17 Interpolation

Recommended Problems

P17.1

Suppose we have the system in Figures P17.1-1 and P17.1-2, in which $x(t)$ is sampled with an impulse train. Sketch $x_p(t)$, $y(t)$, and $w(t)$.

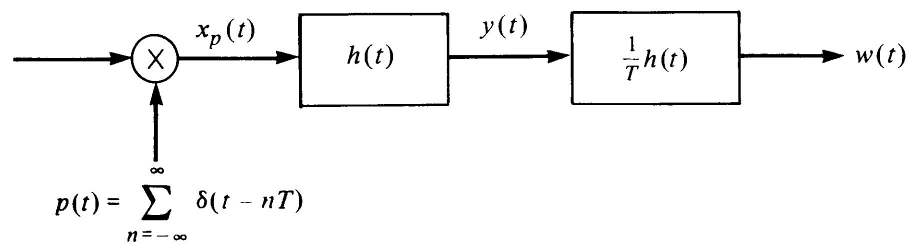

Figure P17.1-1

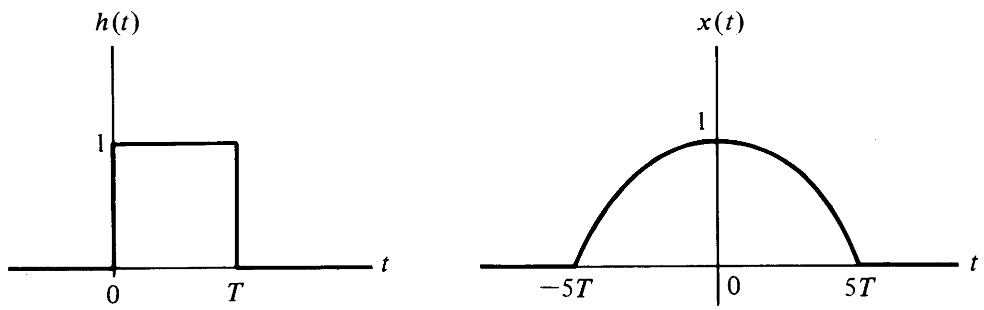

Figure P17.1-2

P17.2

Consider the signal $x(t) = \delta(t - 1) + \frac{1}{2}\delta(t - 2)$, which we would like to interpolate using the system given in Figure P17.2-1.

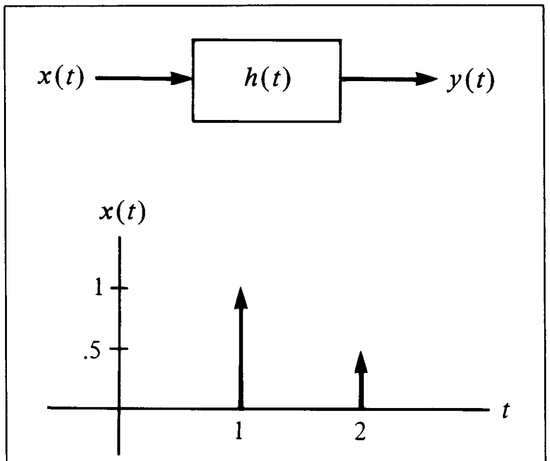

Figure P17.2-1

For the following choices of $h(t)$, sketch $y(t)$.

(a)

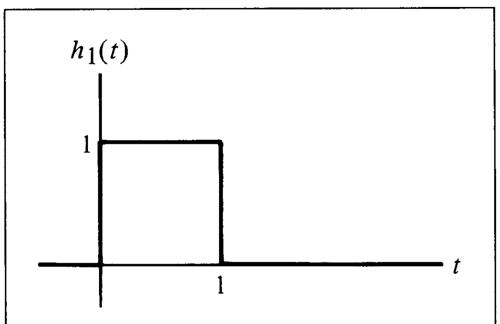

Figure P17.2-2

(b)

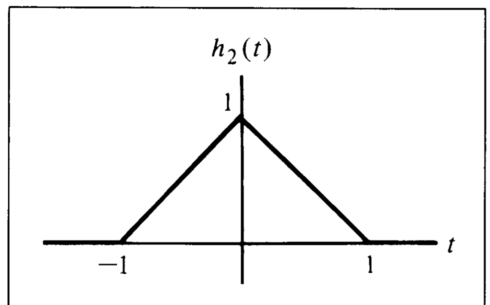

Figure P17.2-3

(c) $h_3(t) = \dfrac{\sin \pi t}{\pi t}$

P17.3

Consider the system in Figure P17.3-1, with $p(t)$ an impulse train with period T.

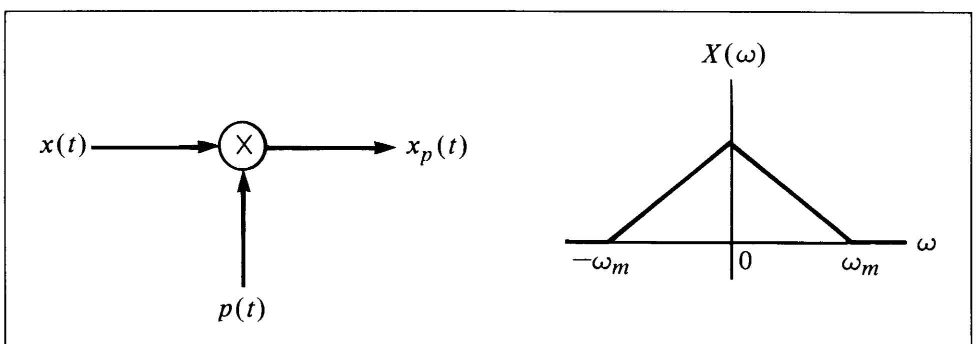

Figure P17.3-1

(a) Sketch $P(\omega)$ and $X_p(\omega)$, assuming that no aliasing is present. What is the relation between T and the highest frequency present in $X(\omega)$ to guarantee that no aliasing occurs?

(b) Consider recovering $x(t)$ from $x_p(t)$, assuming that no aliasing has occurred. For example, assume that $T = 2\pi/4\omega_m$. We know that $x(t)$ is recovered by interpolating $x_p(t)$, as shown in Figure P17.3-2.

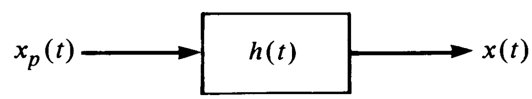

Figure P17.3-2

Is the specification of $h(t)$ unique so that $x(t)$ can be exactly recovered from $x_p(t)$? Why not?

(c) Using the convolution integral, show that if the original sampling period was T and if the filter is an ideal lowpass filter with cutoff ω_c, then the recovered signal $x_r(t)$ is

$$x_r(t) = \frac{T\omega_c}{\pi} \sum_{n=-\infty}^{\infty} x(nT) \operatorname{sinc} \frac{\omega_c(t - nT)}{\pi}$$

P17.4

In the system in Figure P17.4, $p(t)$ is an impulse train with period Δ, and the impulse response $g(t)$ is as indicated. Determine $H(\omega)$ so that $y(t) = x(t)$, assuming that no aliasing has occurred.

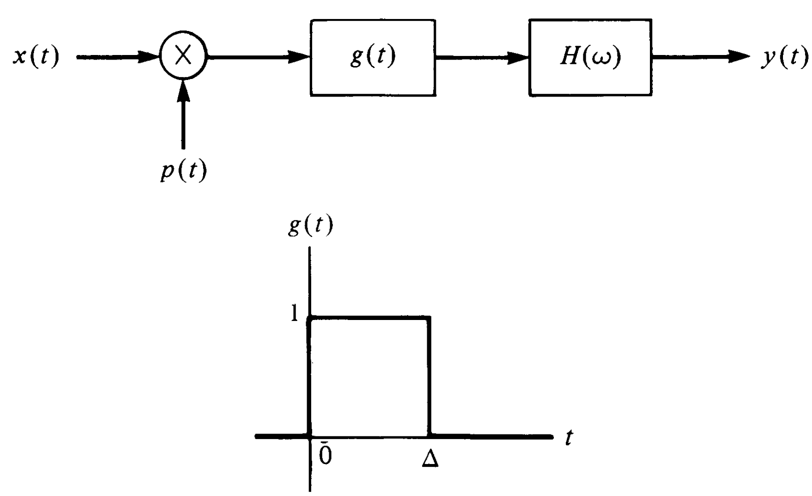

Figure P17.4

P17.5

Figure P17.5-1 shows the overall system for filtering a continuous-time signal using a discrete-time filter. If $X_c(\omega)$ and $H_1(\Omega)$ are as shown in Figure P17.5-2, with $1/T = 20$ kHz, sketch $X_p(\omega)$, $X(\Omega)$, $Y(\Omega)$, and $Y_c(\omega)$.

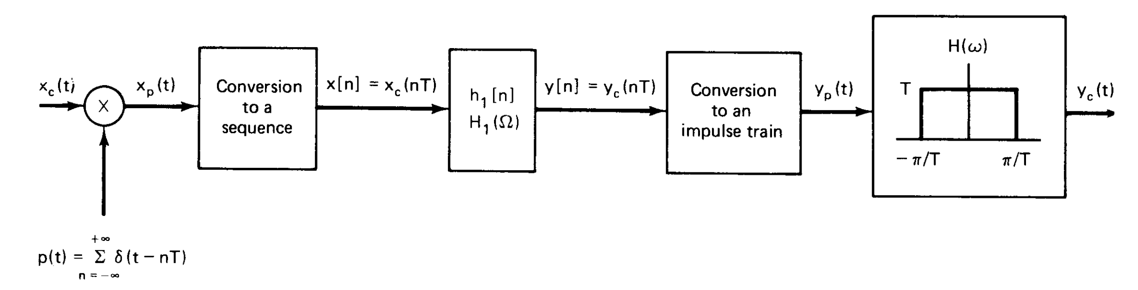

Figure P17.5-1

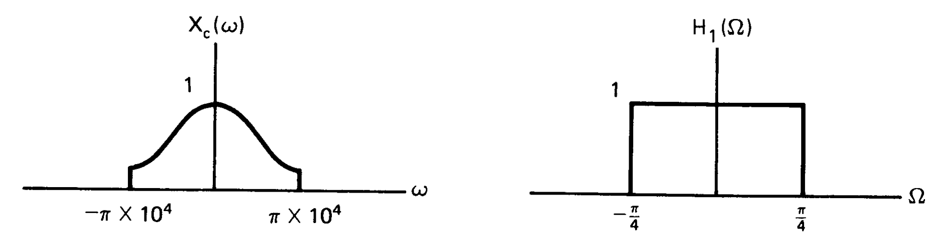

Figure P17.5-2

Optional Problems

P17.6

We consider a sequence $x[n]$ to which discrete-time sampling, as illustrated in Figure 8.32 of the text, has been applied. We assume that the conditions of the discrete-time sampling theorem are satisfied; that is, $\Omega_S > 2\Omega_M$, where Ω_S is the sampling frequency and $X(\Omega) = 0$, $\Omega_M < |\Omega| \leq \pi$. The original sequence $x[n]$ is then recoverable from $x_p[n]$ by ideal lowpass filtering, which, as discussed in Section 8.6 of the text, corresponds to bandlimited interpolation.

The ZOH represents an approximate interpolation whereby each sample value is repeated (or held) $N - 1$ successive times, as illustrated in Figure P17.6-1 for $N = 3$. The FOH represents a linear interpolation between samples, as illustrated in Figure P17.6-1.

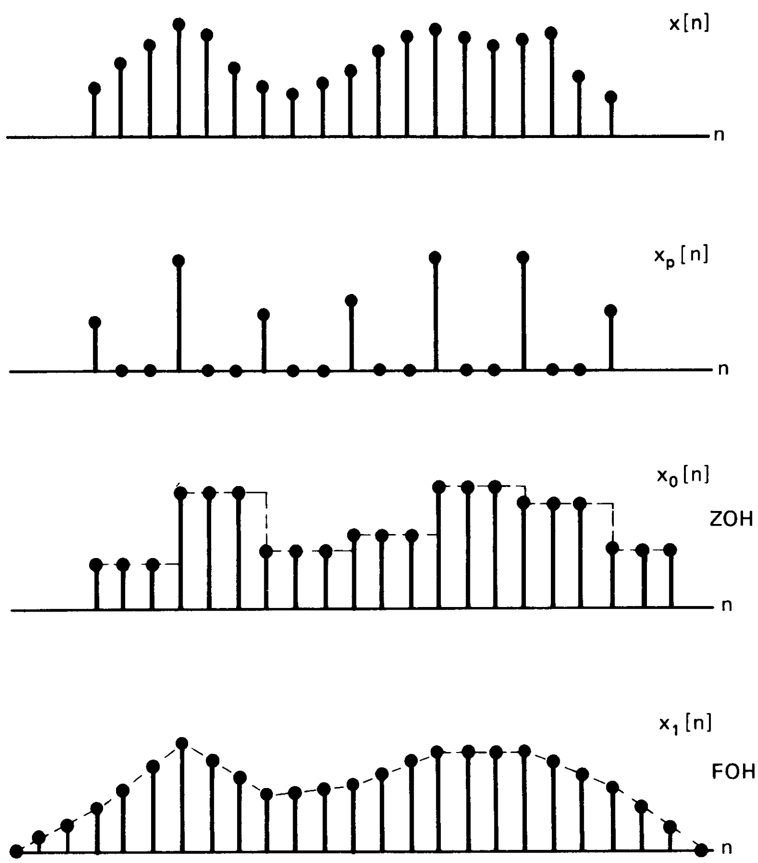

Figure P17.6-1

(a) The ZOH can be represented as an interpolation in the form of eq. (8.51) of the text (page 545) and the system in Figure P17.6-2. Determine and sketch $h_0[n]$ for the general case of a sampling period N.

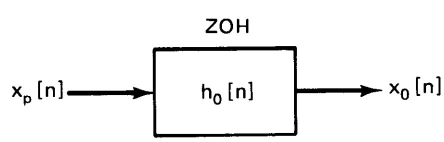

Figure P17.6-2

(b) $x[n]$ can be exactly recovered from the ZOH sequence $x_0[n]$ using an appropriate LTI filter $H(\Omega)$, as indicated in Figure P17.6-3. Determine $H(\Omega)$.

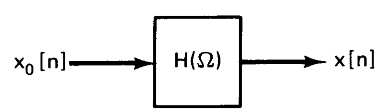

Figure P17.6-3

(c) The FOH (linear interpolation) can be represented as an interpolation in the form of eq. (8.51) of the text and, equivalently, the system in Figure P17.6-4. Determine and sketch $h_1[n]$ for the general case of a sampling period N.

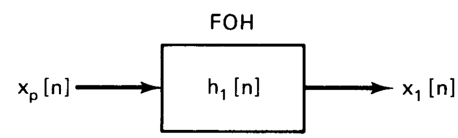

Figure P17.6-4

(d) $x[n]$ can be exactly recovered from the FOH sequence $x_1[n]$ using an appropriate LTI filter with frequency response $H(\Omega)$, as illustrated in Figure P17.6-5. Determine $H(\Omega)$.

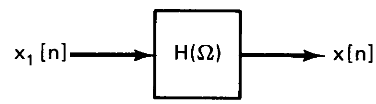

Figure P17.6-5

P17.7

Figure P17.7 shows a system consisting of a continuous-time linear time-invariant system followed by a sampler, conversion to a sequence, and a discrete-time linear time-invariant system. The continuous-time LTI system is causal and satisfies the LCCDE:

$$\frac{dy_c(t)}{dt} + y_c(t) = x_c(t)$$

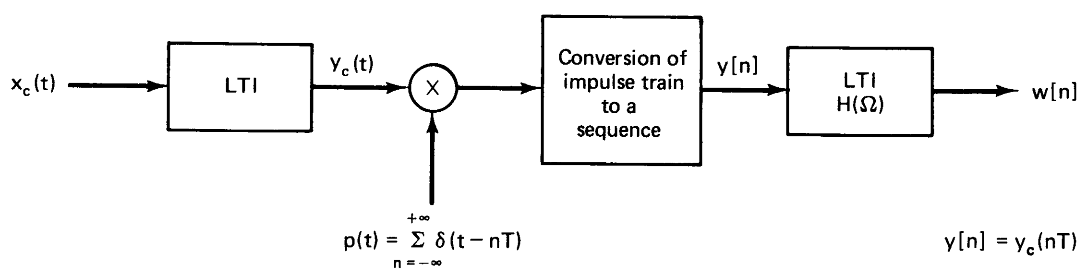

Figure P17.7

The input $x_c(t)$ is a unit impulse $\delta(t)$.

(a) Determine $y_c(t)$.

(b) Determine the frequency response $H(\Omega)$ and the impulse $h[n]$ such that $w[n] = \delta[n]$.

P17.8

Consider a signal $x(t)$ that is nonzero only in an interval $[-T, T]$. This problem deals with the sampling of the Fourier transform of $x(t)$.

(a) Suppose that we consider sampling the Fourier transform with an impulse train

$$P(\omega) = \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s),$$

as shown in Figure P17.8-1.

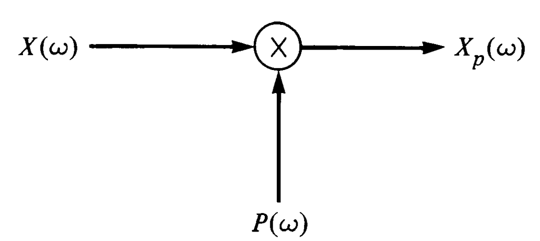

Figure P17.8-1

Sketch $X_p(\omega)$ if $X(\omega)$ is as given in Figure P17.8-2.

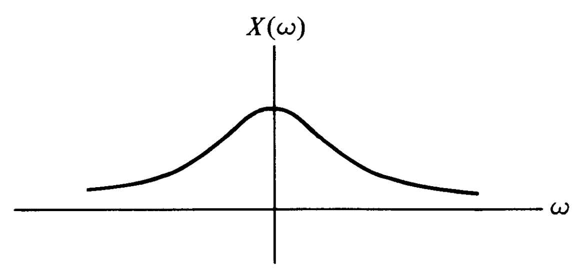

Figure P17.8-2

(b) Determine an expression for $x_p(t)$, the inverse Fourier transform of $X_p(\omega)$. How does $x_p(t)$ relate to $x(t)$?

(c) Determine a relation between ω_s and T such that $x(t)$ is recoverable.

(d) Assuming that ω_s satisfies the condition in part (c), how is $x(t)$ recovered from $x_p(t)$?