Texture Synthesis using Reaction-Diffusion

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Abstract

Reaction-Diffusion (RD) systems from cellular automata theory have been used for synthesizing patterns. In this paper, the pattern formation using RD systems are discussed and shown how they can be used to synthesize textures. A methodology for synthesizing textures with a specific regularity measure using RD is presented. This procedure uses random walk technique. Several examples with varying regularity are shown. The regularity can be measured by computing a variogram. A variogram is a statistical measure that is computed at different pixel spacing. The method presented can be adapted to synthesizing any structural texture pattern.

1. Introduction

Textures can be synthesized by different techniques such as fractals, random fields, reaction-diffusion, morphology, Gabor filters, Eigen-patterns, steerable pyramids, wavelets, tiling and co-occurrence methods. Each method can generate only a particular subset of texture patterns and have their own advantages and disadvantages. Most methods estimate parameters from an observed image and try to synthesize the observed image. They minimize the error between the synthesized image and observed image. The Grey Level Co-occurrence matrices can be used for synthesizing textures [Copeland01], Markov Random Field (MRF) models are used in [Paget98], a tree-structured vector quantization is presented in [Wei01], joint statistics of wavelet coefficient magnitudes are used in [Simoncelli98], minimax entropy learning by diffusion is used in [Shah98], a non-parametric sampling method is used by [Efros99], a set of feature statistics is used in [Zhu99] and a pyramid based synthesis method is presented in [Heeger96]. The type of synthesis method used depends on the application. For example, if the synthesis is for displays, animations, games then color and volume rendering are important. Synthesis by itself is widely studied under scientific computing where fast methods are developed. This paper presents a texture synthesis methods based on reaction diffusion systems in chemical morphogenesis. The paper is organized as follows. Section 2 presents a review of the 2 dimenisonal RD systems. Section 3 presents the algorithms implemented. Section 4 presents experimental results of synthesis and observations. Conclusions are presented in Section 5.

2. Reaction-Diffusion Systems

A dynamic texture pattern evolves based concentrates of chemicals called morphogens in a reaction-diffusion system. The morphogens react together and diffuse throughout the developing tissue. The initial state of the morphogens can be random or homogeneous. After the system has evolved macroscopic concentration patterns appear in the reacting chemicals. Local concentration gradients modify the concentration rates of the morphogens, which react to increase or decrease the amount of chemical within each cell. The equations for the model are:

$$\begin{aligned} \frac{\partial a}{\partial t} &= F(a,b) + Da\nabla^2 a \\ \frac{\partial b}{\partial t} &= G(a,b) + Db\nabla^2 b \end{aligned} \qquad (1)$$

where a and b are concentrations of two diffusing morphogens; F and G are functions determining the production rate of a and b; Da and Db are diffusion rate constants and ∇^2a and ∇^2b are the Laplacians of a and b. A system to approximate the model is built upon an array of cells through which morphogens a and b may diffuse. Within each cell a and b are created or destroyed according to F and G. A discrete system to solve (1) is:

$$\begin{aligned} \Delta a_i &= s(16 - a_i b_i) + Da(a_{i+1} + a_{i-1} - 2a_i) \\ \Delta b_i &= s(a_i b_i - b_i - b_i) + Db(b_{i+1} + b_{i-1} - 2b_i) \end{aligned} \qquad (2)$$

where a_i and b_i are morphogen concentrations in a one dimensional array of cells; βi represents the natural variation between individual cells; and s is the reaction rate constant [Turk91]. Complex patterns can be formed by lateral activation of locally exclusive states [Meinhardt82]. Lateral help can be introduced via diffusible substances in a straightforward manner. g_1 and g_2 are the autocatalytic substances required for the self-stabilization. A common repressor is used to bring about the local mutual exclusion of the two states. The diffusible substances s_1 and s_2 provide the long-ranging help of one feedback system to the other:

$$\frac{\partial g_1}{\partial t} = \frac{cs_2 g_1^2}{r} - ag_1 + D_a \frac{\partial^2 g_1}{\partial x^2} + r_0$$

$$\frac{\partial g_2}{\partial t} = \frac{cs_1 g_2^2}{r} - ag_2 + D_g \frac{\partial^2 g_2}{\partial x^2} + r_0 \qquad (3)$$

$$\frac{\partial r}{\partial t} = cs_2 g_1^2 - cs_1 g_2^2 - br(+D_r \frac{\partial^2 r}{\partial x^2})$$

$$\frac{\partial s_1}{\partial t} = g\,(g_1 - s_1) + D_s \frac{\partial^2 s_1}{\partial x^2} + r_1$$

$$\frac{\partial s_2}{\partial t} = g\,(g_2 - s_2) + D_s \frac{\partial^2 s_2}{\partial x^2} + r_1$$

In stripe like patterns, the stripes can have different width if the strength of the autocatalysis or of the mutual help is different in both feedback loops. Then, the equilibrium between the g_1 or g_2 cells would be shifted in favor of one or the other leading to a corresponding change in the number of high g_1 and high g_2 cells.

3. Synthesis Algorithms

Before presenting the texture synthesis method, a few definitions are in order:

Texture: A property of a surface that has a repetitive structure.

Main properties of texture are its regularity, randomness and directionality.

Texton- local conspicuous features as basis for texture discrimination. Textures of varying regularity can be synthesized using diffusion mechanism. Fig. 2 shows two textures with different regularity.

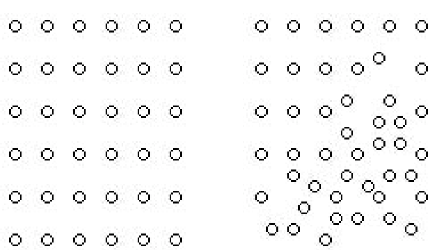


Fig. 2. Textures with varying regularity

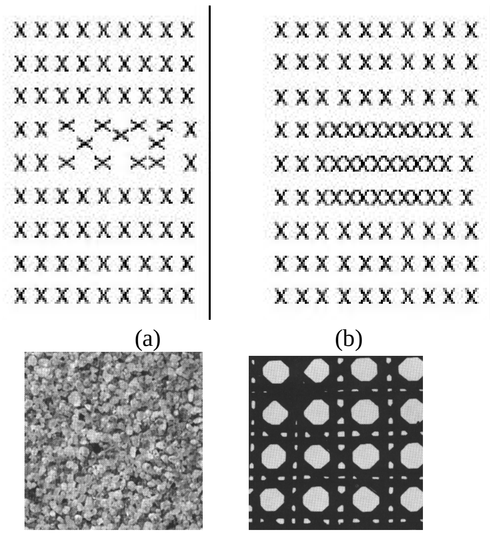
A statistical measure called a variogram that quantifies the regularity in an image is computed and is given by:

$$V(d) = E([t(q) - t(q')]^2) \qquad (4)$$

where q and q' are two points distance d apart. The variogram is a measure of regularity of the texture pattern. For a discrete set of space vectors d, integral value for regularity is computed as:

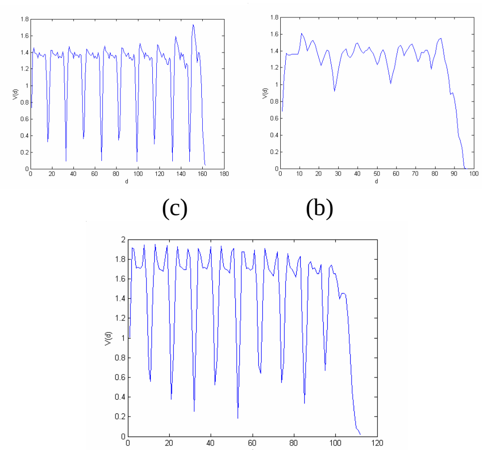
$$REG = \max\{V(d_i)\} \qquad (5)$$

where i=1,2,…, Q, Q is the number of spacing vectors. Fig. 3 shows texture patterns with different regularities. Fig. 4(a) shows the variograms for the basic pattern in Figs. 3(a) and (b). Figs. 4(b) and (c) shows the variograms of the two center patterns in Figs. 3(a) and (b), respectively. The REG measures computed from plots is shown in Table 1 for the textures in Figs. 2 and 3. As can be seen the least regular pattern is the center pattern in Fig. 3(a) with corresponding REG measure of 1.2031. Figs. 3(c) and (d) show two textures scanned at 300dpi from Brodatz textures [Brodatz66] of size 256x256. Their corresponding REG measures are 1.7988 and 1.9608.



(a) (b) (c) (d)

Fig. 3. Texture patterns



(c) (b) (c)

Fig. 4. Variograms

As repetitiveness of a texture is governed by the placement of texton features on a 2D grid, the synthesis algorithm employs placement rules. The placement rules follow random walk diffusion mechanism, as the key is to generate textures with incremental changes to regularity. The following Section describes the random walk model.

3.1 Random-Walk Model of Placement

Initial texton elements are placed on a grid of locations called the field. The location of each element is given by its coordinates on this grid (i,j), 1≤ i,j ≤ N where N is the maximum size of the grid. The horizontal distance between neighboring grid locations is assumed as unit distance and all spatial dimensions of the random walk are expressed as multiples of the unit distance. The location of the texton in the grid changes probabilistically at each step. Successive steps represent successive times that are separated by a unit time interval. All times are expressed in terms of the number of steps of the random walk. Parameters are to be defined for the entire field and boundaries. In this experiment only one homogeneous region is assumed. Hence, the parameters are region size, texton step size, directional probabilities and initial texton distribution. The maximum regions size is limited to 400 x 400 locations. The step size defines the distance, that texton elements move in each step of time. Varying the step size simulates varying the diffusion coefficient. At each instant of time, a texton element is at some location in the region. The disposition of the element at the next instant in time is determined by one of six mutually exclusive and collectively exhaustive possibilities. The element can move one step size to the upper left, upper right, lower left or lower right; stay in the same location or be eliminated. The probabilities of each are:

P[expired]=1/L
P[center]=(1-1/L)(1-p-q)
P[upper left]=0.5(1-1/L)q
P[upper right]=0.5(1-1/L)p (6)
P[lower left]=0.5(1-1/L)q
P[lower right]=0.5(1-1/L)p

Where L is the average lifetime of the element (average number of time steps to expiration). P is the conditional probability that the element moves to the right given that it has not expired; q is the conditional probability that the element moves to the left given that it has not expired. The six probabilities sum to unity. The random walk defined by assuming p-q = 0.5 is the simple, unbiased random walk. This is used in this paper. The initial distribution of texton elements in the grid can be distributed randomly following a uniform distribution or can have maximum regularity. The grid has vertical perimeter boundaries as there is only one region. An element that would have crossed a vertical perimeter boundary is placed on the boundary and then subject to the boundary conditions with directional probabilities to move to upper or lower right given in Eqn. (6). Other parameters that change the number of elements can be set. The number of elements is not changed in this implementation.

4. Experimental Results

Several examples of synthesized textures are presented in this Section. The left images are generated after 500 iterations. The right images are generated after 1000 iterations.

Fig. 5. Reaction type – inhibition of an inhibition
Initial condition: local advantage of G1

Fig. 6. Reaction type – Multiplicative help
Initial condition: Homogeneous,
except random fluctuations

The patter formation mechanism of lateral activation results in formation of stripes as shown in Figs. 5 and 6. If the pattern formation is initiated by random fluctuations, the orientation of the stripes is somewhat irregular. They consist of long narrow ridges. Perfect stripes are formed if some initial spatial cues are present. For example, if the pattern formation starts at one side of the field. Both types of stripes can have a different width if the strength of the autocatalysis or of the mutual help is different in both feedback loops. The equilibrium between the g_1 or g_2 cells would be shifted in favor of one or the other leading to a corresponding change in the number of high g_1 or high g_2 cells.

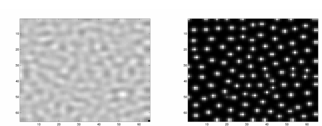


Fig. 7. Reaction type – Autocatalytic
and lateral inhibition
Initial condition: Homogeneous,
except random fluctuations

Figs. 5-7 shows how patterns evolve with time due to reaction and diffusion. Reaction involves presence of two or more chemical concentrations. Other patterns such as net like structure can be formed using RD systems. In the case of texton textures there is no reaction as there is only one type of element. Random walk examples of texture synthesis are shown below. Fig. 8 is generated with 100 steps with a step size of 2. Fig. 9 is generated with 200 steps and a step size of 1.

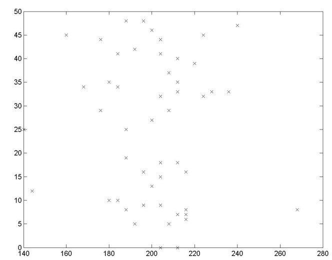


Fig. 8. Texture pattern in 100 steps

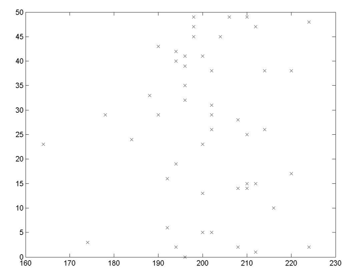


Fig. 9. Texture pattern in 200 steps

The number of steps can be increased to get a widely dispersed pattern. Also, the number of elements can be increased. The current number of elements is 50. Both patterns in Figs. 8 and 9 are irregular compared to the patterns in Figs. 3(a) and 3(b). The regularity of these patterns can be computed using the variogram. The initial distribution of elements is chosen as pulsative. Later experiments will start with a regular pattern with equally spaced elements and the randomness of the walk changes the regularity.

5. Conclusions

In this paper, synthesis of textures using RD systems was presented. These textures resemble patterns found in nature. A procedure to synthesize textures employing random walk diffusion method was presented for textures composed of texton elements. The textures have varying regularity depending on the randomness of the walk. A method was shown to quantify the regularity of the synthesized textures.

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