MULTIRATE BEAMFORMING FOR TWO DIMENSIONAL SENSOR ARRAYS A THEORETICAL APROACH TO DESIGN

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ABSTRACT

This paper proposes the implementation of multirate concepts using sensor arrays on digital signal processor (DSP) units such as TMS320C6711 DSK. This implementation based on Kronecker products formulation for mapping from hardware configurations to software algorithms, centers on a scalable and modular approach. The scalable approach to this implementation implies that the function and structure of each algorithmic formulation should adapt to changes in the size of the hardware system and on the length and dimensions of the signal to be processed. The modularity approach implies that each system can be composed by a set of modules with flexible interconnectivity, and reconfigurability could be obtained. The sensor arrays for this approach were made with micro-microphones and the A/D converter is performed by the TI (Texas Instruments) EVM (Evaluation Module) ADS8364 for data acquisition from six A/D converters because we are interested in sound applications.

1. INTRODUCTION

The normal structure of sensor arrays at the moment is based on a regular grid, that produces aperture-smoothing functions, which are relatively simple to analyze and allows the use of fast algorithms, such as Fast Fourier Transform to compute spatial spectra and array output signals [1]. Now when we want large-scale sensor arrays, it is imperative to develop robust hardware and software applications able to drive different physical structures for sensors location. In addition the sampling rate of sensors could be synchronized to different rates depending on the specific signal interest and desired information extraction. Finally when we are talking about large-scale structures (array sensors) it could be exposed to external damage and the software and hardware needs to drive different configurations in order to minimize the impact of damage.

The new applications are demanding real time processing test beds, offering modularity, scalability, reconfigurability, features to effect a faster delivery of computational hardware prototype system. Therefore, the hardware and software used with these applications must be strong enough to meet its requirements.

2. COMPUTATIONAL KRONECKER PRODUCTS

The Kronecker product between the matrices A and B of size $N \times N$ and $M \times M$ respectively, is defined as follows:

$$C = A \otimes B = \left\lfloor a_{(i,j)} \cdot B \right\rfloor$$

Thus,

$$C = A \otimes B = \begin{bmatrix} a_{(0,0)}B & \cdots & a_{(0,s-1)}B \\ a_{(1,0)}B & \cdots & a_{(1,s-1)}B \\ \vdots & \ddots & \vdots \\ a_{(r-1,0)}B & \cdots & a_{(r-1,s-1)}B \end{bmatrix}$$

Which is a resulting matrix C of size $NM \times NM$. As demonstrated in [2], the FFT matrix F_N, of order N can be written as the product:

$$F_N = (F_R \otimes I_s)T_{N,S}(I_R \otimes F_S)P_{N,R} \quad (1)$$

Where $N = R \cdot S$, I_N identity matrix has a size $N \times N$.

The first step for mapping FFT algorithms to computational structures by performing a Kronecker formulation of a Fourier matrix of order 4th figure 1, as a Cooley-Tukey FFT in time algorithm. It contains the smallest order Fourier computational elements which are F_2:

$$F_4 \cdot x = (F_2 \otimes I_2)T_{4,2}(I_2 \otimes F_2)P_{4,2} \cdot x \quad (2)$$

Each factor in the formulation represents a stage in the FFT algorithm. For high order FFTs, the method is applied recursively until functional primitive factors are reached.

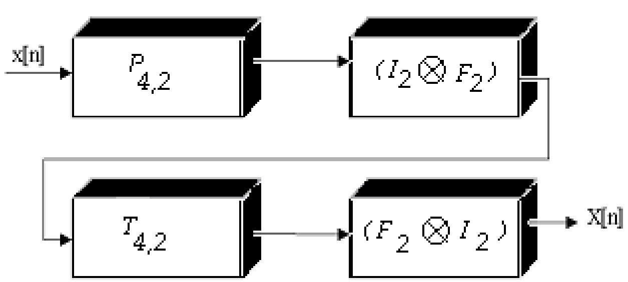

Figure 1. F4 Computational Structure.

3. RECTANGULAR SENSOR ARRAYS

Figure 2 shows the structure of a rectangular array [4]. The antenna elements are denoted by (n_1, n_2), where $0 \le n_1 \le N_1 - 1$ and $0 \le n_2 \le N_2 - 1$. The plane wave is received by this array with $N_1 \times N_2$ elements which are placed at constant array distances d_1 and d_2 on the direction of n_1 and n_2 respectively. Assuming that the phase reference point is located at $(n_1 = 0, n_2 = 0)$, the phase of the wave at the element denoted by (n_1, n_2) is:

$$f(n_1, n_2) = \frac{w}{c}(d_2 n_2 \sin q - d_1 n_1 \cos q) \quad (3)$$

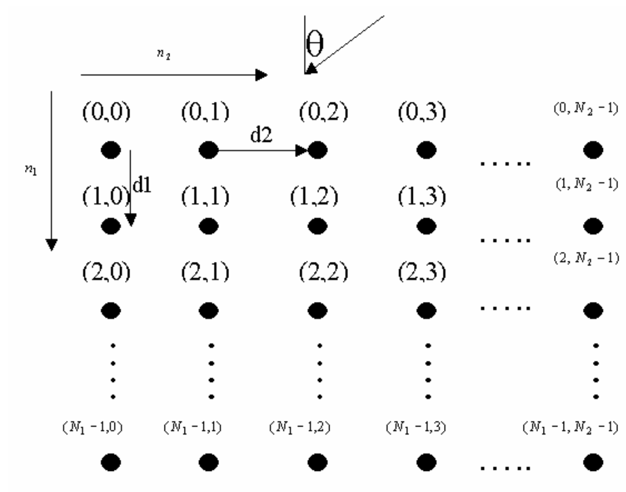

Figure 2. Uniform Rectangular Array

By assuming that each sensor element is connected to only one single coefficient $C_{n1,n2}$. Hence, the response of the array with respect to frequency and DOA (Direction of the Arrive) is as follows:

$$H(w,q) = \sum_{n_1=0}^{N_1-1} \sum_{n_2}^{N_2-1} C_{n_1,n_2} e^{j\frac{w}{c}(d_2 n_2 \sin q - d_1 n_1 \cos q)} \quad (4)$$

Where q is the azimuth angle of the incoming wave and is considered the angle of DOA, $w = 2pf$ is the operating frequency and c is the propagation speed. Here we assume that the elevation angle of the arriving signal is small and almost the same, hence the effects of the elevation is neglected.

4. MODEL FORMULATION

Beamforming is a signal processing operation used widely in wireless Communication, Radar, and Sonar applications to estimate the DOA of a propagating waveform source when the waveform is received by an array of sensors [3].

A plane wave can be spatially sampled using an array of omni-directional sensors in order to extract information about its propagating direction. Features of the sensors array, such as the number of sensors and distance between sensors are related with the wavelength of the incident wave. This paper consider the case of a rectangular sensor array, where the samples along the array are a finite, discrete input signal to be processed by a Multirate Preprocessing in order to obtain different rates of the frequency sample and then the output signal is evaluated by the beamforming processor to identify the phase and then the angle of the incident wave as shown in figure 3.

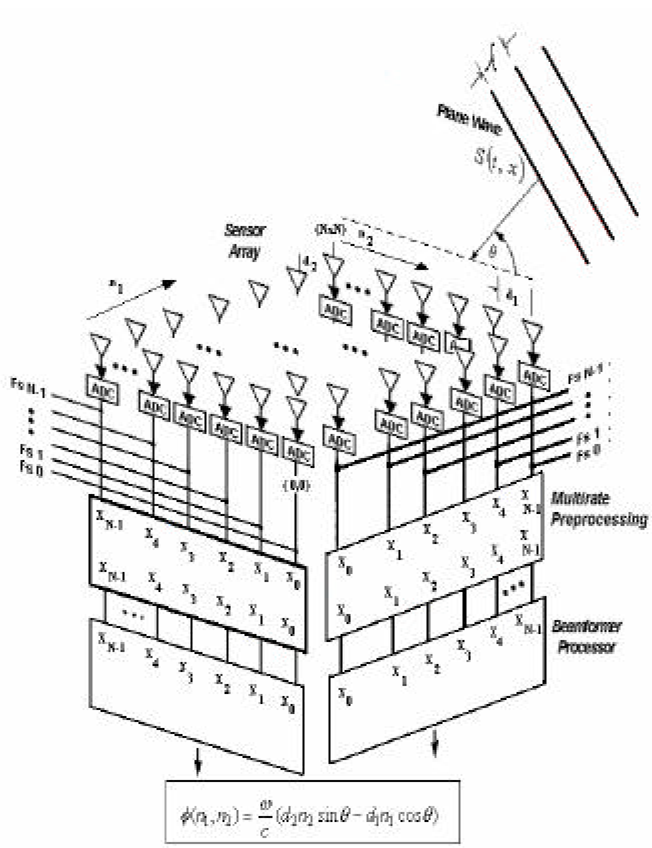

Figure 3. Two Dimensional Digital Beamforming

5. HARDWARE IMPLEMENTATION

The implementation step starts with the design of 3 x 5 array of sensors using micro-microphones, this array of micro-microphones is connected to the A/D converter ADS8364 EVM of TI, and this A/D converter sends data from six channels to the DSP processor DSK TMS320C6711. By using the RTDX (Real Time Data Exchange) interface we can be done on Matlab environmental and the data can be evaluated. But finally all operations should be performed under the DSP platform and Matlab is only used as a graphic interface. This field of work is the sound spectrum to obtain practical applications on vibration and moving targets.

5.1 Kronecker Multirate Beamforming.

Partial beamforming is performed decomposing the input row or column vector x of length $L = MN$ as shown in equation (2), in M segments of length N:

$$(I_M \otimes F_N)x$$

Thus we define a Kronecker parallel factor as a diagonal matrix:

$$I_M \otimes F_N = \begin{bmatrix} F_N & 0 & \cdots & 0 \\ 0 & F_N & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & F_N \end{bmatrix}$$

Multi beamforming is performed collecting and combining the output information, channel by channel, of every DFT:

$$\left(U_M^T \otimes I_N\right) = [I_N, I_N, \cdots, I_N] \ ; \quad U_M^T = [1,1,\cdots,1]$$

Thus, we obtain a matrix of size MN :

$$U^T{}_M \otimes I_N = \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots & 0 & 1 & \ddots & \vdots & \cdots & 0 & 1 & \ddots & \vdots \\ 0 & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 & \cdots & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 1 & \cdots & 0 & 0 & \cdots & 1 \end{bmatrix}$$

The description formulation using Kronecker products for the multirate beamforming operations is shown on figure 4.

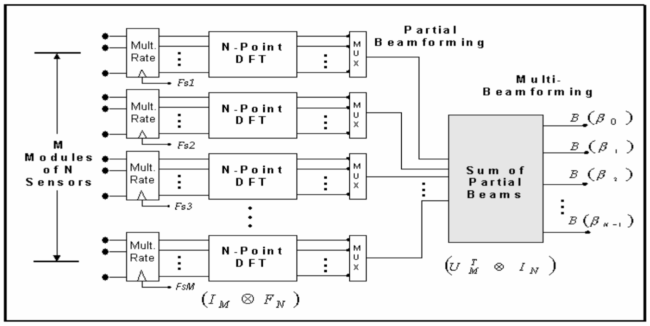

Figure 4. Kronecker Multirate Beamforming

5.2 **Filter Multirate Model**

For each row or column of the figure 3, divided in M modules of N sensors as shown in figure 4, a filter multirate could be implemented as shown in figure 5, SN is the nth sensor in our case micro-microphones; A/D is the Analog to digital converters with frequency sampling at FsN; DN is the decimation filter; $\downarrow$ MN is the down sampling process to reduce the sampling rate of the incoming signal; Hn(f) is the filter with cut frequency proportional to new sampling rate and removes undesired spectral signal; $\uparrow$ MN is the up sampling process that recovers the original rate of the signal; IN(f) is an interpolation filter. Then the output of the multi beamforming is sent through RTDX to Matlab environmental on PC (Personal Computer) or Host, for a graphical interface.

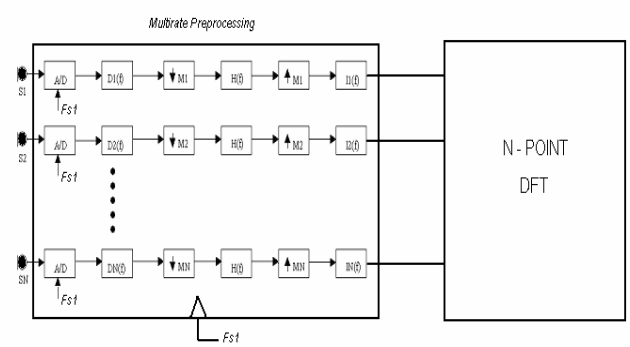

Figure 5. Block diagram of multirate Preprocessing

6. CONCLUSIONS AND FUTURE WORK

This paper presented an approach to the implementation of beamforming using two dimensional sensor arrays and multirate concepts, based on Kronecker products as an important tool for DFT mapping from hardware to software implementation.

In addition, the new approach of design will be implemented on a real DSP processor DSK TMS320C6711, using array sensors for sound applications. Finally, the use of new technologies, such as the Internet, can be interesting in gathering real data provide from sensors arrays.
This approach contemplates this possibility because all results of the design have to be displayed on a PC or Host Server, and it could be accessed by different users through the www (World Wide Web).

8. REFERENCES

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