

Magnetic Resonance Imaging Heat Deposition In Non-uniform Ellipsoidal Objects

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Abstract

In this work we compute the Specific Absorption Rate (SAR) of radio frequency fields in ellipsoidal geometries representing regions of the human head under Magnetic Resonance Imaging (MRI) studies. The E & M fields were calculated by the T-Matrix algorithm. The SAR presented here shows strong conductivity and resonance effects.

I. Introduction

The T-Matrix or Extended Boundary Condition theory was developed in 1969 by P.C. Waterman¹⁻² for the description of acoustic and electromagnetic scattering from a single homogeneous scatterer. Peterson and Ström³ extended the range of applicability of the method to scatterers consisting of a collection of homogenous layers each of which has constant electric and magnetic properties.

Although this theory proposes a method to solve the problem of electromagnetic scattering of a general monochromatic wave by objects of arbitrary shape, applications have been limited to the scattering of single ellipsoids or clusters. This is due to the fact that in actual implementations of the algorithm as computer code, the evaluation of general surface differentials becomes prohibitively time consuming.

T-Matrix expresses the fields as multipolar vector expansions. It fully takes into account phase-retardation effects, which are critical in the cases where the size of the scatterers is of the order of the expressed of the incident field.

Each of those fields is expanded in terms of a vector spherical basis which, in turn, satisfies the Helmholtz Equation.wavelength in the medium. The scattered and internal fields are in terms

$$\nabla \times \nabla \times \mathbf{E} - k^2 \mathbf{E} = 0 \quad (1)$$

The surface currents at the interfaces of the scatterers are used to link the expansion coefficients of the various fields by means of non-homogeneous, linear equations. In particular, internal field coefficients are expressed in terms of linear combinations of incident and scattered field coefficients. That linear combination contains information of surface topology, electromagnetical properties and the basis functions.

The surface topology enters through an integral of surface differentials. It is in this calculation where T-Matrix has the most stringent limitations. On one hand, numerical evaluation of surface differentials is computationally expensive. In addition, analytical expressions of surface differentials, which speed up the computer code, can only be accomplished in the most simple situations.^{2,4,5}

In this work, we have added the possibility to deal with non-homogenous ellipsoidal objects made up of non-concentric layers. In that configuration (Fig I), an inner object is centered at the origin. An outer object, completely covering the inner one is located such that its center lies along the z axis.

Our interest is to represent a non-homogeneous organ to visualize the radio frequency electromagnetic field (EMRF) profiles, specific absorption rates (SAR) for configurations relevant to magnetic resonance imaging (MRI) as used in clinical settings.

With this T-Matrix code we obtain the EMRF and SAR for a variety of shapes and compositions at various frequencies.

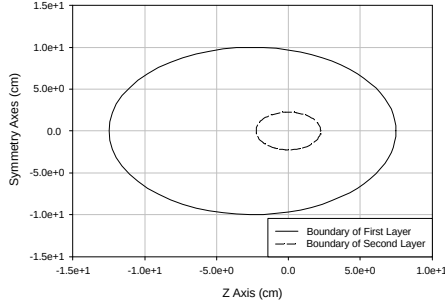


Figure I

Knowledge of EMRF is critical to reliably assess image reconstruction algorithms. SAR, on the other hand, is proportional to the power deposited in the tissue, and is important to gauge the safety of the radiation.

In what follows we demonstrate the portability of the code to different computer architectures and operating systems, and benchmark the program by logging the various execution times.

II. Theoretical Background

T-Matrix considers three kinds of fields: incident, scattered, and internal. In our specific application the expansion of the scattered field is in spherical Hankel functions of the first kind (non-regular). The incident and the inner layer internal field, are expressed in terms of spherical Bessel functions of the first kind (regular). The internal field in the region inside the outer layer (but outside the inner) is a combination of the regular and non-regular solutions. The expansion coefficients are stored in the so called Q-Matrices².

We generalize the T-Matrix code beyond relating incident and scattered waves fields allowing for the possibility to evaluate internal fields. This turns out to be important for our applications in MRI since the image is strongly dependent on the internal fields. In the past this issue was somewhat less important since in the problem of scattering of light by nanoparticles, the knowledge of the internal fields was more a matter of curiosity than a relevant issue for applications.

The main aim of our program is to obtain

the expansion coefficients of the fields in all the regions in terms of the coefficients of the incident field.

At the heart of the computer code are the Q-Matrices, which contain geometrical and electrical information as well as frequency. The speed of the program depends on the dimensions of the Q-Matrices (some of which need to be inverted). These dimensions are a quadratic function of the multipolar order, as shown in Table I.

Table I

Order	Dimension
1	6
2	16
3	30
4	48
5	70
6	96
7	126
8	160
9	198
10	240
11	286
12	336
13	390

Our program uses four matrices in addition to the four Q-Matrices. The former represents the smaller set needed to store linear combinations of the Q-Matrices and their inverses, which are needed to relate any set of field coefficients to the incident one.

At the beginning of the program and its subroutines all the arrays are defined, and space is allocated in random access memory. In the poorer computer architectures, convergence is achieved only after more than eleven hours. Therefore it is of utmost importance to optimize computer resources. One way we do that is by saving the coefficients in the diagonal of the intermediate matrices.

The program also needs two kinds of arrays of dimension $3 \times$ multipolar order which contain the expansion coefficients in terms of the basis set, of the incident, scattered, and internal fields.

In addition to the possibility to calculate internal fields in a two-layer scatterer, we also contribute a new application that allows the code

to deal with more general geometries. We changed the integration subroutine, so that it can handle scattering objects with ellipsoidal outer boundaries. In particular, we improved the code by calculating the surface differentials referred to a generic point, not necessarily the center of the ellipsoid. In doing that, we had in mind the need to tackle problems in which the internal layers were not concentric with the center surface.

III. Results of simulation

In this section we present results along a direction perpendicular to the symmetry axis. The inset shows the lower and upper limit of the coordinates of the small ellipsoid. We can see that SAR dependence on the small ellipsoid is only in the neighborhood of it. Heat deposition for small ellipsoid is independent of the characteristics of the small object. The reduction of SAR inside the large ellipsoid is due to conductivity damping of the fields. The enhancement of the signal at 700MHz at the outer of the small ellipsoid is due to cavity resonance. All the figures were calculated for $\epsilon_1 = 46.25$, $\sigma_1 = 2$ S/m, $\epsilon_2 = 46.25$, $\sigma_2 = 1$ S/m.

Figure II shows SAR along a direction perpendicular to the symmetry axis at 20MHz. Most of the heat deposition is in the incoming region of the E & M field. As the field moves into the scatterer, conductivity effects reduce its magnitude.

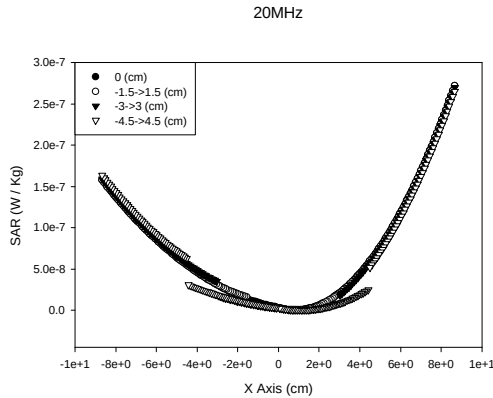


Figure II

Figure III shows SAR along a direction perpendicular to the symmetry axis at 64MHz. Most of the heat deposition is in the incoming region of the E & M field. As the field moves into the scatterer, conductivity effects reduce its magnitude.

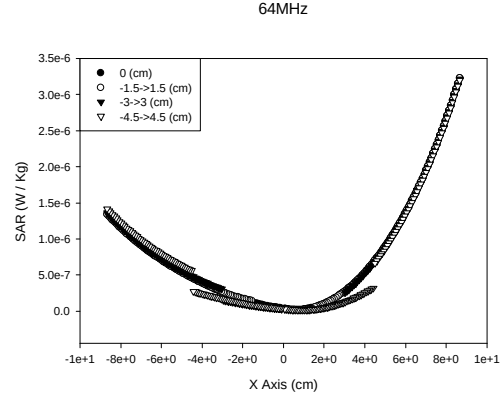


Figure III

Figure IV shows SAR along a direction perpendicular to the symmetry axis at 170MHz. Most of the heat deposition is in the incoming region of the E & M field. As the field moves into the scatterer, conductivity effects reduce its magnitude.

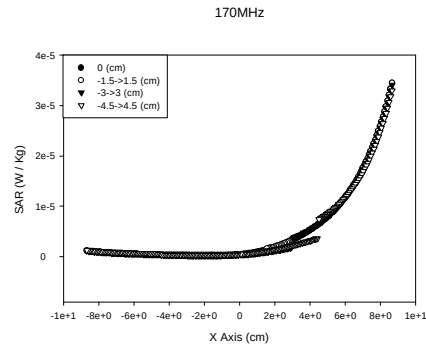


Figure IV

Figure V shows SAR along a direction perpendicular to the symmetry axis at 700MHz. Most of the heat deposition is in the incoming region of the E & M field. As the field moves into the scatterer, conductivity effects reduce its magnitude. One can see that SAR is a quadratic function of the frequency.

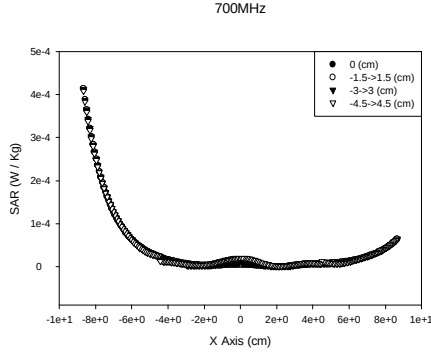


Figure V

IV. Conclusions

We have described improvements to the T-Matrix computer code. These improvements allow for the evaluation of electromagnetic fields scattered by objects made up of elliptical non-concentric layers. In addition, the code now can calculate internal fields. This is a new important improvement, relevant for situation in which the size of the object allows for internal measurements. Finally we have extended the traditional use of T-Matrix in the nanometers wavelengths to the meter size scales. We found the method useful in MRI applications, in particular in the detection of soft tissue inside the human body.

V. Future Work

We are setting up more realistic configurations of the human body to make full use of the versatility of this enhanced T-Matrix code. Concretely we are aiming to study the fields with arbitrary and irregular shapes like brain, lung, kidney and other.

VI. Acknowledgment

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