ICOM 4036: PROGRAMMING LANGUAGES

Lecture 5

Logic Programming

5/11/2004

What is Prolog

- **Prolog is a ‘typeless’ language with a very simple syntax.**
- **Prolog is declarative: you describe the relationship between input and output, not how to construct the output from the input (“specify *what* you want, not *how* to compute it”)**
- **Prolog uses a subset of first-order logic**

First-Order Logic

- **Simplest form of logical statement is an *atomic formula*. An assertion about objects.**

  Examples:

  is-man(tom)

  is-woman(mary)

  married-to(tom,mary)

  mother-of(mary,john)

First Order Logic

- More complex formulas can be built up using *logical connectives:*
  - **Men and Women are humans**
    - $\forall$ X [is-men(X) v is-woman(x) $\rightarrow$ is-human(x)]
  - **Somebody is married to Tom**
    - $\exists$ X married-to(tom,X)
  - **Some woman is married to Tom**
    - $\exists$ X [married-to(tom,X) $\wedge$ is-woman(X)]
  - **John has a mother**
    - $\exists$ X mother-of(X, john)
  - **Two offspring of the same mother are siblings**
    - $\forall$ X $\forall$ Y $\forall$ Z [mother-of(Z, X) $\wedge$ mother-of(Z, Y) $\rightarrow$ siblings(X, Y)]

$\exists$ is the Existential quantifier

$\forall$ is the Universal quantifier

Logical Inference

- **Example 2: Given these facts:**

  is-man(carlos)

  is-man(pedro)

and this rule:

$\forall$ **X [is-mortal(X)** $\leftarrow$ **is-man(X)]**

derive:

is-mortal(carlos), is-mortal(pedro).

Logical Inference

Logic programming is based on a simple idea: From facts and inferences try to prove more facts or inferences.

Prolog Notation

- **A rule:**

  $\forall X\ [p(X) \leftarrow (q(X) \wedge r(X))]$

is written as

  $p(X) \leftarrow q(X), r(X).$

- **Prolog conventions:**
  - *variables* begin with upper case (A, B, X, Y, Big, Small, ACE)
  - *constants* begin with lower case (a, b, x, y, plato, aristotle)

Prolog Assertions

```
/* list of facts in prolog, stored in an ascii file, ‘family.pl’*/
mother-of(mary, ann).
mother-of(mary, joe).
mother-of(sue, mary).
father-of(mike, ann).
father-of(mike, joe).
grandparent-of(sue, ann).

/* reading the facts from a file */
?-  consult ( ‘family.pl’ ).
family.pl compiled, 0.00 sec, 828 bytes
```

Prolog Evaluation

?- mother-of(sue, mary).

Yes

?- mother-of(sue, ann).

no

***?-* father-of(X, Y).**

X = mike;

***Y = joe* ;**

no

% Prolog returns these solutions one at a time, in the order it finds them. You can press semicolon (;) to repeat the query and find the next solution. Prolog responds "no" when no more valid variable bindings of the variables can be found.

Prolog Inference Rulesc

/* Rules */

parent-of(X , Y) :– mother-of(X , Y).

% if mother(X,Y) then parent(X,Y)

parent-of(X , Y) :– father-of(X , Y).

% if father(X,Y) then parent(X,Y)

grandparent(X , Z) :– parent-of(X , Y), parent-of(Y, Z).

% if parent(X,Y) and parent(Y,Z) then grandparent(X,Z)

:=	**means**	←

Prolog Inference Rule Evaluation

```
?- parent-of( X , ann), parent-of( X , joe).
X = mary;
X = mike;
no

?- grandparent-of(sue, Y ).
Y = ann;
Y = joe;
no
```

Factorial in Prolog

```
/* specification of factorial n! */
factorial(0,1).
factorial(N, M) :– N1 is N – 1,
                   factorial (N1, M1),
                   M is N*M1.
```

Takes 1 assertion and 1 inference

Factorial in Prolog - Evaluation

```
?- factorial (2, X).

factorial (0, 1).          /* fails */
factorial (2, M) :- 1 is 2 - 1,
                   factorial (1, M1),
                   M is N * M1.
```

Bindings: M = X, N = 2, N1 = 1

```
?- factorial (1, X1).
(X1 is the M1 above)

factorial (0, 1).          /* fails */
factorial (1, M) :- 0 is 1 - 1,
                   factorial (0, M1),
                   M is N * M1.
```

Bindings: M = X1, N = 1, N1 = 0

```
?- factorial (0, X2).
(X2 is the M1 above)

factorial (0, 1).          /* succeeds */
/* after the first rule succeeds, the
   second rule is not used */
```

Bindings: X2 = 1

Lists in Prolog

```
mylength( [ ], 0).
mylength( [X | Y], N):– mylength(Y, Nx), N is Nx+1.
```

? – mylength([1, 7, 9], X).
X = 3

? - mylength(jim, X).
No

? - mylength(Jim, X).
Jim = [ ]
X = 0

List Membership

```
mymember( X , [X | Y] ).
mymember( X , [W | Z ] ) :– mymember( X , Z ).
```

?–mymember(a, [b, c, 6]).
no

? – mymember(a, [b, a, 6]).
yes

? – mymember(X , [b, c, 6]).
X = b;
X = c;
X = 6;
no

Appending Lists

The Problem: Define a relation `append(X,Y,Z)` as X appended to Y yields Z

Appending Lists

- **The Problem: Define a relation `append(X,Y,Z)` to mean that X appended to Y yields Z**

```
append([], Y, Y).
append([H|X], Y, [H|Z]) :-
                        append(X,Y,Z).
```

Appending Lists

```
?- append([1,2,3,4,5],[a,b,c,d],Z).
Z = [1,2,3,4,5,a,b,c,d];
no

?- append(X,Y,[1,2,3]).
X = [] Y = [1,2,3];
X = [1] Y = [2,3];
X = [1,2] Y = [3];
X = [1,2,3] Y = [];
no
```

Prolog Computes ALL Possible Bindings!

Control in Prolog

Prolog tries to solve the clauses from left to right. If there is a database file around, it will be used in a similarly sequential fashion.

1. Goal Order: Solve goals from left to right.
2. Rule Order: Select the first applicable rule, where first refers to their order of appearance in the program/file/database

Control in Prolog

The actual search algorithm is:

1. start with a query as the current goal.
2. WHILE the current goal is non-empty

```
DO choose the leftmost subgoal ;
      IF a rule applies to the subgoal
      THEN select the first applicable rule;
              form a new current goal;
      ELSE backtrack;
ENDWHILE
SUCCEED
```

Control in Prolog

- Thus the order of the queries is of paramount importance .
- The general paradigm in Prolog is Guess then Verify: Clauses with the fewest solutions should come first, followed by those that filter or verify these few solutions

Fibonacci in Prolog

```
fib1(1, 1).
fib1(2, 1).
fib1(N1, F1) :-
        N1 > 2,
        N2 is N1 - 1,
        N3 is N1 - 2,
        fib1(N2, F2),
        fib1(N3, F3),
        F1 is F2 + F3.
```

More List Processing

remove(X, L1, L2) ~ sets L2 to the list obtained by removing the first occurrence of X from list L1

```
remove(X, [X|Rest], Rest).

remove(X, [Y|Rest], [Y|Rest2]) :-
            X \ == Y,
            remove(X, Rest, Rest2).
```

More List Processing

replace(X, Y, L1, L2) ~ sets L2 to the list obtained by replacing all occurrences of X in list L1 with Y

```
replace(_, _, [], []).
replace(X, Y, [X|Rest], [Y|Rest2]) :-
                    replace(X, Y, Rest, Rest2).
replace(X, Y, [Z|Rest], [Z|Rest2]) :-
                    Z \== X,
replace(X, Y, Rest, Rest2).
```

More List Processing

Write a predicate insert(X, Y, Z) that can be used to generate in Z all of the ways of inserting the value X into the list Y.

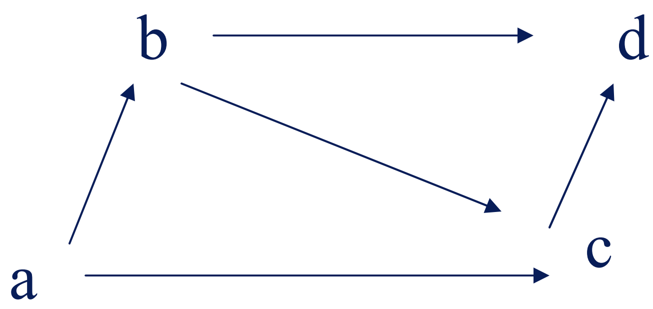
```
insert(X, [], [X]).

insert(X, [Y|Rest], [X,Y|Rest]).

insert(X, [Y|Rest], [Y|Rest2]) :-
                    insert(X,Rest, Rest2).
```

More List Processing

Write a predicate permutation(X, Y) that can be used to generate in Y all of the permutations of list X

```
permutation([], []).

permutation([X|Rest], Y) :-

        permutation(Rest, Z), insert(X, Z, Y).
```

Graphs in Prolog



Write a predicate route(X,Y) that success if there is a connection between X and Y

path(a,b).

path(b,c).

path(c,d).

path(d,b).

path(a,c).

Route(X,X).

Route(X,Y):- path(X,Z), route(Z,Y).

Binary Search Trees in Prolog

```
<bstree>::= empty
            node(<number>, <bstree>, <bstree>)
```

```
node(15,node(2,node(0,empty,empty),
node(10,node(9,node(3,empty,empty),
                    empty),
                node(12,empty,empty))),
node(16,empty,node(19,empty,empty)))
```

Binary Search Trees

```
isbtree(empty).
isbtree(node(N,L,R)):- number(N),isbtree(L),isbtree(R),
   smaller(N,R),bigger(N,L).

smaller(N,empty).
smaller(N, node(M,L,R)):- N < M, smaller(N,L),
                                   smaller(N,R).

bigger(N, empty).
bigger(N, node(M,L,R)):- N > M, bigger(N,L),
                                 bigger(N,R).
```

Binary Search Trees

```
?- [btree].
?-
 isbtree(node(6,node(9,empty,empty),empty)).
no

?-
 isbtree(node(9,node(6,empty,empty),empty)).
yes
```

Binary Search Trees

Define a relation which tells whether a particular number is in a binary search tree .

`mymember(N,T)` should be true if the number N is in the tree T.

```
mymember(K,node(K,_,_)).
mymember(K,node(N,S,_)) :-
            K <N, mymember(K,S).
mymember(K,node(N,_,T)) :-
            K >T, mymember(K,T).
```

Binary search Trees

```
?-
  mymember(3,node(10,node(9,node(3,empty,empty),empty)
  , node(12,empty,empty))).
yes
```

Sublists (Goal Order)

```
myappend([], Y, Y).
myappend([H|X], Y, [H|Z]) :-
                myappend(X,Y,Z).
myprefix(X,Z) :- myappend(X,Y,Z).
mysuffix(Y,Z) :- myappend(X,Y,Z).
```

Version 1

```
sublist1(S,Z) :-
        myprefix(X,Z), mysuffix(S,X).
```

Version 2

```
sublist2(S,Z) :-
        mysuffix(S,X), myprefix(X,Z).
```

Sublists

```
?- [sublist].

?- sublist1([e], [a,b,c]).
   no

?- sublist2([e], [a,b,c]).
   Fatal Error: global stack
   overflow …
```

Version 1

So what's happening? If we ask the question:

```
sublist1([e], [a,b,c]).
```

this becomes

```
prefix(X,[a,b,c]), suffix([e],X).
```

and using the *guess-query* idea we see that the first goal will generate four guesses:

```
[]    [a]    [a,b]    [a,b,c]
```

none of which pass the *verify* goal, so we fail.

Version 2

On the other hand, if we ask the question:

```
sublist2([e], [a,b,c]).
```

this becomes

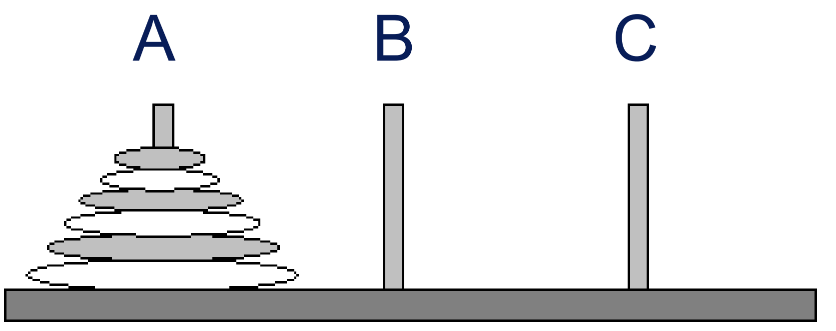
```
suffix([e],X),prefix(X,[a,b,c]).
```

using the guess-query idea note that the goal will generate an *infinite* number of guesses.

```
[e] [_,e] [_,_,e] [_,_,_,e] [_,_,_,_,e]
```

None of which pass the verify goal, so we never terminate!!

Towers of Hanoi



- You can move N disks from A to C in three general recursive steps.
    - Move N-1 disks from A pole to the B pole using C as auxiliary.
    - Move the last (Nth) disk directly over to the C pole.
    - Move N-1 disks from the B pole to the C pole using A as auxiliary.

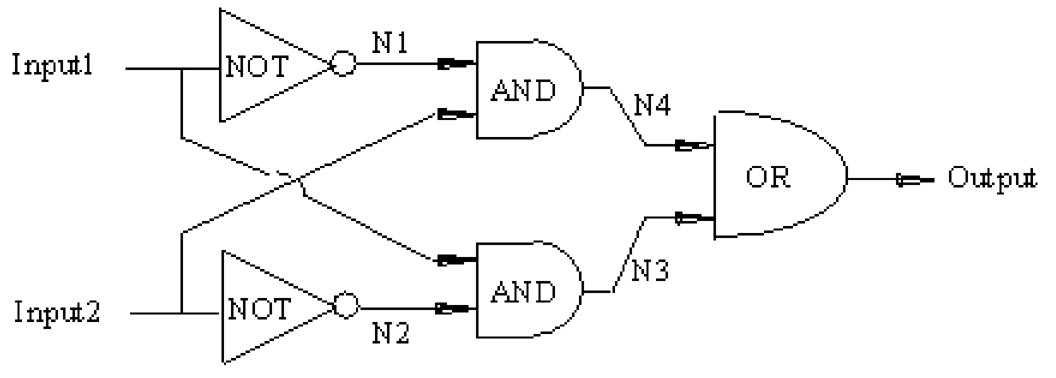
Towers of Hanoi

```
loc := right;middle;left

hanoi(integer)
move(integer,loc,loc,loc)
inform(loc,loc)

inform(Loc1, Loc2):-
    write("\nMove a disk from ", Loc1, " to ", Loc2).
```

Towers of Hanoi

```
hanoi(N):-
        move(N,left,middle,right).

 move(1,A,_,C) :- inform(A,C),!.
 move(N,A,B,C):-
        N1 is N-1,
        move(N1,A,C,B),
        inform(A,C),
        move(N1,B,A,C).
```

Logic Circuits



construct an exclusive OR circuit from AND, OR, and NOT circuits, and then check its operation

Logic Circuits: Prolog Model

```
not_(D,D)
and_(D,D,D)
or_(D,D,D)
xor_(D,D,D)
```

```
not_(1,0).      not_(0,1).
and_(0,0,0).    and_(0,1,0).
and_(1,0,0).    and_(1,1,1).
or_(0,0,0).     or_(0,1,1).
or_(1,0,1).     or_(1,1,1).
```

```
xor_(Input1,Input2,Output):-
        not_(Input1,N1),
        not_(Input2,N2),
        and_(Input1,N2,N3),
        and_(Input2,N1,N4),
        or_(N3,N4,Output).
```

Symbolic Differentiation

```
EXP :=      var(STRING);
            int(INTEGER);
            plus(EXP,EXP);
            minus(EXP,EXP);
            mult(EXP,EXP);
            div(EXP,EXP);
            ln(EXP);
            potens(EXP, EXP);
            cos(EXP);
            sin(EXP);
            tan(EXP);
            sec(EXP).
```

Symbolic Differentiation

```
d(int(_),_,int(0)).

d(var(X),X,int(1)) :- !.

d(var(_),_,int(0)).

d(plus(U,V),X,plus(U1,V1)) :- d(U,X,U1), d(V,X,V1).

d(minus(U,V),X,minus(U1,V1)) :- d(U,X,U1), d(V,X,V1).
```

Symbolic Differentiation

```
d(mult(U,V),X,plus(mult(U1,V),mult(U,V1))):-
      d(U,X,U1),
      d(V,X,V1).

d(div(U,V),X,div(minus(mult(U1,V),mult(U,V1)),mult(V,V))) :-
      d(U,X,U1),
      d(V,X,V1).

d(ln(U),X,mult(div(int(1),U),U1)) :- d(U,X,U1).

d(potens(E1,int(I)),X,mult(mult(int(I),potens(E1,int(I1))),EXP)) :-
      I1=I-1,
```

Symbolic Differentiation

```
d(E1,X,EXP).

d(sin(U),X,mult(cos(U),U1)) :- d(U,X,U1).

d(cos(U),X,minus(int(0),mult(sin(U),U1))) :- d(U,X,U1).

d(tan(U),X,mult(potens(sec(U),int(2)),U1)) :- d(U,X,U1).
```

Insertion Sort

```
isort([ ],[ ]).
isort([X|UnSorted],AllSorted) :-
                    isort(UnSorted,Sorted),
                    insert(X,Sorted,AllSorted).
insert(X,[ ],[X]).
insert(X,[Y|L],[X,Y|L]) :- X <= Y.
insert(X,[Y|L],[Y|IL]) :- X > Y, insert(X,L,IL).
```

Tail Recursion

Recursive

```
reverse([ ],[ ]).
reverse([XL],Rev) :- reverse(L,RL), append(RL,[X],Rev).
```

Tail Recursive (Iterative)

```
reverse([ ],[ ]).
reverse(L,RL) :- reverse(L,[ ],RL).
reverse([ ],RL,RL).
reverse([XL],PRL,RL) :- reverse(L,[XPRL],RL).
```

Prolog Applications

- Aviation, Airline and Airports
  - Airport Capacity Planning (SCORE)
  - Aircraft Rotation Schedule Optimization (OPUS)
  - Resource Optimization for Ramp Handling (LIMBO II)
  - Baggage Sorter Planning (CHUTE)
- Industry, Trade
  - Shop Floor Scheduling (CAPS)
  - Shop Floor Scheduling (CIM.REFLEX)
  - Production, Stock & Transportation Planning (LOGIPLAN)
- Health, Public
  - Staff Scheduling (STAFFPLAN)
  - Hospital Resource Management & Booking (IDEAL)
  - Integrated Hospital Resource Management (REALISE)