by the linear constant-coefficient difference equation

$$\frac{d^2y_c(t)}{dt^2} + 4\frac{dy_c(t)}{dt} + 3y_c(t) = x_c(t)$$

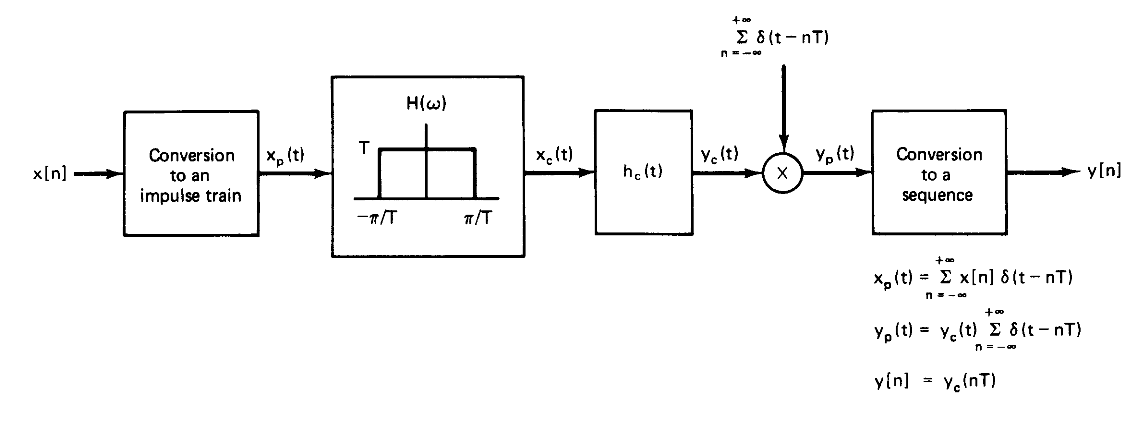


**Figure P18.8-1**

The overall system is equivalent to a causal discrete-time LTI system, as indicated in Figure P18.8-2. Determine the frequency response $H(\Omega)$ of the equivalent LTI system.

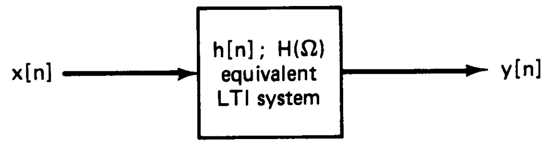


**Figure P18.8-2**

## P18.9

We wish to design a continuous-time sinusoidal signal generator that is capable of producing sinusoidal signals at any frequency satisfying $\omega_1 \leq \omega \leq \omega_2$, where $\omega_1$ and $\omega_2$ are positive numbers.

Our design is to take the following form. We have stored a discrete-time cosine wave of period $N$; that is, we have stored $x[0], \ldots, x[N-1]$, where

$$x[k] = \cos\left(\frac{2\pi k}{N}\right)$$

Every $T$ seconds we output an impulse weighted by a value of $x[k]$, where we proceed through the values of $k = 0, 1, \ldots, N-1$ in a cyclic fashion. That is,

$$\begin{aligned} y_p(t) &= \sum_{k=-\infty}^{\infty} x[k \text{ modulo } N]\, \delta(t-kT) \\ &= \sum_{k=-\infty}^{\infty} \cos\left(\frac{2\pi k}{N}\right) \delta(t-kT) \end{aligned}$$

**(a)** Show that by adjusting $T$ we can adjust the frequency of the cosine signal being sampled. Specifically, show that

$$y_p(t) = (\cos \omega_0 t) \sum_{k=-\infty}^{\infty} \delta(t-kT),$$

where $\omega_0 = 2\pi/NT$. Determine a range of values for $T$ so that $y_p(t)$ can represent samples of a cosine signal with a frequency that is variable over the full range $\omega_1 \leq \omega \leq \omega_2$.

**(b)** Sketch $Y_p(\omega)$.

The overall system for generating a continuous-time sinusoid is depicted in Figure P18.9-1. $H(\omega)$ is an ideal lowpass filter with unity gain in its passband:

$$H(\omega) = \begin{cases} 1, & |\omega| < \omega_c, \\ 0, & \text{otherwise} \end{cases}$$

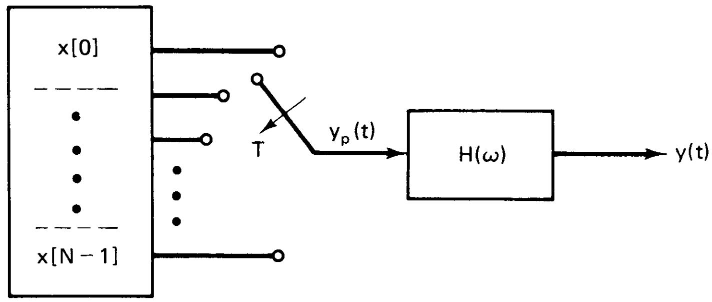


Figure P18.9-1

The parameter $\omega_c$ is to be determined such that $y(t)$ is a continuous-time cosine signal in the desired frequency band.

**(c)** Consider any value of $T$ in the range determined in part (a). Determine the minimum value of $N$ and some value for $\omega_c$ such that $y(t)$ is a cosine signal in the range $\omega_1 \leq \omega \leq \omega_2$.

**(d)** The amplitude of $y(t)$ will vary depending on the value of $\omega$ chosen between $\omega_1$ and $\omega_2$. Determine the amplitude of $y(t)$ as a function of $\omega$ and as a function of $N$.

## P18.10

In many practical situations, a signal is recorded in the presence of an echo, which we would like to remove by appropriate processing. For example, Figure P18.10-1 illustrates a system in which a receiver receives simultaneously a signal $x(t)$ and an echo represented by an attenuated delayed replication of $x(t)$. Thus, the receiver output is $s(t) = x(t) + \alpha x(t - T_0)$, where $|\alpha| < 1$. The receiver output is to be processed to recover $x(t)$ by first converting to a sequence and using an appropriate digital filter $h[n]$ as indicated in Figure P18.10-2.

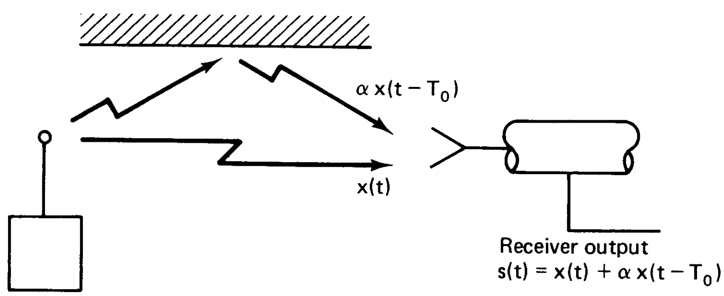


Figure P18.10-1

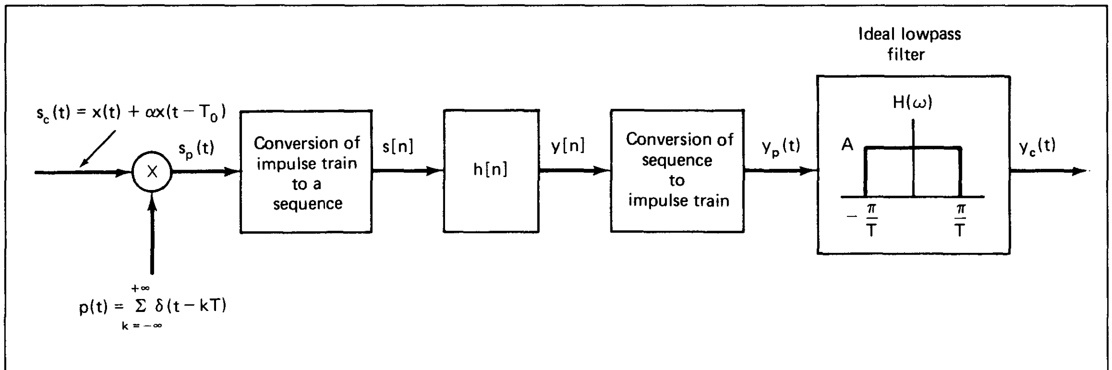


**Figure P18.10-2**

Assume that $x(t)$ is bandlimited, i.e., $X(\omega) = 0$ for $|\omega| > \omega_M$, and that $|\alpha| < 1$.

**(a)** If $T_0 < \pi/\omega_M$ and the sampling period is taken equal to $T_0$ (i.e., $T = T_0$), determine the difference equation for the digital filter $h[n]$ so that $y_c(t)$ is proportional to $x(t)$.

**(b)** With the assumptions of part (a), specify the gain $A$ of the ideal lowpass filter so that $y_c(t) = x(t)$.

**(c)** Now suppose that $\pi/\omega_M < T_0 < 2\pi/\omega_M$. Determine a choice for the sampling period $T$, the lowpass filter gain $A$, and the frequency response for the digital filter $h[n]$ such that $y_c(t)$ is equal to $x(t)$.