# Modeling and Simulation of Point Spread Functions for Advanced SAR Systems


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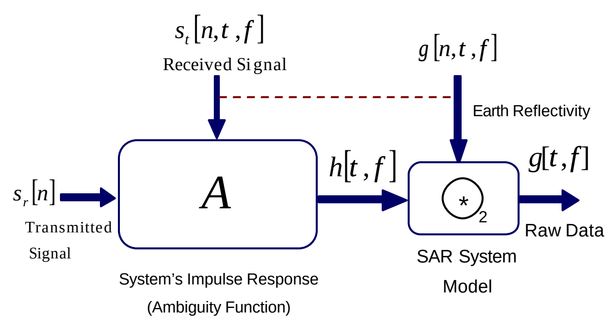
## Abstract

This work deals with discrete cross-ambiguity function modeling and simulation for the study of characteristics of SAR point spread functions (PSF) and associated spatial unit cell resolution, when subjected to active microwave radiation for prescribed scientific and engineering applications such as synthetic aperture radar (SAR) systems. A model for Point Spread Function (PSF) simulation and SAR Raw Data generation is presented. This model is used to analyzes the characteristics of PSF, also known as point target response functions, in a time-frequency context, through the cross-ambiguity function computation, in order to contribute to the on going work on spatial cell resolution enhancements for fine resolution image formation operations. Simulations have been performed on a Symmetric Multiprocessor System (SMP) using computational methods. Recent results are presented of large-scale algorithms computations. The overall terrain surface reflectivity is study through multidimensional weighted correlation operations.


## 1. Introduction

This work is concentrated on the modeling and simulation of imaging radar systems. We give special attention to synthetic aperture radar (SAR) systems as opposed to real aperture imaging radar systems (RAR). The differences between these two imaging radar systems reside in their spatial resolution capability. The spatial resolution of any imaging radar system can be defined as its ability to resolve smallest distance between two or more point targets that are sufficiently separated so as allow individual data measurements among them. This important feature of the imaging radar systems is characterized by its point spread function (PSF) or impulse response function.

One of the original contributions of this work is the modeling and simulation of ambiguity function as PSF of imaging radar systems. This work follows the theoretical formulations on imaging radar systems presented by R. Blahut on his work on remote surveillance algorithms [Blahut91]. According with these formulations, the expected output of an imaging radar system can be viewed as a two-dimensional convolution operation of the point reflectivity density function with the radar ambiguity function. For the particular case of SAR processing, we present a model for PSF simulations and SAR raw data generation (see Figure 1).




$$A\{s_t, s_r\}[t,f] = \left|\sum_{n=0}^{N-1} s_t[n]\, s_r^*[n-t]\, e^{-\frac{j2pfn}{N}}\right|$$

*N = Transmitted Signal Length*
*t = Time delay*
*f = Spectral Shift*

***Figure 1.*** SAR Raw Data Generation Model

In this work, modeling is defined as a set of mathematical structures and equations designed to correspond to a physical system or entity based on a set of prescribed assumptions [Rodríguez02]. A system in general is defined as a set of elements and its interrelationships. Simulation is defined here as the execution of a modeling system through computer programming [Rodriguez02]. Complex and large scale simulations require high performance computers.

## 2. Theoretical Formulations

As is established by R. Blahut in [Blahut91], an imaging radar system can be viewed as a device for forming a two-dimensional convolution of the reflectivity density function of an illuminated scene with the ambiguity function of the radar waveform. This filtering operation describes the output of the system preprocessor. The radar ambiguity function plays the role of the point spread function of the imaging radar system.

### 2.1. Discrete Ambiguity Function (DAF)

The discrete ambiguity function (DAF) defined in this work can be viewed as a generalized signal autocorrelation tool to simultaneously estimate time delay and Doppler frequency offset parameters between a transmitted signal and its returned echo. This function accepts the transmitted and received signal as input and generates a two-dimensional surface, one dimension being time and the other frequency. This simultaneous parameters determination allows for a time-frequency representation of this tool. These parameters can, in turn, be used to estimate, through basic algebraic transformations, range and cross-range (azimuth) parameters in a *spatial object domain*. The combined range and cross-range differentials or parameter increments determine a given unit resolution cell for a particular point in the spatial object domain. Fourier transform methods are normally used to study the properties of the associated *spatial spectral domain*.

### 2.2. DAF Derivation

Let $s_t(t)$ be the transmitted signal of a radar system given by: $s_t(t) = u(t)e^{-j2pf_c t}$, where $u(t)$ is some modulating signal. After a time $t_d$, the signal arrives at the receiver. If the target was moving with respect to the antenna, the received signal will have a frequency shift $f_d$ due to the Doppler effect. Then, a unit response can be modeled by the received signal as follows: $s_r(t) = u(t-t_d)e^{-j2p(f_c-f_d)(t-t_d)}$. We can rewrite $s_r(t)$ as:

$$s_r(t) = u(t-t_d)e^{-j2p f_c(t-t_d)}e^{j2p f_d(t-t_d)} \qquad (1)$$

After the demodulation process, the received signal becomes:

$$s_r(t) = u(t-t_d)e^{j2p f_d(t-t_d)} \qquad (2)$$

Once the transmitted signal is sampled, its discrete-time representation can be expressed as:

$$s_r[n] = u[n-n_d]e^{jw_d[n-n_d]} \qquad (3)$$

For a zero Doppler shift, we can obtain $n_d$ by performing a correlation between $s_r[n]$ and the discrete-time version of the original modulating signal $u[n]$. The index at which the magnitude of the correlation is a maximum is the discrete-time delay $n_d$. This correlation is performed as follows:

$$C[m] = \sum_{n=-\infty}^{\infty} u[n]s_r^*[n+m] \qquad (4)$$

We can rewrite the above expression as follows:

$$C[m] = \sum_{n=-\infty}^{\infty} u[n]u^*[n+m-n_d] \qquad (5)$$

From this equation it can be noticed that the maximum correlation will occur when $m = n_d$.

If a Doppler shift is present the correlation between $u[n]$ and $s_r[n]$ becomes:

$$C[m] = \sum_{n=-\infty}^{\infty} u[n]u[n+m-n_d]e^{-j2pf_d[n+m-n_d]} \qquad (6)$$

Since this new correlation results in a complex function, no guarantee can be made that the maximum magnitude will occur when $m = n_d$. To resolve this problem we generate a new function to try to cancel out the effect of the exponential. The new function is:

$$G[m,f] = \sum_{n=-\infty}^{\infty} u[n]u[n+m-n_d]e^{-j2pf_d[n+m-n_d]}e^{j2pfn} \qquad (7)$$

Where the notation $G[m,f]$ is used to indicate that f is a continuous variable. The first complex exponential can be separated into two exponentials and (7) can be rewritten as:

$$G[m,f] = e^{-j2pf_d[m-n_d]}\sum_{n=-\infty}^{\infty} u[n]u[n+m-n_d]e^{j2p(f_d-f)n} \qquad (8)$$

We can notice that when $f = f_d$, the magnitude of $G[m, f)$ is maximum at $m = n_d$. To determine the parameters $f_d$ and $n_d$, we only need to find the $m$ and $f$ that make the magnitude of $G[m, f)$ a maximum. For a length $N$ sequence $u[n]$, the function $G[m, f)$ then becomes:

$$G[m, f) = \sum_{n=0}^{N-1} s_t[n] s_r^*[n+m] e^{j2pfn} \qquad (9)$$

Since $f$ is a continuous variable, there is a problem in computing $G[m, f)$ and finding its maximum magnitude. To avoid this problem, the variable $f$ is approximated by $k / N$. Now the function becomes:

$$G[m, k] = \sum_{n=0}^{N-1} s_t[n] s_r^*[n+m] e^{j2pk/N} \qquad (10)$$

### 2.3. Discrete Ambiguity Function Definition

For a transmitted signal $s_t \in l^2(Z_N)$ and a received signal $s_r \in l^2(Z_N)$, where $Z_N = \{0,1,2...,N-1\}$ and $l^2(Z_N)$ is the Hilbert space of discrete complex signals of length $N$, the discrete finite cross-ambiguity function of $s_t$ and $s_r$ can be defined as the absolute value of the following expression:

$$A\{s_t, s_r\}[m,k] = \left| \sum_{n=0}^{N-1} \left( s_t[n] s_r^*[<n+m>_N] W_N^{kn} \right) \right| \qquad (11)$$

Where $<n+m>_N$ denotes the remainder of the quotient $\left(\frac{n+m}{N}\right)$ and $W_N = \exp\{-j2p/N\}$.

### 2.4. DFT Method for DAF Computation

This method allows us to compute the cross-ambiguity function directly by means of the discrete Fourier transform (DFT), as follows:

$$b_m[n] = s_r[<n+m>_N]; m \in Z_N \qquad (12)$$

$$v_m[n] = s_t[n] b_m[n]; \; n \in Z_N \qquad (13)$$

Finally, the DAF can be computed by means of multiple one-dimensional Fourier transforms:

$$A\{s_t, s_r\}[m,k] = \sum_{n=0}^{N-1} v_m[n] W_N^{kn} = V_m[k] \qquad (14)$$

Figure 2 shows the algorithm to implement the DFT method for the ambiguity function computation.

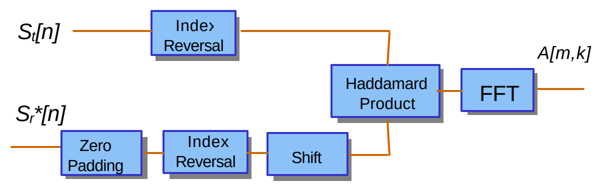



***Figure 2.*** DFT Method Algorithm for DAF Computation

## 3. Modeling and Simulation Results

To study the physical properties of a particular terrain we need a set of calibrated images of that terrain. These images are obtained from raw data through image formation algorithms (See ***Figure 3***). The attributes of the PSF of a SAR system determine, to a great extent, the better quality of the image formation process.

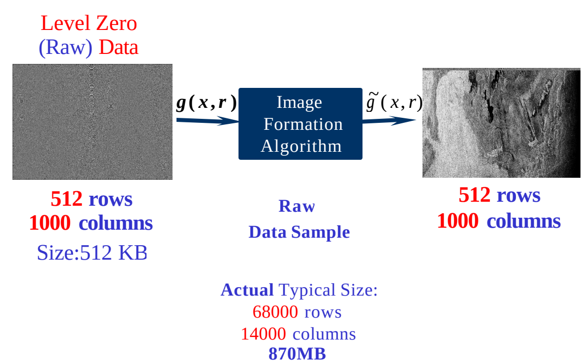



***Figure 3.*** SAR Image Formation Process

Large scale algorithm implementations for point spread functions computation were implemented with Fortran and C codes and tested for several signals lengths in a Symmetric Multiprocessor (SMP) architecture composed of four *UltraSPARC II*[TM] processors running at 400 MHz each, with 4 MB of local, high-speed external cache memory, running the Solaris v. 8 (UNIX) operating system. Serial implementations results are shown in Tables 1 and 2. Figure 4 shows run times of both implementations for comparison:

**Table 1.** Timing Measurements of PSF Implementation in Fortran

| Signal Length | Real Execution Time (sec) | User Time (sec) | System Time (sec) |
|---|---|---|---|
| 64 | 0.006 | 0.010 | 0.01 |
| 128 | 0.006 | 0.100 | 0.010 |
| 256 | 0.426 | 0.400 | 0.020 |
| 512 | 1.458 | 1.350 | 0.090 |
| 1024 | 5.906 | 5.410 | 0.420 |
| 2048 | 22.402 | 20.230 | 1.850 |
| 4096 | 92.220 | 82.870 | 7.650 |

**Table 2.** Timing Measurements of PSF Implementation in C language

| Signal Length | Real Execution Time (sec) | User Time (sec) | System Time (sec) |
|---|---|---|---|
| 64 | 0.052 | 0.040 | 0.01 |
| 128 | 0.134 | 0.110 | 0.010 |
| 256 | 0.397 | 0.340 | 0.020 |
| 512 | 1.354 | 1.190 | 0.090 |
| 1024 | 5.144 | 4.500 | 0.420 |
| 2048 | 20.993 | 16.720 | 1.850 |
| 4096 | 101.481 | 65.220 | 7.650 |

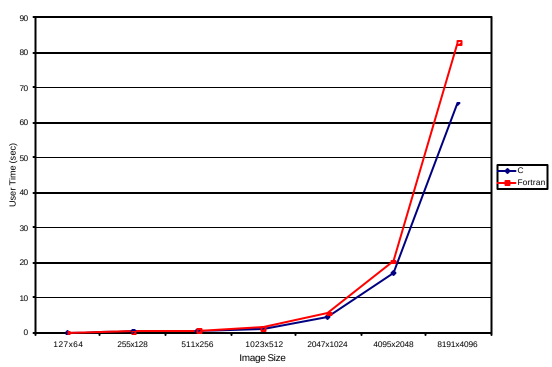



**Figure 4.** Running Times Comparison

Graphical representations of the results were obtained using Matlab. For a transmitted chirp (LFM) signal the ambiguity function surface is shown in Figure 5. We give special attention to the chirp signal and its fine resolution capability, giving rise to pulse compression techniques in a matched filter setting. For a transmitted rectangular pulse and cosine signals the ambiguity surface is depicted in Figure 6.

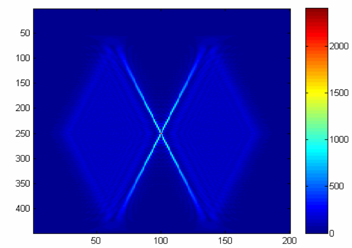


**Figure 5.** PSF of a Linear Frequency Modulated (LFM) Signal

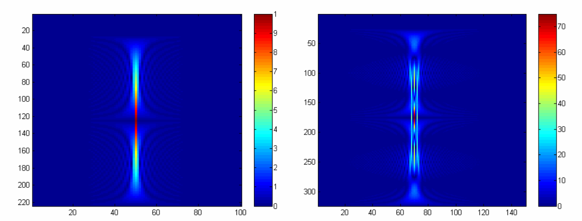


**Figure 6.** PSF of a Rectangular Pulse Signal (left) and Cosine signal (right)

## 3. Conclusion

A model for SAR raw data generation was presented, which we are using to study the characteristics of Point Spread Functions (PSF), in a time-frequency context, through the efficient computation of the discrete ambiguity function. A large scale algorithm for Point Spread Functions (PSF) computation using the DFT (Discrete Fourier Transform) method has been implemented in FORTRAN and C languages. Due to the nature of the large-scale computation involve in this type of modeling and simulation the PSF simulations have been performed in an advanced computer architecture based on symmetric multiprocessor system. Profiling and performance tuning tools were use to enhance the processing effort. Our continuing work will concentrate on developing more efficient algorithms for cross-ambiguity function and parallel implementations.

## References

[Rodríguez02] D. Rodríguez, "Computational Signal Processing and Sensor Array Signal Algebra. A Representation Development Approach", first book draft, Sept. 2002.

[Blahut91] R. E. Blahut, "Theory of Remote Surveillance Algorithms," Radar and Sonar, Part I, Springer-Verlag, (New York, 1991).